Math 1452 Final Exam Spring 2015

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.

1. Consider the region bounded by $y = \sqrt{x}$ and $y = \frac{1}{2}x$. **Set up** (but do not solve) integrals to find
   (a) The area of this region.
   (b) The volume of the solid generated by rotating this region about the vertical line $x = 5$ using both shells and washers.
   (c) The center of mass of this region, assuming the density is 1.

2. Express the point $(-1, \sqrt{3})$ in polar coordinates with $0 \le \theta < 2\pi$.

3. Consider a cylindrical tank with radius 3 ft and height 10 ft. If it is filled with a fluid with weight density 94 lb/ft^3, **set up** an integral to find the work required to pump all the fluid out of the top of the tank.

4. Evaluate the following integrals.
   (a) $\displaystyle\int \frac{x-1}{x(x^2+1)}\,dx$ (b) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$
   (c) $\displaystyle\int \cos(x)\ \cot(\sin x)\,dx$ (d) $\displaystyle\int x(e^x + e^{-x})\,dx$

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.
   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{1+\ln k}{k}$ (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k}{k!}$
   (c) $\displaystyle\sum_{k=2}^{\infty} \frac{k!}{(k-1)^3}$ (d) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k} - \frac{2}{k+1}$

6. Does the series $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{5^k}$ converge? If so, find the sum. If not, explain why not.
   Does the sequence $\left\{\dfrac{2^k}{5^k}\right\}$ converge? If so, find the limit. If not, explain why not.

7. Suppose $a_k, b_k > 0$, $\displaystyle\lim_{k\to\infty} \frac{a_k}{b_k} = 0$, and $\sum_{k=1}^{\infty} b_k$ converges. Does $\displaystyle\sum_{k=1}^{\infty} a_k$ converge? Why or why not?

8. Find the radius of convergence of the power series $\displaystyle\sum_{k=3}^{\infty} \frac{3}{k\,2^k}\,(x-5)^k$.

9. Find the first 3 terms of the Taylor series for $f(x) = \sqrt{x}$ centered at 9.

10. Let $\mathbf{u} = < -2, 0, 1 >$ and $\mathbf{v} = < 0, 2, -3 >$.
    (a) Find the number a so that $< 1, 1, a >$ is orthogonal to $\mathbf{u}$.
    (b) Find $||\mathbf{u} \times \mathbf{v}||$.