Mathematics 2450, Calculus 3 with applications

Fall 2018, version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the equation of the plane parallel to the intersecting lines $\langle 1, 2+3t, -3+4t\rangle$ and $\langle 1+3t, 2+3t, -3+3t\rangle$, and passing through the origin $O = (0,0,0)$.

   a) $12x - 12y - 3z = 0$
   b) $-3x + 12y - 9z = 0$
   c) $9y + 12z = 0$
   d) $3x + 3y + 3z = 0$
   e) $z = 0$

2. Let $\mathbf{F}(t) = \dfrac{\sin(5t)}{\sin(4t)}\boldsymbol{i} + \dfrac{\ln(\tan(4t))}{\ln(\sin(5t))}\boldsymbol{j} + (t-3)\cos(5t)\,\boldsymbol{k}$. Find $\lim_{t\to 0}\mathbf{F}(t)$.

   a) $\left\langle \dfrac{5}{4}, 1, -3\right\rangle$
   b) $\left\langle \dfrac{0}{0}, \dfrac{-\infty}{-\infty}, -3\right\rangle$
   c) $1 + \dfrac{4}{5} - 3$
   d) $\left\langle 1, \dfrac{4}{5}, -3\right\rangle$
   e) The limit does not exist

3. Let $f(x,y) = \dfrac{x-y}{3x^2 + xy - 4y^2}$. Find the limit $\lim_{(x,y)\to(2,2)} f(x,y)$.

   a) $\dfrac{0}{0}$
   b) $\dfrac{1}{2}$
   c) $\dfrac{1}{28}$
   d) $\dfrac{1}{14}$
   e) The limit does not exist

4. Let $f(x,y) = \sin(3x + 6y)$ and $P = \left(\dfrac{\pi}{3}, \dfrac{\pi}{3}\right)$. Find the maximum rate of change of the function f at the point P.

   a) $\langle -3, -6\rangle$
   b) $\sqrt{45}\langle -1, -1\rangle$
   c) $\sqrt{45}$
   d) $-\sqrt{45}$
   e) $\dfrac{1}{\sqrt{45}}\langle -3, -6\rangle$

5. For the function $f(x, y) = 2x^2 + 3xy + 2y^2 - 7x - 7y + 3$, find and classify all critical points.

a) $(0, 0)$, Saddle

b) $(1, 1)$, Relative Minimum

c) $(1, 1)$, Saddle

d) $(0, 0)$, Relative Maximum

e) $\begin{cases} (1,1), & \text{Relative Minimum} \\ (0,0), & \text{Relative Maximum} \end{cases}$

6. Find the area inside the limacon $r = (8 + 4\cos(\theta))$.

a) $\dfrac{72\pi}{3}$

b) 144π

c) $\dfrac{80\pi}{3}$

d) 80π

e) 72π

7. Evaluate the triple integral $I = \iiint_D y\, dV$ where D is the region in the first octant ($x \geq 0, y \geq 0, z \geq 0$), below the plane $z = 3 - y$ and with $x \leq 1$.

a) $I = 0$

b) $I = 9$

c) $I = 3$

d) $I = 27$

e) $I = \dfrac{9}{2}$

8. Evaluate the triple integral $I = \iiint_D 3(x^2+y^2)\, dV$ where D is the region inside the paraboloid $z = 9 - x^2 - y^2$ and inside the first octant $x \geq 0, y \geq 0, z \geq 0$.

a) $I = \left(\dfrac{\pi}{4}\right) 3^6$

b) $I = \left(\dfrac{\pi}{8}\right) 3^6$

c) $I = 0$

d) $I = (3\pi)\, 3^6$

e) $I = \left(\dfrac{\pi}{2}\right) 3^6$

9. Find the curl $\mathbf{F}$ where $\mathbf{F} = \langle \sin(x), y^3 + \sin(4y), \cos(5z^5) \rangle$.

a) $\nabla \times \mathbf{F} = \left\langle \cos(x), 3y^2 + 4\cos(4y), -25z^4 \sin\left(5z^5\right) \right\rangle$

b) $\nabla \cdot \mathbf{F} = 0$

c) $\nabla \times \mathbf{F} = 0$

d) $\nabla \times \mathbf{F} = \langle 0, 0, 0 \rangle$

e) $\nabla \cdot \mathbf{F} = \cos(x) + 3y^2 + 4\cos(4y) - 25z^4 \sin\left(5z^5\right)$

