Multiple choice questions.

Follow the directions of the instructor.

1. Find the equation of the plane parallel to the intersecting lines \( \langle 1, 2 + 3t, -3 + 4t \rangle \) and \( \langle 1 + 3t, 2 + 3t, -3 + 3t \rangle \), and passing through the origin \( O = (0, 0, 0) \).

a) \( 12x - 12y - 3z = 0 \)  
   b) \( -3x + 12y - 9z = 0 \)  
   c) \( 9y + 12z = 0 \)  
   d) \( 3x + 3y + 3z = 0 \)  
   e) \( z = 0 \)

2. Let \( \mathbf{F}(t) = \frac{\sin(5t)}{\sin(4t)} \mathbf{i} + \frac{\ln(\tan(4t))}{\ln(\sin(5t))} \mathbf{j} + (t - 3) \cos(5t) \mathbf{k} \). Find \( \lim_{t \to 0} \mathbf{F}(t) \).

a) \( \langle \frac{5}{4}, 1, -3 \rangle \)  
   b) \( \langle 0, -\infty, -3 \rangle \)  
   c) \( 1 + \frac{4}{5} - 3 \)  
   d) \( \langle 1, \frac{4}{5}, -3 \rangle \)  
   e) The limit does not exist

3. Let \( f(x, y) = \frac{x - y}{3x^2 + xy - 4y^2} \). Find the limit \( \lim_{(x,y) \to (2,2)} f(x, y) \).

a) \( 0 \)  
   b) \( \frac{1}{2} \)  
   c) \( \frac{1}{28} \)  
   d) \( \frac{1}{14} \)  
   e) The limit does not exist

4. Let \( f(x, y) = \sin(3x + 6y) \) and \( P = \left( \frac{\pi}{3}, \frac{\pi}{3} \right) \). Find the maximum rate of change of the function \( f \) at the point \( P \).

a) \( \langle -3, -6 \rangle \)  
   b) \( \sqrt{45} \langle -1, -1 \rangle \)  
   c) \( \sqrt{45} \)  
   d) \( -\sqrt{45} \)  
   e) \( \frac{1}{\sqrt{45}} \langle -3, -6 \rangle \)
5. For the function \( f(x, y) = 2x^2 + 3xy + 2y^2 - 7x - 7y + 3 \), find and classify all critical points.

a) (0, 0), Saddle  
b) (1, 1), Relative Minimum  
c) (1, 1), Saddle  
d) (0, 0), Relative Maximum  
e) \( \{ (1, 1), \text{Relative Minimum} \) \( (0, 0), \text{Relative Maximum} \)

6. Find the area inside the limacon \( r = (8 + 4 \cos(\theta)) \).

a) \( \frac{72 \pi}{3} \)  
b) 144\pi  
c) \( \frac{80 \pi}{3} \)  
d) 80\pi  
e) 72\pi

7. Evaluate the triple integral \( I = \iiint_D y \, dV \) where \( D \) is the region in the first octant \( (x \geq 0, y \geq 0, z \geq 0) \), below the plane \( z = 3 - y \) and with \( x \leq 1 \).

a) \( I = 0 \)  
b) \( I = 9 \)  
c) \( I = 3 \)  
d) \( I = 27 \)  
e) \( I = \frac{9}{2} \)

8. Evaluate the triple integral \( I = \iiint_D 3(x^2 + y^2) \, dV \) where \( D \) is the region inside the paraboloid \( z = 9 - x^2 - y^2 \) and inside the first octant \( x \geq 0, y \geq 0, z \geq 0 \).

a) \( I = \left( \frac{\pi}{4} \right) 3^6 \)  
b) \( I = \left( \frac{\pi}{8} \right) 3^6 \)  
c) \( I = 0 \)  
d) \( I = (3\pi) 3^6 \)  
e) \( I = \left( \frac{\pi}{2} \right) 3^6 \)

9. Find the curl \( \mathbf{F} \) where \( \mathbf{F} = \langle \sin(x), y^3 + \sin(4y), \cos(5z^5) \rangle \).

a) \( \nabla \times \mathbf{F} = \langle \cos(x), 3y^2 + 4\cos(4y), -25z^4 \sin(5z^5) \rangle \)  
b) \( \nabla \cdot \mathbf{F} = 0 \)  
c) \( \nabla \times \mathbf{F} = 0 \)  
d) \( \nabla \times \mathbf{F} = \langle 0, 0, 0 \rangle \)  
e) \( \nabla \cdot \mathbf{F} = \cos(x) + 3y^2 + 4\cos(4y) - 25z^4 \sin(5z^5) \)
10. Let S be the part of the plane $z = 4 - x - y$ which lies in the first octant, oriented upward. Evaluate the flux integral

$$I = \int\int_S \mathbf{F} \cdot \mathbf{N} \, dS,$$

of the vector field $\mathbf{F} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ across the surface S (with $\mathbf{N}$ being the unit upward vector normal to the plane).

