Mathematics 2450, Calculus 3 with applications

Fall 2015, version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the **parametric** equations for the line passing through the point $P = (1, 2, 3)$ and perpendicular to the plane $3x - 2y + 5z = 4$.

   a) $t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k}$
   b) $\langle 3 + t, -2 + 2t, 5 + 3t \rangle$
   c) $\dfrac{x-3}{1} = \dfrac{y+2}{2} = \dfrac{z-5}{3}$
   d) $x + 2y + 3z = 14$
   e) $\langle 1 + 3t, 2 - 2t, 3 + 5t \rangle$

2. Let the velocity vector be $\mathbf{v}(t) = t^2\mathbf{i} + \cos t\mathbf{j} + e^{2t}\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$.

   a) $\left(\dfrac{t^3}{3} + c_1\right)\mathbf{i} + (\sin t + c_2)\mathbf{j} + (\dfrac{1}{2}e^{2t} + c_3)\mathbf{k}$
   b) $2t\mathbf{i} - \sin t\mathbf{j} + 2e^{2t}\mathbf{k}$
   c) $\dfrac{t^3}{3}\mathbf{i} + \sin t\mathbf{j} + \dfrac{1}{2}e^{2t}\mathbf{k}$
   d) $2t - \sin t + 2e^{2t}$
   e) $(2t + c_1)\mathbf{i} + (-\sin t + c_2)\mathbf{j} + \left(2e^{2t} + c_3\right)\mathbf{k}$

3. Find the value of the following limit

$$A = \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2},$$

   if it exists.

   a) The limit does not exist
   b) $A = \dfrac{1}{2}$
   c) $A = +\infty$
   d) $A = \dfrac{0}{0}$
   e) $A = 0$

4. Given $F(x, y) = \cos(xy)$ where $x = u^2 + v^2$ and $y = u^2 - v^2$. Use the chain rule (do not substitute for x and y!) to find $\frac{\partial F}{\partial v}$. Express the result in terms of x, y, u, and v.

a) $\frac{\partial F}{\partial v} = -\sin(xy)(y - x)2v$

b) $\frac{\partial F}{\partial v} = -\sin(xy)(y + x)2u$

c) The function is not differentiable

d) $\frac{\partial F}{\partial v} = -\sin(xy)(2yu - 2xv)$

e) $\frac{\partial F}{\partial v} = -\sin(xy)(2yv + 2xu)$

5. Let $f(x, y) = \cos(x + 3y)$, $P = \left(\frac{\pi}{2}, \frac{\pi}{3}\right)$ and $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$. Find the directional derivative of f at P in the direction of $\mathbf{v}$.

a) 9

b) 0

c) $\frac{9}{5}$

d) $\frac{1}{5}\langle -3, 4\rangle$

e) $\langle 1, 3\rangle$

6. Evaluate the integral by reversing the order of integration.

$$I = \int_0^1 \int_y^1 e^{x^2}\, dx dy.$$

a) $I = 1$

b) $I = e$

c) $I = (e - 1)$

d) $I = 0$

e) $I = \frac{1}{2}(e - 1)$

7. Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 4$.

a) $S = 4\sqrt{2}\,\pi$

b) $S = 0$

c) $S = 4\pi$

d) $S = \frac{\pi}{6}(17^{3/2} - 1)$

f) $S = \pi$

8. Find curl **F** , where

$$\mathbf{F}(x,y,z) = (x^3+2x)\mathbf{i} + \cos(y)\mathbf{j} + e^{z^2}\mathbf{k}.$$

a) $\nabla \times \mathbf{F} = 3x^2 + 2 - \sin(y) + 2e^{z^2}$ b) $\nabla \times \mathbf{F} = \langle 3x^2+2, -\sin(y), 2e^{z^2}\rangle$

c) $\nabla \times \mathbf{F} = \langle 0,0,0\rangle$ d) $\nabla \times \mathbf{F} = \sqrt{(3x^2+2)^2 + \sin^2(y) + (2e^{z^2})^2}$

e) $\nabla \times \mathbf{F} = 0$

9. Verify if the vector field $\mathbf{F} = \langle x\cos(2y), -x^2\sin(2y)\rangle$ is conservative and evaluate the line integral

$$I = \int_C \mathbf{F}\cdot d\mathbf{R},$$

where C is the curve parametrized by $\mathbf{R}(t) = \langle t, \pi t^2\rangle$, for $0 \le t \le 1$.

a) $I = \frac{1}{4}$ b) $I = \frac{1}{2}$

c) $I = 0$ d) $I = 1$

e) $I = 2$

10. Use Green's theorem to evaluate

$$I = \oint_C \left(-y+y^2\right) dx + (x+2xy)\, dy,$$

where C is the rectangle with vertices in $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$, traversed counterclockwise.

a) $I = 4$ b) $I = 0$

c) $I = 1$ d) $I = 2$

e) $I = 8$

11. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F}\cdot\mathbf{N}\, dS,$$

where $\mathbf{F} = \langle xz, yx, zy\rangle$, and $\mathbf{N}$ is the unit outward normal to the surface S which encloses the box $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$.

a) $I = 3$ b) $I = 0$

c) $I = 1/2$ d) $I = 3/2$

e) $I = 1$

Essay questions.

Show all your work. A correct answer with no work counts as 0.

12. Let the position vector be $\mathbf{R}(t) = 8t\,\mathbf{i} + 3\sin(2t)\,\mathbf{j} - 3\cos(2t)\mathbf{k}$. Find the unit tangent vector $\mathbf{T}(t)$ and the principal unit normal vector $\mathbf{N}(t)$.

13. Find and classify all the critical points for the function

$$f(x, y) = 2x^3 + 3xy - 2y^3 + 7 \ .$$

14. Use either cylindrical or spherical coordinates to evaluate the triple integral

$$I = \iiint_{\mathbf{D}} z\, dV,$$

where $\mathbf{D}$ is the portion of the ball, $x^2 + y^2 + z^2 \leq 4$, in the first octant, $x \geq 0,\ y \geq 0$ and $z \geq 0$.

15. Use Stokes' theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{R}$, where

$$\mathbf{F} = (\mathrm{e}^{x^2} + 3y)\mathbf{i} + (\cos y + x)\mathbf{j} + z^2\mathbf{k}$$

and C is the closed curve given by the line segments connecting the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ traversed in the given order.