When It Comes To Your Future

DON'T GUESS

Department Of Mathematics PlacemeNT Study GUide TExaS Tech UniverSity

ALGEBRA I MATERIAL

The problems included in pages 1 – 4 are generally discussed in an algebra course. The algebra course at Texas Tech University is Math 0301. Mastering this material is a requirement in order to meet the prerequisites for entering Math 0302.

and

ALGEBRA II MATERIAL

The problems included in pages 5 – 31 are generally discussed in an Algebra II course. The Algebra II course at Texas Tech University is Math 0302. Mastering this material is necessary in order to meet the prequisites for entering Math 1330/1331 (Business Math) and/or Math 1320 (College Algebra).

This study guide has been made available to help students prepare for the Mathematics Placement Exam at Texas Tech University. The workbook provides practice problems to review material needed for various freshman level mathematics courses offered at Texas Tech University. A student who has difficulty with any of the problem sets should consult a mathematics textbook to find discussion and worked examples.

The sections listed below, in conjunction with the prerequisite flow chart on the next page, provides a self-guidance system intended to aid you in matching your level of preparation with the appropriate mathematics course at Texas Tech.

ALGEBRA I — Prerequisite for Math 0302

ALGEBRA II — Prerequisite for Math 1320/1330/1331

COLLEGE ALGEBRA — Prerequisite for Math 1321

PRE-CALCULUS — Prerequisite for Math 1350/1351

ANSWERS

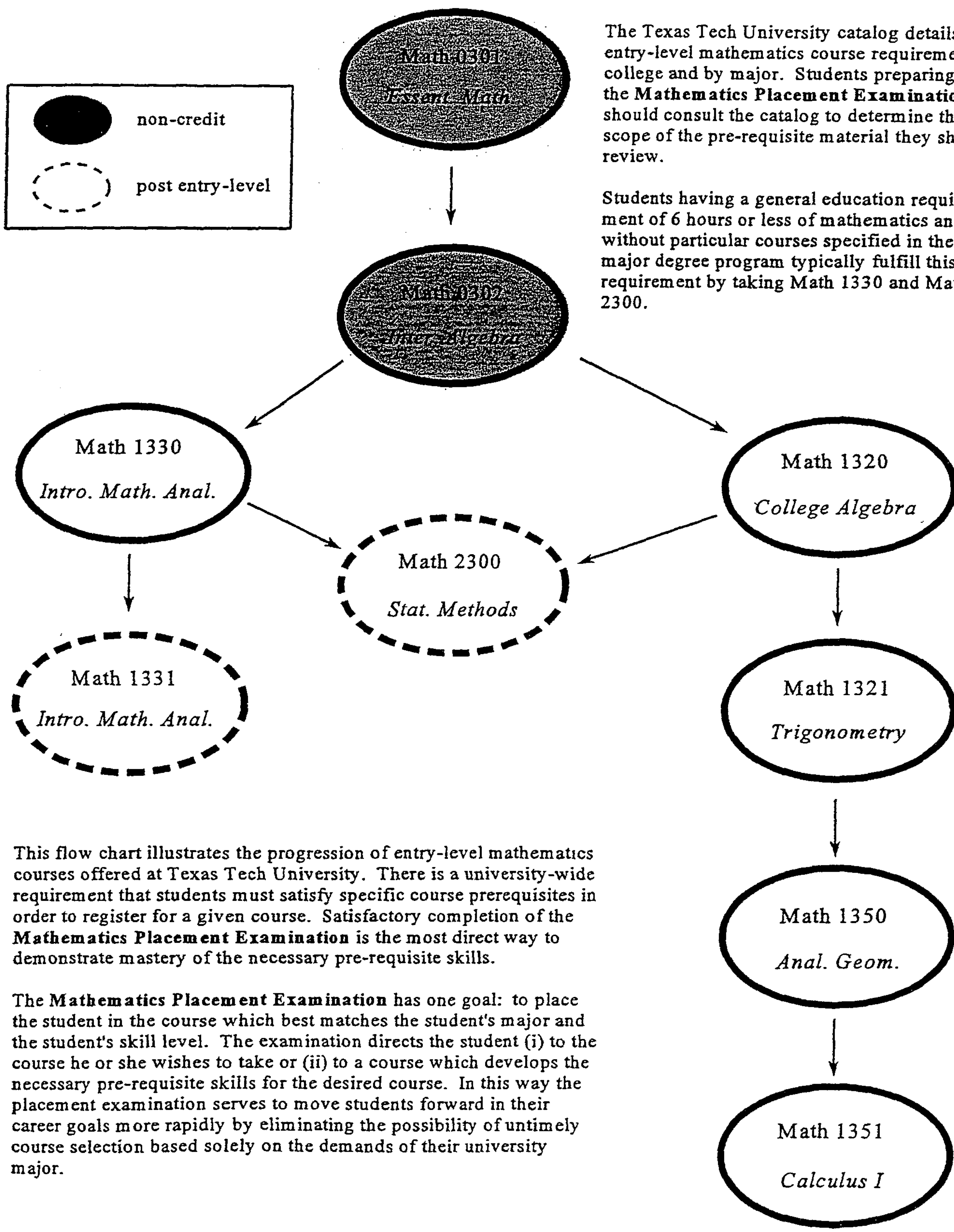
SAMPLE PLACEMENT EXAM

Entry-Level Mathematics Courses

Texas Tech University

The Texas Tech University catalog details entry-level mathematics course requirements by college and by major. Students preparing to take the **Mathematics Placement Examination** should consult the catalog to determine the scope of the pre-requisite material they should review.