a) $I = 48$  

b) $I = 96$  

c) $I = 0$  

d) $I = 72$  

e) $I = 24$

11. Use the divergence theorem to evaluate

$$I = \int\int_S \mathbf{F} \cdot \mathbf{N} \, dS,$$

where $\mathbf{F} = \langle xy^2, yz^2, zx^2 \rangle$, and $\mathbf{N}$ is the the unit outward normal to the surface S given by $x^2 + y^2 + z^2 = 25$.

a) $I = 5^\frac{5}{5} \left(\frac{2}{5}\right) \pi$  

b) $I = 5^\frac{5}{5} \left(\frac{6}{5}\right) \pi$  

c) $I = 0$  

d) $I = 5^\frac{5}{5} \left(\frac{8}{5}\right) \pi$  

e) $I = 5^\frac{5}{5} \left(\frac{4}{5}\right) \pi$

Show work questions.

Show all your work. A correct answer with no work counts as 0.

12. Let the velocity vector be $\mathbf{v}(t) = t^2 \mathbf{i} - \sin(2t)\mathbf{j} + 2te^{t^2}\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = \mathbf{i} - \frac{1}{2}\mathbf{j} + 2\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$, and the position vector $\mathbf{r}(t)$.

13. Find the coordinates of the point $(x, y, z)$ on the plane $x + y + z = 1$, which is closest to the origin.

14. Evaluate the integral

$$I = \int\int_D \frac{3}{4 + y^3} \, dA,$$

where $D$ is the region bounded by the curves $y = \sqrt{x}$, $x = 0$, $y = 1$.

15. Use Green’s theorem to evaluate

$$\oint_C (x^2 \cos x - y^3) \, dx + (x^3 + e^y \sin y) \, dy,$$

where C is the positively oriented circle $x^2 + y^2 = 1$. 
Multiple choice questions.

Follow the directions of the instructor.

1. Find the equation of the plane parallel to the intersecting lines \( \langle 1, 2 - 2t, -3 + 3t \rangle \) and \( \langle 1 - t, 2 - t, -3 + 4t \rangle \), and passing through the origin \( O = (0, 0, 0) \).
   
   a) \(-x - y + 4z = 0\)  
   b) \(2y + 12z = 0\)  
   c) \(-3x + 3y - 5z = 0\)  
   d) \(-5x - 3y - 2z = 0\)  
   e) \(z = 0\)

2. Let \( \mathbf{F}(t) = \frac{\sin(5t)}{\sin(8t)} \mathbf{i} + \frac{\ln(\tan(8t))}{\ln(\sin(5t))} \mathbf{j} + (t + 1) \cos(5t) \mathbf{k} \). Find \( \lim_{t \to 0} \mathbf{F}(t) \).
   
   a) \(\langle \frac{1}{5}, \frac{8}{5}, 1 \rangle\)  
   b) \(\langle 0, -\infty, 1 \rangle\)  
   c) \(1 + \frac{8}{5} + 1\)  
   d) \(\langle \frac{5}{8}, 1, 1 \rangle\)  
   e) The limit does not exist

3. Let \( f(x, y) = \frac{x - y}{2x^2 + 2xy - 4y^2} \). Find the limit \( \lim_{(x,y)\to(2,2)} f(x, y) \).
   
   a) \(\frac{1}{4}\)  
   b) \(\frac{1}{24}\)  
   c) \(\frac{1}{12}\)  
   d) \(0\)  
   e) The limit does not exist

4. Let \( f(x, y) = \sin(4x + 5y) \) and \( P = \left( \frac{\pi}{4}, \pi \right) \). Find the maximum rate of change of the function \( f \) at the point \( P \).
   
   a) \(-\sqrt{41}\)  
   b) \(\sqrt{41}\)  
   c) \(\langle 4, 5 \rangle\)  
   d) \(\sqrt{41} \langle 1, 1 \rangle\)  
   e) \(\frac{1}{\sqrt{41}} \langle 4, 5 \rangle\)
5. For the function \( f(x, y) = -2x^2 + 3xy - 2y^2 + x + y + 4 \), find and classify all critical points.

