Math 1452 Final Exam Fall 2013

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.

1. Consider the region bounded by $y = x^2$, $y = \sqrt{8x}$. **Set up** (but do not solve) integrals to find
   (a) The area of this region.
   (b) The volume of the solid generated by rotating this region about the y-axis using both shells and washers.
   (c) The volume of the solid generated by rotating this region about the horizontal line $y = 5$ using any method.
   (d) The moment about the x-axis and the moment about the y-axis of this region.

2. **Set up** (but do not solve) an integral to find the arc length of $y = \sqrt{8x}$, for $0 \leq x \leq 2$.

3. Graph the cardioid $r = 3(1 - \cos(\theta))$ and **set up** an integral to find the area it encloses.

4. Evaluate the following integrals.

   (a) $\displaystyle\int x^2 \sin(3x)\, dx$ (b) $\displaystyle\int \frac{2x}{(x-3)^2}\, dx$

   (c) $\displaystyle\int \frac{\csc(\ln(3x))}{2x}\, dx$ (d) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\, dx$

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{\sqrt{k}-3}{k^2}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(2k)!}{e^k}$

   (c) $\displaystyle\sum_{k=2}^{\infty} \frac{\sin(3k)}{k!}$ (d) $\displaystyle\sum_{k=2}^{\infty} \frac{2}{k} - \frac{2}{k+1}$

6. Find the interval and radius of convergence of the power series $\displaystyle\sum_{k=3}^{\infty} \frac{2}{3^k\, k}(x-4)^k$.

7. Find the first 3 terms of the Maclaurin series for $f(x) = \sqrt{x+3}$.

8. If $\mathbf{u} =< 1, 0, -2 >$ and $\mathbf{v} =< 2, -3, 0 >$, find
   (a) $\mathbf{u} - 3\mathbf{v}$
   (b) The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$
   (c) The area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.