Mathematics 2450, Calculus 3 with applications

Spring 2013, Version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. If $\mathbf{F}(t) = 2t\,\mathbf{i} - 5\,\mathbf{j} + t^2\,\mathbf{k}$ and $\mathbf{H}(t) = \sin(t)\,\mathbf{i} + e^t\,\mathbf{j}$, compute the dot product $\mathbf{F}(t) \cdot \mathbf{H}(t)$.

   a) $2t\sin(t)\mathbf{i} - 5e^t\mathbf{j}$
   b) $-t^2e^t + t^2\sin(t) + (2te^t + 5\sin(t))$
   c) $-t^2e^t\mathbf{i} + t^2\sin(t)\mathbf{j} + (2te^t + 5\sin(t))\mathbf{k}$
   d) $2t\sin(t) - 5e^t$
   e) $2\sin(t) + 2t\cos(t) - 5e^t$

2. Find the limit, if it exists.

$$\mathbf{L} = \lim_{t\to 1}\left[\frac{(t^3-1)}{(t-1)}\,\mathbf{i} + \frac{\tan(t-1)}{(t-1)}\,\mathbf{j} + (t^2+1)e^{t-1}\,\mathbf{k}\right]$$

   a) DNE (Does Not Exist)
   b) $\langle 3, 1, 2\rangle$
   c) $\langle \frac{0}{0}, \frac{0}{0}, 2\rangle$
   d) $\langle 0, 0, 2\rangle$
   e) $\langle 1, 2, 3\rangle$

3. Let the velocity vector be $\mathbf{v}\,(t) = 2t\mathbf{i} + \cos t\mathbf{j} + e^{2t}\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$.

   a) $\left(t^2 + c_1\right)\mathbf{i} + (\sin t + c_2)\mathbf{j} + (\frac{1}{2}e^{2t} + c_3)\mathbf{k}$
   b) $2\mathbf{i} - \sin t\mathbf{j} + 2e^{2t}\mathbf{k}$
   c) $t^2\mathbf{i} + \sin t\mathbf{j} + \frac{1}{2}e^{2t}\mathbf{k}$
   d) $2 - \sin t + 2e^{2t}$
   e) $(2 + c_1)\,\mathbf{i} + (-\sin t + c_2)\,\mathbf{j} + \left(2e^{2t} + c_3\right)\mathbf{k}$

4. Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2} = A.$$

Compute the limit along the two lines $y = 0$ and $y = x^2$. From the results in the two cases, get to the correct conclusion.

a) $A = \frac{0}{0}$

b) $A = 0$

c) $A = 1$

d) The limit does not exist

e) $A = \frac{1}{2}$

5. Find the equation of the plane tangent to the surface $G(x, y, z) = x^2 + y^2 + z^2 = 14$ at the point $(2, 1, 3)$.

a) $z = 3$

b) $2x(x-2) + 2y(y-1) + 2z(z-3) = 0$

c) $\langle 4, 2, 6\rangle$

d) $2x + y + 3z = 14$

e) $x + y + z = 1$

6. Let $f(x, y) = \cos(x + 3y)$ and $P = (\pi/2, \pi/3)$. Find the maximum rate of change of the function f at the point P.

a) $\langle 1, 3\rangle$

b) 0

c) $\sqrt{10}$

d) 4

e) $\frac{1}{\sqrt{10}}\langle 1, 3\rangle$

7. Evaluate the integral by reversing the order of integration.

$$I = \int_0^1 \int_x^1 e^{y^2}\, dy dx.$$

a) $I = \frac{1}{2}(e-1)$

b) $I = 1$

c) $I = 0$

d) $I = (e-1)$

e) $I = e$

8. Evaluate the triple integral

$$I = \iiint_{\mathbf{D}} (x^2 + y^2)\, dV,$$

where $\mathbf{D}$ is the region inside the cone $z = \sqrt{x^2 + y^2}$, below the plane $z = 1$ and inside the first octant ($x \geq 0$, $y \geq 0$ and $z \geq 0$).

a) $I = \pi$

b) $I = 1$

c) $I = 0$

d) $I = \frac{\pi}{10}$

e) $I = \frac{\pi}{40}$

9. Find div**F** and curl**F**, where

$$\mathbf{F}(x, y, z) = \cos y\, \mathbf{i} + \sin x\, \mathbf{j} + xz\, \mathbf{k}.$$

a) $\begin{cases} \text{div}\mathbf{F} = x\,\mathbf{k} \\ \text{curl}\mathbf{F} = 0 \end{cases}$

b) $\begin{cases} \text{div}\mathbf{F} = x\,\mathbf{k} \\ \text{curl}\mathbf{F} = \mathbf{0} \end{cases}$

c) $\begin{cases} \text{div}\mathbf{F} = 0 \\ \text{curl}\mathbf{F} = -z\,\mathbf{j} + (\cos x + \sin y)\,\mathbf{k} \end{cases}$

d) $\begin{cases} \text{div}\mathbf{F} = x \\ \text{curl}\mathbf{F} = -z\,\mathbf{j} + (\cos x + \sin y)\,\mathbf{k} \end{cases}$

e) $\begin{cases} \text{div}\mathbf{F} = -z\,\mathbf{j} + (\cos x + \sin y)\,\mathbf{k} \\ \text{curl}\mathbf{F} = x \end{cases}$

10. Evaluate

$$I = \int_C \mathbf{F} \cdot d\mathbf{R},$$

where $\mathbf{F} = \langle ye^{xy}, xe^{xy} \rangle$ and C is the curve parametrized by $\mathbf{R} = \langle \cos t, \sin t \rangle$ for $0 \leq t \leq 2\pi$.

a) $I = \pi$
b) $I = 2\pi$
c) $I = 0$
d) $I = 1$
e) $I = \frac{1}{2}$

11. Let S be the part of the plane $x + y + z = 3$ which lies in the first octant, oriented upward. Evaluate the flux integral

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

of the vector field $\mathbf{F} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ across the surface S ($\mathbf{N}$ is the unit upward vector normal to the plane).

a) $I = 9$
b) $I = 8$
c) $I = 0$
d) $I = 36$
e) $I = -36$

12. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} = \langle xz, yx, zy \rangle$, and $\mathbf{N}$ is the unit outward normal to the surface S which encloses the box $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 1$.

a) $I = 3$
b) $I = 0$
c) $I = 1/2$
d) $I = 3/2$
e) $I = 1$

Essay questions.

Show all your work. A correct answer with no work counts as 0.

13. Let the position vector be $\mathbf{R}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j}$. Find the unit tangent vector $\mathbf{T}(t)$, the principal unit normal vector $\mathbf{N}(t)$, and the curvature $\kappa(t)$.

14. Find and classify all the critical points for the function

$$f(x, y) = 2x^3 + 3xy - 2y^3 + 7\ .$$

15. Find the surface area of the portion of the paraboloid $z = 4 - (x^2 + y^2)$ above the $xy-$plane.

16. Use Stokes' theorem to evaluate the flux integral

$$\iint_S (\operatorname{curl} \mathbf{F} \cdot \mathbf{N})\, dS,$$

where $\mathbf{F} = (\cos z + 4y)\mathbf{i} + (\sin z + 4x)\mathbf{j} + (x^2 z^2)\mathbf{k}$, S is part of the surface $z = 9 - x^2 - y^2$ with $z \geq 0$, and $\mathbf{N}$ points upward.