Math 1452 Final Exam Spring 2017

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.

1. Consider the region bounded by the graphs of the functions $y = \sqrt{x}$ and $y = \frac{1}{2}x$.
   Set up (but do not evaluate) integrals to find
   (a) The area of this region.
   (b) The volume of the solid generated by rotating this region about the horizontal line $y = 5$ using washers.
   (c) The volume of the solid generated by rotating this region about the horizontal line $y = -3$ using shells.
2. **Set up** (but do not evaluate) an integral to find the arc length of the graph of $y = \sqrt{x},\ 0 \leq x \leq 4$.

3. **Set up** (but do not evaluate) an integral to find the work done in pumping the water out of a cylindrical tank of height 10 ft and radius 3 ft. Recall that water weighs 62.4 lb/ft^3.

4. Evaluate the following integrals.

   (a) $\displaystyle\int \sqrt{4 - x^2}\, dx$ (b) $\displaystyle\int \frac{4x - 1}{(x-1)(x^2+4)}\, dx$

   (c) $\displaystyle\int e^x \cos(3x)\, dx$ (d) $\displaystyle\int_0^\infty e^{-3x}\, dx$

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k!}$ (b) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k}$

   (c) $\displaystyle\sum_{k=0}^{\infty} \frac{2\sqrt{k}}{k+5}$ (d) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{1}{k+3} - \frac{1}{k+2}\right)$

6. If $a_k > 0$ and $\displaystyle\lim_{k\to\infty} \frac{a_k}{\frac{1}{\sqrt{k}}} = \frac{1}{2}$, does $\displaystyle\sum_{k=1}^{\infty} a_k$ converge? Why or why not?

7. Find the radius and interval of convergence of the power series $\displaystyle\sum_{k=0}^{\infty} \frac{4}{3^k}\,(x-5)^k$.

8. Find the McLaurin series for $f(x) = x^3 \cos(4x)$.

9. Let $\mathbf{u} =< 0, 2, -3 >$ and $\mathbf{v} =< -1, 0, 4 >$.
   (a) Find $\mathbf{u} + \mathbf{2v}$.
   (b) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (c) Find $||\mathbf{u} \times \mathbf{v}||$.