Math 1452 Final Exam Spring 2016

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.

1. Answer the following (give short mathematical reasons):
   (a) Is $\int_0^{\pi} \tan x \, dx$ an improper integral? Do not try to compute this integral.
   (b) Convert $(\pi, -\pi)$ to Polar form (r, θ).
   (c) It is known that the power series $\sum c_n (x-3)^n$ converges for $x = 8$ and diverges for $x = -5$. Does the series converge for $x = -1$?
   (d) Is it possible for two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ to add up to the zero vector, but have $\mathbf{v} \times \mathbf{w} \neq 0$?

2. Consider the curves $r = \sin 2\theta$ and $r = \cos\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Solve **all** of the following problems.
   (i) *Sketch* the curves. Your sketch does not have to be exact.
   (ii) Find the points of intersection between the two curves in polar form for $0 \leq \theta \leq \frac{\pi}{2}$.
   (iii) *Set up the integral* to find the area of the region bounded between the two curves.

3. A typical olympic-size swimming pool is 50m long, 25m wide, and 3m deep. The pool is filled to 0.5m from the top. Find the expression for the work (in Joules) required to pump all of the water over the side. The density of water = 1000 Kg/m^3, and acceleration due to gravity = 9.8 m/s^2.

4. Evaluate the following integrals.
   (a) $\displaystyle\int e^{-\cos^2(x)} \sin(2x)\, dx$ (b) $\displaystyle\int \arctan(x)\, dx$ (c) $\displaystyle\int \sec^3(x)\tan(x)\, dx$
   (d) $\displaystyle\int \frac{1}{\sqrt{x^2 - 6x + 10}}\, dx$ (e) $\displaystyle\int \frac{x-1}{x\,(x+1)^2}\, dx$

5. Find the limit of the sequence if it exists: $\displaystyle\left(\frac{\arctan(k)}{k+1}\right)$

6. Indicate whether the following series converge or diverge. *Note that in case of alternating series, absolute convergence must be checked first.* You must identify all the tests you use.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(k)}{k^2+1}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k!\,(2k)!}{(3k)!}$
   (c) $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^{-k}$. (d) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{\ln(k)}{k}$.

7. Find the interval of convergence for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{2^k (x-1)^k}{k+1}$.

8. Find the first four terms of the Taylor series for $f(x) = \sin(2\pi x)$ about the point $c = 1$.

9. Given $\mathbf{u} = \langle 2, -1, 1\rangle$ and $\mathbf{v} = \langle 1, 1, 3\rangle$.
   (i) Find $\frac{\mathbf{u}\cdot\mathbf{v}}{\|u\|\,\|v\|}$.
   (ii) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (iii) Find a unit vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
   (iv) If $\mathbf{w} = \langle 1, 1, 1\rangle$, find $\mathbf{w} \times (\mathbf{u} \times \mathbf{v})$ and show that it is equal to $(\mathbf{w}\cdot\mathbf{v})\,\mathbf{u} - (\mathbf{w}\cdot\mathbf{u})\,\mathbf{v}$.