MATH 1550 – Departmental Final Exam – Spring 2011

- Show all the details leading to the solution of each problem, and show all your work. Use enough significant decimal digits so that you get a precise and accurate answer. Use the appropriate notation.
- All problems are weighted equally.
- Write all your answers in the blue book. Only the answers in the blue book will be graded. Turn in your exam with your blue book.
- You are only allowed to have a scientific calculator (NOT a graphic calculator). No cellphones are allowed. No sharing of calculators is allowed.

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1. Prove that $\cos^4 x - \sin^4 x = \cos(2x)$.

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2. Find all solutions in the interval $[0, 2\pi)$ of the equation $\sin x - \sin(2x) = 0$.

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3. Let $\mathbf{a} = \langle 3, 5\rangle$, $\mathbf{b} = \langle 2, 1\rangle$, and $\mathbf{c} = \langle 0, 3\rangle$. Compute the following:
(i) $\mathbf{a} - \mathbf{b}$; (ii) $|\mathbf{b}|$; (iii) $3\mathbf{c} + 2\mathbf{a}$.

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4. In an isosceles triangle, the two base angles are each 35 degrees and the length of the base is 120cm. Find the area of the triangle.

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5. Convert the following polar equation into a rectangular form:

$$r = 2 - 2\cos\theta.$$

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6. Assuming $a > 5$, simplify the expression

$$\left[\ln(a^2 - 25) - \ln(a + 5)\right] \log_{(a-5)}(a + 5).$$

(Note: For $\log_{(a-5)}$ the quantity $a - 5$ is the base of the logarithm.)

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7. A certain type of bacterium, given a favorable growth medium, doubles its population every 6.5 hours. Given that there were 100 bacteria to start with, how many bacteria will there be in a day and half?

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8. A rectangle is inscribed in a circle with radius 12cm. Express the perimeter of the rectangle as a function of its width w.

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9. Given $\cos\theta = \frac{5}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin\theta$ and $\tan\theta$.

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10. Consider the function

$$y = 3\sin\left(\frac{1}{2}x + \frac{\pi}{6}\right).$$

Determine the amplitude, period and phase shift of the above function.

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11. A circle has radius 3m. A sector of the circle has an angle of $135°$ between two radii. Find the perimeter and the area of the sector.

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12. Find the product and the sum of the roots of the following quadratic equation:

$$(x-5)(x+3) = 1.$$

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13. Let $z = 6 + 8i$ and $w = 1 - 2i$. Find the quotient $\frac{z}{w}$.

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14. Determine **all** real number-solutions for the following equation:

$$\sqrt{x-5} - \sqrt{x+4} + 1 = 0.$$

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15. Use factoring and key numbers (critical points) to solve the following inequality:

$$\frac{x^2 - x - 2}{x^2 - 3x + 2} > 0.$$

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16. Let $f(x) = \frac{x+2}{x-3}$.

(a) Find the domain and range of the function f.

(b) Find the inverse function $f^{-1}(x)$.

(c) Find the domain and the range of the inverse function f^{-1}.

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17. Graph the function $y = -|4 - x| + 1$.

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18. Assume $F(x) = \frac{3x-4}{3x+3}$ and $G(x) = \frac{x+1}{x-1}$. Compute $(G \circ F)(2)$.

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19. Determine the constants a and b, given that the parabola $y = ax^2 + bx + 3$ passes through $(-3, 36)$ and $(4, 1)$.

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20. From the point $(5, -1)$, tangent lines are drawn to the circle $(x-4)^2 + (y-3)^2 = 16$. Find the slopes of these lines. (*Hint*: Find the distance of the center of the circle to each tangent line; this distance must equal the radius of the circle. Otherwise, use any method your teacher taught you!)

Formulas for the final exam of MATH 1550 – Spring 2001

This is the set of formulas that would be provided to each student for the final exam of MATH 1550. The student is responsible for memorizing any other formula from the textbook (see, for example, the inside covers of the textbook, even though there are many more formulas in the various chapters of the book).

Compound Interest Formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$
Continuous Compound Interest Formula: $A = Pe^{rt}$

Product to Sum Formulas:
(a) $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(b) $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$
(c) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

Sum to Product Formulas:
(a) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
(b) $\sin A - \sin \mathrm{B} = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
(c) $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
(d) $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Formulas for logarithms:
(a) $\log_b PQ = \log_b P + \log_b Q$
(b) $\log_b \frac{P}{Q} = \log_b P - \log_b Q$
(c) $\log_b P^n = n \log_b P$
(d) $\log_a x = \frac{\log_b x}{\log_b a}$

Formulas related to radian measure:
(a) $s = r\theta$
(b) $A = \frac{1}{2}r^2\theta$

Some more trigonometric formulas:
(a) $\cos(A+B) = \cos A \cos B - \sin A \sin B$
(b) $\sin(A+B) = \sin A \cos B + \cos A \sin B$
(c) $\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$
(d) $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$
(e) $\tan \frac{x}{2} = \frac{\sin x}{1+\cos x}$

Formulas for the area of a triangle:
(a) Area $= \frac{1}{2}ab\sin C$
(b) Area $= \frac{1}{2}(\text{base} \times \text{height})$

Formulas for a triangle:
(a) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
(b) $c^2 = a^2 + b^2 - 2ab\cos C$

Double-angle formulas:
(a) $\sin(2\theta) = 2\sin\theta\cos\theta$
(b) $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
(c) $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Distance of a point (x_0, y_0) to the line $Ax + By + C = 0$:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$
