Mathematics 2450, Calculus 3 with applications

Spring 2018, version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the equations of the plane passing through the point $P = (1, 1, 2)$ and perpendicular to the line $\langle -2 + 3t, 1 - t, 3 + 2t\rangle$.

   a) $\langle 1 + 3t, 1 - t, 2 + 2t\rangle$
   b) $x + y + 2z = 5$
   c) $-2x + y + 3z = 5$
   d) $\dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{z-2}{2}$
   e) $3x - y + 2z = 6$

2. Find the following limit

$$\lim_{t\to 0}\left(\ln(t+1)\mathbf{i} + \frac{2t^2 + t^4}{t^2 + 3t^3}\mathbf{j} + \frac{\sin(t)}{t^2}\mathbf{k}\right),$$

   if it exists.

   a) $\langle 1, 2, 1\rangle$
   b) $\left\langle 0, \frac{0}{0}, \frac{0}{0}\right\rangle$
   c) the limit does not exist
   d) $\langle 1, +\infty, 1\rangle$
   e) $\langle 0, 2, 1\rangle$

3. Find the arclength of the curve described by $\mathbf{R}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$ for $0 \le t \le \pi$.

   a) $\dfrac{1}{5}$
   b) 5π
   c) $\dfrac{1}{5\pi}$
   d) $\langle -3\sin t, 3\cos t, 4\rangle$
   e) $\dfrac{1}{5}\langle -3\sin t, 3\cos t, 4\rangle$

4. Given $F(x,y)=\ln(xy)$ where $x=e^{uv^2}$ and $y=uv$. Use the chain rule to find $\dfrac{\partial F}{\partial v}$, where it exists. Express your result in terms of u and v only.

a) $\dfrac{\partial F}{\partial v}=2uv+\dfrac{1}{v}$

b) $\dfrac{\partial F}{\partial v}=\dfrac{1}{u}$

c) The function is not differentiable

d) $\dfrac{\partial F}{\partial v}=v^2+\dfrac{1}{u}$

e) $\dfrac{\partial F}{\partial v}=2uv$

5. Let $f(x,y)=\cos(x+3y)$, $P=(\pi/2,\pi/3)$ and $\mathbf{v}=-3\mathbf{i}+4\mathbf{j}$. Find the directional derivative of f at P in the direction of $\mathbf{v}$.

a) 9

b) 0

c) $\dfrac{9}{5}$

d) $\dfrac{1}{5}\langle -3,4\rangle$

e) $\langle 1,3\rangle$

6. Reverse the order of integration for the following integral

$$\int_0^4\int_x^{2\sqrt{x}} f(x,y)\,dy\,dx\ .$$

a) $\displaystyle\int_x^{2\sqrt{x}}\int_0^4 f(x,y)\,dx\,dy$

b) $\displaystyle\int_0^4\int_{\frac{y^2}{4}}^{y} f(x,y)\,dx\,dy$

c) $\displaystyle\int_0^4\int_y^{\frac{y^2}{4}} f(x,y)\,dx\,dy$

d) $\displaystyle\int_y^{\frac{y^2}{4}}\int_0^4 f(x,y)\,dx\,dy$

e) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{4\cos(\theta)} f(r,\theta)\,r dr\,d\theta$

7. Find the surface area of the portion of the cone $z=\sqrt{x^2+y^2}$ in between the planes $z=1$ and $z=2$.

a) $S=4\pi$

b) $S=0$

c) $S=3\sqrt{2}\,\pi$

d) $S=5\sqrt{2}\,\pi$

f) $S=\pi$

8. Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2+y^2+z^2 = 4$.

a) $I = 1 - \frac{\sqrt{2}}{2}$

b) $I = 1$

c) $I = 0$

d) $I = \frac{16\pi}{3}$

e) $I = \left(1 - \frac{\sqrt{2}}{2}\right)\frac{16\pi}{3}$

9. Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$, where

$$\mathbf{F} = (y\cos(xy) + 2x\mathrm{e}^{x^2})\mathbf{i} + (x\cos(xy) + 3y^2)\mathbf{j},$$

and C is the curve parametrized by $\mathbf{R}(t) = \langle 2\cos t, 3\sin t\rangle$ for $0 \le t \le \frac{\pi}{2}$.