10. Let S be the part of the plane $z = 4 - x - y$ which lies in the first octant, oriented upward. Evaluate the flux integral

$$I = \iint_S \mathbf{F} \cdot \mathbf{N} \, dS,$$

of the vector field $\mathbf{F} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ across the surface S (with $\mathbf{N}$ being the unit upward vector normal to the plane).

a) $I = 48$
b) $I = 96$
c) $I = 0$
d) $I = 72$
e) $I = 24$

11. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F} \cdot \mathbf{N} \, dS,$$

where $\mathbf{F} = \langle xy^2, yz^2, zx^2 \rangle$, and $\mathbf{N}$ is the the unit outward normal to the surface S given by $x^2 + y^2 + z^2 = 25$.

a) $I = 5^5 \left(\frac{2}{5}\right) \pi$
b) $I = 5^5 \left(\frac{6}{5}\right) \pi$
c) $I = 0$
d) $I = 5^5 \left(\frac{8}{5}\right) \pi$
e) $I = 5^5 \left(\frac{4}{5}\right) \pi$

Show work questions.

Show all your work. A correct answer with no work counts as 0.

12. Let the velocity vector be $\mathbf{v}(t) = t^2\mathbf{i} - \sin(2t)\mathbf{j} + 2te^{t^2}\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = \mathbf{i} - \frac{1}{2}\mathbf{j} + 2\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$, and the position vector $\mathbf{r}(t)$.

13. Find the coordinates of the point (x, y, z) on the plane $x + y + z = 1$, which is closest to the origin.

14. Evaluate the integral

$$I = \iint_D \frac{3}{4 + y^3} dA,$$

where D is the region bounded by the curves $y = \sqrt{x}$, $x = 0$, $y = 1$.

15. Use Green's theorem to evaluate

$$\oint_C \left(x^2 \cos x - y^3\right) dx + \left(x^3 + e^y \sin y\right) dy,$$

where C is the positively oriented circle $x^2 + y^2 = 1$.

Mathematics 2450, Calculus 3 with applications

Fall 2018, version B

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the equation of the plane parallel to the intersecting lines $\langle 1, 2-2t, -3+3t\rangle$ and $\langle 1-t, 2-t, -3+4t\rangle$, and passing through the origin $O=(0,0,0)$.

   a) $-x-y+4z=0$ b) $2y+12z=0$
   c) $-3x+3y-5z=0$ d) $-5x-3y-2z=0$
   e) $z=0$

2. Let $\mathbf{F}(t)=\dfrac{\sin(5t)}{\sin(8t)}\boldsymbol{i}+\dfrac{\ln(\tan(8t))}{\ln(\sin(5t))}\boldsymbol{j}+(t+1)\cos(5t)\,\boldsymbol{k}$. Find $\lim\limits_{t\to 0}\mathbf{F}(t)$.

   a) $\left\langle 1, \dfrac{8}{5}, 1\right\rangle$ b) $\left\langle \dfrac{0}{0}, \dfrac{-\infty}{-\infty}, 1\right\rangle$
   c) $1+\dfrac{8}{5}+1$ d) $\left\langle \dfrac{5}{8}, 1, 1\right\rangle$
   e) The limit does not exist

3. Let $f(x,y)=\dfrac{x-y}{2x^2+2xy-4y^2}$. Find the limit $\lim\limits_{(x,y)\to(2,2)} f(x,y)$.

   a) $\dfrac{1}{4}$ b) $\dfrac{1}{24}$
   c) $\dfrac{1}{12}$ d) $\dfrac{0}{0}$
   e) The limit does not exist

