

Math 1452 Final Exam Fall 2016

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class. Copyright 2016 Dept of Mathematics and Statistics, Texas Tech University. Unauthorized reproduction prohibited.

1. Consider the region bounded by $y = x^2$, $y = 2x$, and the y -axis. **Set up** (but do not solve) integrals to find
 - (a) The area of this region.
 - (b) The volume of the solid generated by rotating this region about the line $x = -3$ using washers.
2. **Set up** an integral to find the arc length of $y = e^{3x}$, for $0 \leq x \leq 2$.
3. Graph $r = \sin(\theta)$ and $r = \cos(\theta)$. **Set up** integrals to find the area enclosed by the graphs.
4. Evaluate the following integrals.

(a) $\int \frac{x^2}{\sqrt{9-x^2}} dx$

(b) $\int \frac{x-3}{x(x+1)^2} dx$

(c) $\int x^2 \cos(3x) dx$

(d) $\int \frac{\sin(x)}{1+\cos^2(x)} dx$

5. Consider the following calculation. Identify and correct any mistakes.

$$\int_{-1}^1 \frac{1}{x^2} dx = \ln(x^2)|_{-1}^1 = \ln(1) - \ln(1) = 0$$

6. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

(a) $\sum_{k=1}^{\infty} 2^{1/k}$

(b) $\sum_{k=3}^{\infty} \frac{k + \ln k}{\sqrt{k}}$

(c) $\sum_{k=0}^{\infty} \frac{2^k}{k!}$

(d) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

7. Does the series $\sum_{k=1}^{\infty} \frac{2^k}{3^k}$ converge? If so, find the sum. If not, explain why not.

Does the sequence $\left\{ \frac{2^k}{3^k} \right\}$ converge? If so, find the limit. If not, explain why not.

8. Find the first 3 terms of the Taylor series for $f(x) = x \ln(x)$ centered at 4.
9. Let $\mathbf{u} = \langle 1, 4, 3 \rangle$ and $\mathbf{v} = \langle 2, 0, -1 \rangle$.
 - (a) Find a unit vector that points in the same direction as $\mathbf{u} - 3\mathbf{v}$.
 - (b) Are \mathbf{u} and \mathbf{v} orthogonal? Explain.
 - (c) Find $\mathbf{u} \times \mathbf{v}$.