a) the vector field is non conservative

b) $2 - \mathrm{e}$

c) $\sin(xy) + \mathrm{e}^{x^2} + y^3$

d) 0

e) $28 - \mathrm{e}^4$

10. Evaluate

$$I = \oint_C \mathbf{F} \cdot d\mathbf{R},$$

where $\mathbf{F} = \langle -y^3, x^3\rangle$ and C is the curve parametrized by $\mathbf{R} = \langle \cos t, \sin t\rangle$ for $0 \le t \le 2\pi$.

a) $I = \pi$

b) $I = 2\pi$

c) $I = \frac{5}{3}\pi$

d) $I = 1$

e) $I = \frac{3}{2}\pi$

11. Use divergence theorem to evaluate the flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} = \langle xz^2, yx^2, zy^2\rangle$ and $\mathbf{N}$ is the outward unit vector normal to the surface S defined by

$$x^2 + y^2 + z^2 = 4.$$

a) $\frac{128}{5}\pi$

b) 0

c) 16π

d) π

e) $\frac{32}{3}\pi$

Essay questions.

Show all your work. A correct answer with no work counts as 0.

12. Let the velocity vector be $\mathbf{v}(t) = e^{3t}\,\mathbf{i} - 2\sin 2t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = \frac{1}{3}\mathbf{i} - \mathbf{j} + \mathbf{k}$. Compute the position vector $\mathbf{r}(t)$.

13. Find the absolute extrema for the function $z = f(x, y) = x\,y$, in the closed disk $x^2 + y^2 \leq 1$.

14. Find the triple integral

$$\iiint_V x\,dz\,dy\,dx,$$

where V is the tetrahedron bounded by the plane $x + y + z = 1$ and by the coordinates planes $x = 0$, $y = 0$ and $z = 0$.

15. Find the line integral

$$\int_C (2x - 3y)\,ds\,,$$

where C is the semicircle $y = \sqrt{4 - x^2}$ traversed from $(2, 0)$ to $(-2, 0)$.

Mathematics 2450, Calculus 3 with applications

Spring 2018, version B

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the equations of the plane passing through the point $P = (2, 1, 1)$ and perpendicular to the line $\langle 3 + 2t, -2 + 3t, 1 - t\rangle$.

   a) $3x - 2y + z = 5$
   b) $2x + y + z = 5$
   c) $\langle 2 + 2t, 1 + 3t, 1 - t\rangle$
   d) $2x + 3y - z = 6$
   e) $\dfrac{x-2}{2} = \dfrac{y-1}{3} = \dfrac{z-1}{-1}$

2. Find the following limit

$$\lim_{t\to 0}\left(\ln(t+1)\mathbf{i} + \frac{\sin(t)}{t^2}\mathbf{j} + \frac{2t^2 + t^4}{t^2 + 3t^3}\mathbf{k}\right),$$

   if it exists.

   a) the limit does not exist
   b) $\langle 1, 1, 2\rangle$
   c) $\langle 0, 1, 2\rangle$
   d) $\langle 1, 1, +\infty, \rangle$
   e) $\left\langle 0, \dfrac{0}{0}, \dfrac{0}{0}\right\rangle$

3. Find the arclength of the curve described by $\mathbf{R}(t) = \langle 4\sin t, 4\cos t, 3t\rangle$ for $0 \le t \le \pi$.

   a) $\dfrac{1}{5\pi}$
   b) $\dfrac{1}{5}\langle 4\cos t, -4\sin t, 3\rangle$
   c) $\dfrac{1}{5}$
   d) $\langle 4\cos t, -4\sin t, 3\rangle$
   e) 5π

4. Given $F(x,y) = \ln(xy)$ where $x = e^{uv^2}$ and $y = uv$. Use chain rule to find $\dfrac{\partial F}{\partial u}$, where it exists. Express your result in terms of u and v only.

a) $\dfrac{\partial F}{\partial u} = 2uv + \dfrac{1}{v}$

b) $\dfrac{\partial F}{\partial u} = \dfrac{1}{u}$

c) The function is not differentiable

d) $\dfrac{\partial F}{\partial u} = v^2 + \dfrac{1}{u}$

e) $\dfrac{\partial F}{\partial u} = 2uv$

5. Let $f(x,y) = \cos(3x+y)$, $P = (\pi/3, \pi/2)$ and $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$. Find the directional derivative of f at P in the direction of $\mathbf{v}$.

