

Show all of your work in your blue book, or you will not receive credit. Work the problems in order. Use proper notation and be organized in your work. Box in your final solutions. State and write out which formulas you use for a given problem. You may use a non-graphing, non-solving scientific calculator. A cell phone or other electronic device may not serve as a calculator for this exam. Each of the following problems have equal value.

1. Express the following equation of a circle in standard form. State the circle's center and radius exactly.

$$4x^2 + 4y^2 + 16x + 24y = 0$$

2. Solve the following rational inequality for  $x$  exactly. State answer in interval notation and graph the interval on a number line.

$$\frac{x + 3}{x^2 - 3x + 3} < 1$$

3. Find the inverse of the following linear function algebraically. Graph the function and its inverse on the same graph, clearly labeling both.

$$f(x) = 1 - \frac{x}{2}$$

4. Given  $f(x) = x^2 + 7$  and  $g(x) = 3x - 2$ , state the following functions. Fully simplify your answers.

(a)  $(f + g)(x)$

(b)  $(fg)(x)$

(c)  $(g \circ f)(x)$

(d)  $(g \circ f)(2)$

5. Suppose a ball is launched from the ground such that its trajectory follows the path  $y = -x^2 + 28x$ . In this problem  $y$  denotes the vertical displacement between the ball and the ground, and  $x$  denotes the horizontal displacement between the firing location and the ball. Assume both displacements are measured in meters.

(a) Find the maximum height the ball attains on this trajectory.

(b) State the horizontal distance the ball travels over the course of the flight. Assume that the ground is flat.

6. Graph the rational function  $f(x) = \frac{x-4}{x+2}$ . Clearly label the graph and state the following:

(a) domain of  $f(x)$

(b) range of  $f(x)$

(c) x-intercept(s) of  $f(x)$

(d) y-intercept of  $f(x)$

(e) asymptotes of  $f(x)$

7. Solve the following for  $x$ :

(a)  $\ln(x - 3) + \ln(x - 2) = \ln(2x + 24)$

(b)  $e^{x^2} = 25$

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8. The element Bohrium has a half-life of 1.5 hours. Suppose that a 500mg sample of this element is left to decay.

- (a) State the function that gives the amount of Bohrium in mg present at the time  $t$ , where  $t$  is in hours.
- (b) How much of the sample will remain 5.5 hours from now? Approximate to 4 decimal places.
- (c) At what point in time will the sample decay to 15% of its original mass? Approximate to 4 decimal places.

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9. Calculate the following exactly.

(a)  $\cos\left(\frac{7\pi}{6}\right)$

(b)  $\sin\left(\frac{5\pi}{12}\right)$

(d)  $\csc^{-1}(2)$

(e)  $\sin\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)$

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10. Graph a full period of  $f(x) = 3\sin(2\pi x) + 3$  in Cartesian coordinates. Clearly label the graph and state the following:

- (a) range of  $f(x)$
- (b) x-intercept(s) of  $f(x)$
- (c) y-intercept of  $f(x)$
- (d) amplitude
- (e) frequency
- (f) phase shift

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11. Reduce the following to a single trigonometric function of  $x$ .

(a)  $\csc(x) - \frac{\csc(x)}{\sec^2(x)}$

(b)  $\frac{\tan(-x)}{\sin(-x)}$

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12. A surveyor determines that the angle of elevation to the top of a building from a point on the ground is  $32^\circ$ . He then moves back 41.3 feet and determines that the angle of elevation is  $26^\circ$ . What is the height of the building? Round your result to the nearest whole foot.

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13. Suppose that a ceiling fan 4.5 feet in diameter is rotating at a rate of 3.5 full rotations per second.

- (a) Calculate the angular speed of the ceiling fan in radians per second.
- (b) Calculate the linear speed at the tip of a ceiling fan blade in feet per second. Approximate to 4 decimal places.
- (c) If the diameter of the fan were to be halved, what angular speed would be necessary for the linear speed at the tip of a fan blade to remain unchanged?

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14. Let  $A = \langle -7, 12 \rangle$  and  $B = \langle 13, -5 \rangle$ .

- (a) Calculate  $4A - 9B$  exactly.
  - (b) Calculate  $-2A \cdot 3B$  exactly.
  - (c) Normalize the vectors  $A$  and  $B$  exactly.
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15. Calculate the following finite series:

(a)  $\sum_{i=1}^{10} (2i - 1)$

(b)  $\sum_{i=1}^5 3^{i-1}$

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16. Find the partial fraction decomposition of the following rational function. State your answers exactly.

$$\frac{3x^2 + 2x + 1}{(x^2 - 1)(x^2 + 1)}$$

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