4. Let $f(x,y)=\sin(4x+5y)$ and $P=\left(\dfrac{\pi}{4},\pi\right)$. Find the maximum rate of change of the fucntion f at the point P.

   a) $-\sqrt{41}$ b) $\sqrt{41}$
   c) $\langle 4,5\rangle$ d) $\sqrt{41}\langle 1,1\rangle$
   e) $\dfrac{1}{\sqrt{41}}\langle 4,5\rangle$

5. For the function $f(x, y) = -2x^2 + 3xy - 2y^2 + x + y + 4$, find and classify all critical points.

a) $(0, 0)$, Saddle

b) $\begin{cases} (1, 1), & \text{Relative Maximum} \\ (0, 0), & \text{Relative Minimum} \end{cases}$

c) $(1, 1)$, Relative Maximum

d) $(1, 1)$, Saddle

e) $(0, 0)$, Relative Minimum

6. Find the area inside the limacon $r = (7 + 4\cos(\theta))$.

a) 65π

b) $\dfrac{65\pi}{3}$

c) 114π

d) $\dfrac{57\pi}{3}$

e) 57π

7. Evaluate the triple integral $I = \iiint_D y\, dV$ where D is the region in the first octant ($x \geq 0, y \geq 0, z \geq 0$), below the plane $z = 2 - y$ and with $x \leq 1$.

a) $I = 0$

b) $I = 8$

c) $I = \dfrac{8}{3}$

d) $I = \dfrac{4}{3}$

e) $I = \dfrac{8}{9}$

8. Evaluate the triple integral $I = \iiint_D 2(x^2+y^2)\, dV$ where D is the region inside the paraboloid $z = 4 - x^2 - y^2$ and inside the first octant $x \geq 0, y \geq 0, z \geq 0$.

a) $I = \left(\dfrac{\pi}{3}\right) 2^6$

b) $I = \left(\dfrac{\pi}{12}\right) 2^6$

c) $I = 0$

d) $I = (2\pi)\, 2^6$

e) $I = \left(\dfrac{\pi}{6}\right) 2^6$

9. Find the curl $\mathbf{F}$ where $\mathbf{F} = \langle 3\sin(3x)\,, y^5 + \sin(4y)\,, \cos(z)\rangle$.

a) $\nabla \times \mathbf{F} = \left\langle 9\cos(3x)\,, 5y^4 + 4\cos(4y)\,, -(\sin(z))\right\rangle$

b) $\nabla \times \mathbf{F} = 0$

c) $\nabla \cdot \mathbf{F} = 9\cos(3x) + 5y^4 + 4\cos(4y) - \sin(z)$

d) $\nabla \cdot \mathbf{F} = 0$

e) $\nabla \times \mathbf{F} = \langle 0, 0, 0\rangle$

10. Let S be the part of the plane $z = 2 - x - y$ which lies in the first octant, oriented upward. Evaluate the flux integral

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

of the vector field $\mathbf{F} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ across the surface S (with $\mathbf{N}$ being the unit upward vector normal to the plane).

a) $I = 12$ b) $I = 24$
c) $I = 6$ d) $I = 0$
e) $I = 18\pi$

11. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} = \langle xy^2, yz^2, zx^2 \rangle$, and $\mathbf{N}$ is the the unit outward normal to the surface S given by $x^2 + y^2 + z^2 = 16$.

a) $I = 0$ b) $I = 4^5 \left(\frac{6}{5}\right) \pi$
c) $I = 4^5 \left(\frac{2}{5}\right) \pi$ d) $I = 4^5 \left(\frac{8}{5}\right) \pi$
e) $I = 4^5 \left(\frac{4}{5}\right) \pi$

Show work questions.

Show all your work. A correct answer with no work counts as 0.

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13. Find the coordinates of the point (x, y, z) on the plane $x + y + z = 1$, which is closest to the origin.

14. Evaluate the integral

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where C is the positively oriented circle $x^2 + y^2 = 1$.