a) $\dfrac{9}{5}$

b) $\dfrac{1}{5}\langle -3, 4\rangle$

c) $\langle 1, 3\rangle$

d) 9

e) 0

6. Reverse the order of integration for the following integral

$$\int_0^4 \int_y^{2\sqrt{y}} f(x,y)\, dx\, dy\ .$$

a) $\displaystyle\int_y^{2\sqrt{y}} \int_0^4 f(x,y)\, dy\, dx$

b) $\displaystyle\int_0^4 \int_x^{\frac{x^2}{4}} f(x,y)\, dy\, dx$

c) $\displaystyle\int_0^4 \int_{\frac{x^2}{4}}^{x} f(x,y)\, dy\, dx$

d) $\displaystyle\int_0^{\frac{\pi}{4}} \int_0^{4\sin(\theta)} f(r,\theta)\, r dr\, d\theta$

e) $\displaystyle\int_x^{\frac{x^2}{4}} \int_0^4 f(x,y)\, dy\, dy$

7. Find the surface area of the portion of the cone $z = \sqrt{x^2+y^2}$ in between the planes $z = 2$ and $z = 3$.

a) $S = 4\pi$

b) $S = 0$

c) $S = 3\sqrt{2}\,\pi$

d) $S = 5\sqrt{2}\,\pi$

f) $S = \pi$

8. Find the volume of the region enclosed by the cone $z = \sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2 = 1$.

a) $I = 1$

b) $I = 1 - \dfrac{\sqrt{2}}{2}$

c) $I = \left(1 - \dfrac{\sqrt{2}}{2}\right)\dfrac{2\pi}{3}$

d) $I = \dfrac{2\pi}{3}$

e) $I = 0$

9. Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$, where

$$\mathbf{F} = (y\cos(xy) + 2x\mathrm{e}^{x^2})\mathbf{i} + (x\cos(xy) + 3y^2)\mathbf{j},$$

and C is the curve parametrized by $\mathbf{R}(t) = \langle 2\cos t, 3\sin t\rangle$ for $0 \le t \le \frac{\pi}{2}$.

a) $28 - \mathrm{e}^4$

b) the vector field is non conservative

c) $2 - \mathrm{e}$

d) $\sin(xy) + \mathrm{e}^{x^2} + y^3$

e) 0

10. Evaluate

$$I = \oint_C \mathbf{F} \cdot d\mathbf{R},$$

where $\mathbf{F} = \langle -3yx^2, 3xy^2\rangle$ and C is the curve parametrized by $\mathbf{R} = \langle \cos t, \sin t\rangle$ for $0 \le t \le 2\pi$.

a) $I = \pi$

b) $I = 2\pi$

c) $I = \dfrac{3}{2}\pi$

d) $I = 1$

e) $I = \dfrac{5}{3}\pi$

11. Use divergence theorem to evaluate the flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} = \langle xy^2, yz^2, zx^2\rangle$ and $\mathbf{N}$ is the outward unit vector normal to the surface S defined by

$$x^2 + y^2 + z^2 = 1.$$

a) $\dfrac{1}{3}\pi$

b) $\dfrac{4}{5}\pi$

c) 0

d) $\dfrac{4}{3}\pi$

e) 2π

Essay questions.

Show all your work. A correct answer with no work counts as 0.

12. Let the velocity vector be $\mathbf{v}\,(t) = e^{3t}\,\mathbf{i} - 2\sin 2t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$, and the initial position vector be $\mathbf{r}\,(0) = \frac{1}{3}\mathbf{i} - \mathbf{j} + \mathbf{k}$. Compute the position vector $\mathbf{r}(t)$.

13. Find the absolute extrema for the function $z = f(x,y) = x\,y$, in the closed disk $x^2 + y^2 \leq 1$.

14. Find the triple integral

$$\iiint_V x\,dz\,dy\,dx,$$

where V is the tetrahedron bounded by the plane $x+y+z=1$ and by the coordinates planes $x=0$, $y=0$ and $z=0$.

15. Find the line integral

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