Math 1452 Final Exam Fall 2014

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.

1. Consider the region bounded by $y = 2x$, $y = 1$, and the y-axis. **Set up** (but do not solve) integrals to find
   (a) The volume of the solid generated by rotating this region about the horizontal line $y = -3$ using both shells and washers.
   (b) The volume of a solid whose base is this region and has the property that each cross section perpendicular to the x-axis is a square.

2. Graph the rose $r = 3\sin(2\theta)$ and **set up** an integral to find the area enclosed by one leaf.

3. A particle moves with speed $f(t) = 3t^2$. Explain in complete detail why the formula ***Distance = Speed * Time*** is inadequate to finding the distance traveled by the particle from time $t = 0$ to $t = 4$ and why an integral can be used to find this distance. **Set up** such an integral.

4. Evaluate the following integrals.
   (a) $\displaystyle\int \frac{1}{x\sqrt{\ln x}}\,dx$
   (b) $\displaystyle\int e^{3x}\cos(2x)\,dx$
   (c) $\displaystyle\int \sin^3(x)\,dx$
   (d) $\displaystyle\int \frac{6x-4}{x(x+4)}\,dx$

5. Evaluate $\displaystyle\int_2^3 \frac{5}{x-2}\,dx$.

6. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.
   (a) $\displaystyle\sum_{k=2}^{\infty} k^{1/k}$
   (b) $\displaystyle\sum_{k=3}^{\infty} \frac{\ln k}{\sqrt{k}}$
   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{(2k)!}$
   (d) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k^2+1}$

7. Explain completely the difference between absolute and conditional convergence.

8. If $a_k > 0$ and $\dfrac{a_{k+1}}{a_k} = \dfrac{3}{k}$ for all $k > 1$, does $\displaystyle\sum_{k=1}^{\infty} a_k$ converge? Why or why not?

9. Find the first 3 terms of the Taylor series for $f(x) = x\ln(x)$ centered at 2.

10. Let $\mathbf{u} =< 1, 4, -3 >$ and $\mathbf{v} =< 2, 0, -1 >$.
    (a) Find a unit vector that points in the same direction as $\mathbf{u} - \mathbf{2v}$.
    (b) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal? Explain.