MATH 2450 Final Examination

1. Find a parametric representation of the line through $(-2, 1, 0)$ and perpendicular to the plane $x - y + 2z = 7$.

2. Find an equation of the plane through $(1, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$.

3. Find the the length of the curve $\mathbf{R} = \cos t\mathbf{i} + \sin t\mathbf{j} + 3t\mathbf{k}$, $0 \le t \le 4\pi$.

4. Let $f(x, y, z) = xz - yz$. Find the directional derivative f at $(0, -1, 1)$ in the direction of $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$.

5. Find all the critical points of the function $f(x, y) = x^3 + xy - x - y^2$ and classify each of them as either relative maximum, relative minimum, or saddle point.

6. Find the absolute maximum and minimum values of the function $f(x, y) = 2xy - x - y$ on the triangular region with vertices $(0, 0)$, $(4, 0)$, $(0, 4)$.

7. Evaluate $\int_0^1 \int_y^1 \cos(x^2)\ dxdy$ by reversing the order of integration.

8. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\ dydx$ by changing to polar coordinates.

9. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 4$.

10. Find the volume of the solid in the first octant bounded by the plane $2x + y + 2z = 4$ and the coordinate planes.

11. Use either cylindrical or spherical coordinates to compute $\iiint_D \frac{1}{\sqrt{x^2 + y^2}}\ dV$ where D is the portion the unit solid ball in the first octant: $x^2 + y^2 + z^2 \le 1$, $x \ge 0$, $y \ge 0$, $z \ge 0$.

12. Evaluate the line integral $\int_C (x^2 + y^2)\ dx - xy\ dy$ where C is the half circle $x^2 + y^2 = 1$, $y \ge 0$, from $(1, 0)$ to $(-1, 0)$.

13. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{N} dS$ where $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + \mathbf{k}$, S is the portion of the unit sphere in the first octant: $z = \sqrt{1 - x^2 - y^2}$, $x \ge 0$, $y \ge 0$, and $\mathbf{N}$ is the outward unit normal vector of S.

14. Use Stokes' theorem to evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{N} dS = \iint_S (\text{curl}\mathbf{F}) \cdot \mathbf{N} dS$ where $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + xyz\mathbf{k}$, S is the portion of the paraboloid $z = 1 - x^2 - y^2$ for $z \ge 0$, and $\mathbf{N}$ is the upward unit normal vector of S.

15. Use divergence theorem to evaluate the surface integral $\oiint_S \mathbf{F} \cdot \mathbf{N} dS$ where $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$, S is the unit sphere $x^2 + y^2 + z^2 = 1$ and $\mathbf{N}$ is the outward unit normal vector of S.