Math 1452 Final Exam Spring 2014

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.
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1. Consider the region bounded by $y = x^2$ and $y = 2x$. **Set up** (but do not evaluate) integrals to find
   (a) The volume of the solid generated by rotating this region about the vertical line $x = -3$ using washers
   (b) The volume of the solid generated by rotating this region about the vertical line $x = -3$ using shells
   (c) The moment about the x-axis of this region

2. **Set up** an integral to find the arc length of the curve $y = -x^2 + 2x + 3$ from $x = 0$ to $x = 3$.

3. Graph the polar curves $r = 2\sin(\theta)$ and $r = 2\cos(\theta)$. **Set up** integrals to find the area bounded by these two curves.

4. Evaluate the following integrals.
   (a) $\int x\ln(x)\,dx$
   (b) $\int \frac{\csc^2(3x)}{\cot(3x)}\,dx$
   (c) $\int \frac{1}{\sqrt{x^2-4}}\,dx$
   (d) $\int \frac{2x-5}{(x+3)(x+4)}\,dx$

5. Is $\int_0^1 \frac{3}{x+2}\,dx$ an improper integral? Explain completely.

6. Does the sequence $\left\{\sqrt[k]{k}\right\}$ converge? If it converges, find the limit. If not, explain why.

7. Answer the following, giving brief mathematically sound reasons for your answers.
   (a) Suppose the power series $\sum_{k=0}^{\infty} c_k(x-5)^k$ converges at $x = 8$. Must the series converge at $x = 4$?
   (b) Suppose $a_k > 0$ for all k and the sequence $\left\{\frac{a_k}{\frac{1}{k^2}}\right\}$ converges to 3. Must the series $\sum_{k=1}^{\infty} a_k$ converge?

8. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

(a) $\sum_{k=1}^{\infty} \frac{k^k}{3^k}$

(b) $\sum_{k=1}^{\infty} \frac{(\ln(k))^2}{k}$

(c) $\sum_{k=2}^{\infty} \frac{(-1)^k}{2k}$

(d) $\sum_{k=0}^{\infty} \frac{k}{k^3+5}$

9. Find the interval and radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{2}{k!}(x-3)^k$.

10. Find the Maclaurin series for $x^3\cos(5x)$.

11. If $\mathbf{u} =< 1, 0, -3 >$ and $\mathbf{v} =< 0, -2, 1 >$, find
    (a) $\mathbf{u} - 3\mathbf{v}$
    (b) The angle between $\mathbf{u}$ and $\mathbf{v}$
    (c) A unit vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$