Mathematics 2350, Calculus 3

FINAL EXAM, Fall 2011

The use of calculator, formula sheet and/or any other electronic device is not allowed.

1. Let the velocity vector be $\mathbf{v}(t) = t^2\mathbf{i} - \sin(2t)\mathbf{j} + 2te^{t^2}\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = \mathbf{i} - \frac{1}{2}\mathbf{j} + 2\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$, and the position vector $\mathbf{r}(t)$.

2. Let the position vector be $\mathbf{R}(t) = 4t\,\mathbf{i} + 3\sin t\,\mathbf{j} - 3\cos t\mathbf{k}$. Find the unit tangent vector $\mathbf{T}(t)$ and the principal unit normal vector $\mathbf{N}(t)$.

3. Find the curvature of the plane curve $y = -\cos(x) + e^{2x}$ at $x = 0$.

4. Find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ and $\dfrac{\partial f}{\partial z}$, if $\quad f(x, y, z) = \cos\left(\dfrac{x}{y}\right) e^{z^2}$.

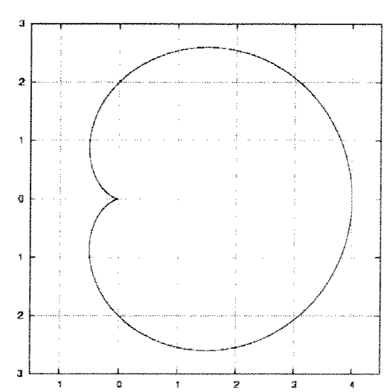
5. Suppose $f(x, y) = \sin(2x + 3y)$, $P = (\pi/2, \pi/3)$ and $\mathbf{v} = 6\mathbf{i} - 8\mathbf{j}$.
   a) Find the directional derivative of f at P in the direction of $\mathbf{v}$.
   b) Find the maximum rate of change of f at P.

6. Find a vector normal to the surface $x^2 + y^2 + z^2 = 14$ at $(3, 1, 2)$. Also find the equation of the tangent plane at the given point.

7. Find and classify all the critical points for the function
$$f(x, y) = 2x^3 + 3xy - 2y^3 + 7 \ .$$

8. Find
$$\iint_D xy \, dA$$
where D is the region enclosed by the curves $y = x$ and $y = x^2$.

9. Find the area of the region bounded by the curve
$$r = 2 + 2\cos(\theta).$$



10. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 8$ inside the cone $z = \sqrt{x^2 + y^2}$.

11. Evaluate the triple integral

$$I = \iiint_D (x^2 + y^2 + z^2)\, dV,$$

where D is the portion of the ball $x^2 + y^2 + z^2 \leq 4$ in the first octant, $x \geq 0$, $y \geq 0$ and $z \geq 0$.

12. Find div**F** and curl**F**, where

$$\mathbf{F}(x, y, z) = \sin x\mathbf{i} + y^3\mathbf{j} + ze^z\mathbf{k}.$$

13. Verify that the vector field **F** is conservative and evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{R} = \int_C (y\cos(xy) + y^2)\ dx + (x\cos(xy) + 2xy)\ dy,$$

where C is the curve parametrized by $< x(t), y(t) >=< t, t^2 >$, for $0 \leq t \leq 1$.

14. Use Green's theorem to evaluate

$$\oint_C \left(x^2 \cos x - y^3\right) dx + \left(x^3 + e^y \sin y\right) dy,$$

where C is the positively oriented circle $x^2 + y^2 = 1$.

15. Use Stokes' theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{R}$, where

$$\mathbf{F} = (e^{x^2} + 3y)\mathbf{i} + (\cos y + x)\mathbf{j} + z^2\mathbf{k}$$

and C is the closed curve given by the line segments connecting the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ traversed in the given order.