\[
\begin{align*}
a) \ (0, 0), \ &\text{Saddle} \quad &b) \ &\begin{cases} (1, 1), &\text{Relative Maximum} \\ (0, 0), &\text{Relative Minimum} \end{cases} \\
c) \ (1, 1), \ &\text{Relative Maximum} \quad &d) \ (1, 1), \ &\text{Saddle} \\
e) \ (0, 0), \ &\text{Relative Minimum} \end{align*}
\]

6. Find the area inside the limacon \( r = (7 + 4\cos(\theta)) \).

\[
\begin{align*}
a) \ 65\pi & \quad &b) \ &\frac{65\pi}{3} \\
c) \ 114\pi & \quad &d) \ &\frac{57\pi}{3} \\
e) \ 57\pi \end{align*}
\]

7. Evaluate the triple integral \( I = \iiint_D y \, dV \) where \( D \) is the region in the first octant \( (x \geq 0, y \geq 0, z \geq 0) \), below the plane \( z = 2 - y \) and with \( x \leq 1 \).

\[
\begin{align*}
a) \ I = 0 & \quad &b) \ I = 8 \\
c) \ I = \frac{8}{3} & \quad &d) \ I = \frac{4}{3} \\
e) \ I = \frac{8}{9} \end{align*}
\]

8. Evaluate the triple integral \( I = \iiint_D 2(x^2 + y^2) \, dV \) where \( D \) is the region inside the paraboloid \( z = 4 - x^2 - y^2 \) and inside the first octant \( x \geq 0, y \geq 0, z \geq 0 \).

\[
\begin{align*}
a) \ I = \left(\frac{\pi}{3}\right) 2^6 & \quad &b) \ I = \left(\frac{\pi}{12}\right) 2^6 \\
c) \ I = 0 & \quad &d) \ I = (2\pi) 2^6 \\
e) \ I = \left(\frac{\pi}{6}\right) 2^6 \end{align*}
\]

9. Find the curl \( \mathbf{F} \) where \( \mathbf{F} = \langle 3 \sin(3x), y^5 + \sin(4y), \cos(z) \rangle \).

\[
\begin{align*}
a) \ \nabla \times \mathbf{F} = \langle 9 \cos(3x), 5y^4 + 4 \cos(4y), -(\sin(z)) \rangle & \quad &b) \ \nabla \times \mathbf{F} = 0 \\
c) \ \nabla \cdot \mathbf{F} = 9 \cos(3x) + 5y^4 + 4 \cos(4y) - \sin(z) & \quad &d) \ \nabla \cdot \mathbf{F} = 0 \\
e) \ \nabla \times \mathbf{F} = \langle 0, 0, 0 \rangle \end{align*}
\]
10. Let $S$ be the part of the plane $z = 2 - x - y$ which lies in the first octant, oriented upward. Evaluate the flux integral

$$ I = \iint_S \mathbf{F} \cdot \mathbf{N} \, dS, $$

of the vector field $\mathbf{F} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ across the surface $S$ (with $\mathbf{N}$ being the unit upward vector normal to the plane).

- a) $I = 12$
- b) $I = 24$
- c) $I = 6$
- d) $I = 0$
- e) $I = 18\pi$

11. Use the divergence theorem to evaluate

$$ I = \iiint_D \nabla \cdot \mathbf{F} \, dV, $$

where $\mathbf{F} = (x^2, y^2, z^2)$, and $\mathbf{N}$ is the the unit outward normal to the surface $S$ given by $x^2 + y^2 + z^2 = 16$.

- a) $I = 0$
- b) $I = 4^5 \left( \frac{6}{5} \right) \pi$
- c) $I = 4^5 \left( \frac{2}{5} \right) \pi$
- d) $I = 4^5 \left( \frac{8}{5} \right) \pi$
- e) $I = 4^5 \left( \frac{4}{5} \right) \pi$

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Show all your work. A correct answer with no work counts as 0.

12. Let the velocity vector be $\mathbf{v}(t) = t^2 \mathbf{i} - \sin(2t) \mathbf{j} + 2te^{t^2} \mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = \mathbf{i} - \frac{1}{2} \mathbf{j} + 2\mathbf{k}$. Compute the acceleration vector $\mathbf{a}(t)$, and the position vector $\mathbf{r}(t)$.

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15. Use Green’s theorem to evaluate

$$ \oint_C \left( x^2 \cos(x) - y^3 \right) \, dx + \left( x^3 + e^y \sin(y) \right) \, dy, $$

where $C$ is the positively oriented circle $x^2 + y^2 = 1$. 