

Math 1452 Final Exam Spring 2013

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class. Copyright 2013 Dept of Mathematics and Statistics, Texas Tech University. Unauthorized reproduction prohibited.

1. Consider the region bounded by $y = x^2$, $x = 2$, and the x -axis. **Set up** (but do not solve) integrals to find
 - (a) The volume of the solid generated by rotating this region about the y -axis using shells.
 - (b) The volume of the solid generated by rotating this region about the horizontal line $y = -3$ using any method.
 - (c) The moment about the x -axis of this region.

2. Graph the rose $r = 5 \cos(2\theta)$ and find the area of one leaf.

3. Evaluate the following integrals.

(a) $\int \sin^3(x) dx$

(b) $\int x \frac{e^x + e^{-x}}{2} dx$

(c) $\int \frac{\sin(3x)}{1 + \cos^2(3x)} dx$

(d) $\int \frac{5x - 5}{(x - 2)(x + 3)} dx$

4. Evaluate $\int_3^{\infty} x e^{-x^2} dx$.

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

(a) $\sum_{k=1}^{\infty} \frac{3 + \ln k}{k}$

(b) $\sum_{k=1}^{\infty} \frac{3^k}{k 2^k}$

(c) $\sum_{k=2}^{\infty} \frac{2k - 3}{k^3 + 2}$

(d) $\sum_{k=0}^{\infty} \frac{3^k}{k!}$

6. Does the series $\sum_{k=1}^{\infty} \frac{(-2)^k}{3^k}$ converge? If it converges, find the sum. If not, explain why not.

7. Find the first 3 terms of the Taylor series for $f(x) = x \ln x$ about $x = 3$.

8. If $\mathbf{u} = \langle 0, 2, -1 \rangle$ and $\mathbf{v} = \langle 3, 1, 0 \rangle$, find

(a) $\|\mathbf{u} - 2\mathbf{v}\|$

(b) $\mathbf{u} \times \mathbf{v}$

- (c) The cosine of the angle between \mathbf{u} and \mathbf{v}