Mathematics 2450, Calculus 3 with applications

Fall 2016, version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the **symmetric** equations for the line passing through the point $P = (3, -1, 2)$ and perpendicular to the plane $-2x + y - 5z = 4$.

   a) $\langle -2 + 3t, 1 - t, -5 + 2t\rangle$
   b) $\dfrac{x+2}{3} = 1 - y = \dfrac{z+5}{2}$
   c) $z = 1$
   d) $\dfrac{3-x}{2} = y + 1 = \dfrac{2-z}{5}$
   e) $-2x + y - 5z = 0$

2. Find the limit, if it exists.

$$\mathbf{L} = \lim_{t\to 1}\left[\frac{(t^3-1)}{(t-1)}\,\mathbf{i} + \frac{\tan(t-1)}{(t-1)}\,\mathbf{j} + (t^2+1)e^{t-1}\,\mathbf{k}\right]$$

   a) DNE (Does Not Exist)
   b) $\langle 3, 1, 2\rangle$
   c) $\langle \frac{0}{0}, \frac{0}{0}, 2\rangle$
   d) $\langle 0, 0, 2\rangle$
   e) $\langle 1, 2, 3\rangle$

3. Let the velocity vector be $\mathbf{v}(t) = e^t\mathbf{i} - \sin(2t)\mathbf{j} + t^2\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$. Compute the position vector $\mathbf{r}(t)$.

   a) $\langle e^t, -2\cos(2t), 2t\rangle$
   b) $\langle e^t + 1, \frac{1}{2}\cos(2t) + \frac{1}{2}, \frac{1}{3}t^3 - 1\rangle$
   c) $\langle e^t, \frac{1}{2}\cos(2t), \frac{1}{3}t^3\rangle$
   d) $\langle e^t + 1, -2\cos(2t) + 3, 2t - 1\rangle$
   e) $\langle 2, 1, -1\rangle$

4. Let $z = z(x, y)$ be a continuous function of x and y defined implicitly by the equation

$$\ln(x^2 + y^2 + xyz^2) = 5.$$

Find the partial derivative $\dfrac{\partial z}{\partial y}$, where it is defined.

a) $\dfrac{\partial z}{\partial y} = -\dfrac{2x + yz^2}{2xyz}$

b) $\dfrac{\partial z}{\partial y} = \ln(2y + xz^2)$

c) $\dfrac{\partial z}{\partial y} = \dfrac{2y + xz^2}{x^2 + y^2 + xyz^2}$

d) $\dfrac{\partial z}{\partial y} = -\dfrac{2y + xz^2}{2xyz}$

e) The function is not differentiable

5. Given $F(x, y) = \ln(xy)$ where $x = e^{uv^2}$ and $y = uv$. Use the chain rule to find $\dfrac{\partial F}{\partial v}$, where it exists. Express your result in terms of u and v only.

a) $\dfrac{\partial F}{\partial v} = 2uv + \dfrac{1}{v}$

b) $\dfrac{\partial F}{\partial v} = \dfrac{1}{u}$

c) The function is not differentiable

d) $\dfrac{\partial F}{\partial v} = v^2 + \dfrac{1}{u}$

e) $\dfrac{\partial F}{\partial v} = 2uv$

6. For the function

$$f(x, y) = e^{-2xy}$$

find and classify all the critical points.

a) $P_0(0, 0)$ Saddle Point

b) $P_0(1, 1)$ Relative Minimum

c) $P_0(0, 0)$ Relative Maximum

d) $\begin{cases} P_0(0,0) \text{ Saddle Point and} \\ P_1(1,1) \text{ Relative Minimum} \end{cases}$

e) $\begin{cases} P_0(1,1) \text{ Relative Maximum and} \\ P_1(0,0) \text{ Saddle Point} \end{cases}$

7. Find the area inside the cardioid $r = 1 + \cos\theta$.

a) $I = \dfrac{3}{2}\pi$

b) $I = 1$

c) $I = 0$

d) $I = \pi$

e) $I = 2\pi$

8. Evaluate the triple integral

$$I = \iiint_{\mathbf{D}} y\, dV,$$

where $\mathbf{D}$ is the region in the first octant ($x \geq 0$, $y \geq 0$ and $z \geq 0$), below the plane $z = 1 - y$ and with $x \leq 1$.

a) $I = 0$ b) $I = 1$

c) $I = \frac{1}{3}$ d) $I = \frac{1}{6}$

e) $I = \frac{1}{9}$

9. Evaluate

$$I = \oint_C \mathbf{F} \cdot d\mathbf{R},$$

where $\mathbf{F} = \langle ye^{xy}, xe^{xy} \rangle$ and C is the curve parametrized by $\mathbf{R} = \langle \cos t, \sin t \rangle$ for $0 \leq t \leq 2\pi$.

a) $I = \pi$ b) $I = 2\pi$

c) $I = 0$ d) $I = 1$

e) $I = \frac{1}{2}$

10. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} = \langle z \sin(y), e^x + y, z + \cos(xy) \rangle$, and $\mathbf{N}$ is the unit outward normal to the surface S defined implicitly by

$$x^2 + y^2 + z^2 = 1.$$

a) $I = 0$ b) $I = \pi$

c) $I = 1$ d) $I = \frac{8}{3}\pi$

e) $I = 4\pi$

Essay questions.

Show all your work. A correct answer with no work counts as 0.

12. Find the curvature of the plane curve $y = -\cos(x) + \mathrm{e}^{2x}$ at $x = 0$.

13. Suppose $f(x, y) = e^{2x+3y}$, $P = (1, 0)$ and $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.
a) Find the directional derivative of f at P in the direction of $\mathbf{v}$.
b) Find the maximum rate of change of f at P.

14. Evaluate the triple integral

$$I = \iiint_{\mathbf{D}} (3x^2 + 3y^2)\, dV,$$

where $\mathbf{D}$ is the region inside the cone $z = \sqrt{4x^2 + 4y^2}$, and below the plane $z = 8$.

15. Use Green's theorem to evaluate

$$\oint_C \left(x^2 \cos x - y^3\right) dx + \left(x^3 + \mathrm{e}^y \sin y\right) dy,$$

where C is the positively oriented circle $x^2 + y^2 = 1$.