Students having a general education requirement of 6 hours or less of mathematics and without particular courses specified in their major degree program typically fulfill this requirement by taking Math 1330 and Math 2300.



This flow chart illustrates the progression of entry-level mathematics courses offered at Texas Tech University. There is a university-wide requirement that students must satisfy specific course prerequisites in order to register for a given course. Satisfactory completion of the **Mathematics Placement Examination** is the most direct way to demonstrate mastery of the necessary pre-requisite skills.

The **Mathematics Placement Examination** has one goal: to place the student in the course which best matches the student's major and the student's skill level. The examination directs the student (i) to the course he or she wishes to take or (ii) to a course which develops the necessary pre-requisite skills for the desired course. In this way the placement examination serves to move students forward in their career goals more rapidly by eliminating the possibility of untimely course selection based solely on the demands of their university major.

Basic Facts

Inequality symbols are used when comparing or ordering mathematical values and expressions. The inequality symbols are:

$<$ which is read "is less than",
$>$ which is read "is greater than",
$\geq$ which is read " is greater than or equal to" and
$\leq$ which is read " is less than or equal to".

Use the symbols $<$ and $>$ to make a true statement about the pairs of numbers below.

1. $2 \quad -7$ 2. $-100 \quad -87$ 3. $\frac{2}{3} \quad \frac{3}{4}$

The **absolute value** of a number is the distance from zero to the point on the number line which represents that number. Since distance is always nonnegative, the absolute value of any number is nonnegative. The symbol | | is used to indicate absolute value. Simplify the following expressions.

4. $|5|$ 5. $\left|\frac{-1}{7}\right|$ 6. $|0|$

Fill in the following table with the equivalent decimals, fractions, and percents.

	fraction	decimal	percent
7.	$\frac{3}{8}$	__________	__________
8.	__________	2.7	__________
9.	__________	__________	3.9%

10. What is 8.3% of 126?

11. If the tax rate is 4.5%, what is the tax on a $3 purchase?

Basic Facts

Perform the indicated operations.

12. $-20+(-67)$

13. $235-(-91)$

14. $\frac{3}{4}+\frac{-2}{3}$

15. $-6.5-3.7$

16. $7(-43)$

17. $\frac{-45}{0}$

18. $\frac{12}{\frac{-3}{4}}$

19. $-135+89$

20. $\frac{-7}{8}\cdot\frac{-12}{21}$

21. $\frac{-0.0518}{-0.14}$

22. $\left(\frac{-2}{3}\right)^2$

23. $(2.5)^4$

The order in which mathematical operations must be performed within a problem can more easily be remembered using the following memory aid.

Please $\longrightarrow$ Parentheses (or any grouping symbols)
Excuse $\longrightarrow$ Exponents
My Dear $\longrightarrow$ Multiplication and Division in order from left to right
Aunt Sally $\longrightarrow$ Addition and Subtraction in order from left to right

Compute:

24. $-9\left(\frac{-2}{3}\right) \div \left(\frac{-6}{5}\right)\left(\frac{12}{5}\right)$

25. $9 + (-5)(-8)$

26. $\frac{7}{9} - \frac{3}{5} \cdot \frac{-5}{3} + \frac{1}{6}$

27. $(-2)^4 + 3(4) - 5(6) + \frac{4(-7) - 8}{5 - 2(4)}$

28. $[8 - 7(-5)] - |7 - 13|$

29. $[4 - 3^2(5 - 7)] - [1 - 2(3)]$

Evaluate the following if $x = -5$, $y = 3.7$, and $z = \frac{-5}{4}$.

30. $\frac{x}{z}$

31. $3y - x$

32. $\frac{1}{2}x - 2y + z$

33. $xy + 3z$

34. Find the perimeter of a rectangle ($P = 2l + 2w$) with a length of $7\frac{5}{8}$ *feet* and a width of $4\frac{1}{2}$ *feet*.

35. Find the area of a triangle ($A = \frac{1}{2}bh$) if the base, b, is 1.2 *centimeters* and the height, h, is 9.8 *centimeters*.

Like terms are terms which contain exactly the same variables with exactly the same exponents, but they may have different numerical coefficients. When simplifying an expression, only combine like terms. Simplify the following.

36. $\frac{7}{9}x + \frac{3}{5}y - x + y$

37. $2a^2 - 5a + 3 - a^2 + 5a - 7$

38. $12 - (3y + 10)$

39. $4(x - y) - \frac{2x + 3y}{3}$

40. $4\left\{\frac{2}{3}\left[5m - \frac{1}{8}(6)\right] + \frac{3}{10}|4n - 12n|\right\}$

41. $1.2(r - t) - 3.5[r - (2.7r - t)]$

Linear Equations and Inequalities in One Variable

A **solution** to an equation is a value that makes the equation a true statement.

1. Is 63, 6.3, or 0.63 a solution to the equation $0.4x - 0.96 = -0.708$?

The following guidelines may be useful when solving equations.

- Remove grouping symbols by using the distributive property or the rules for order of operations.
- Clear fractions - optional
- Use the addition principle to isolate the variable on one side of the equation and the constant on the other side.
- Use the multiplication principle to make the coefficient of the variable equal one.
- Check the solution in the original equation if time permits.

Solve the following equations. Check the solutions.

2. $x + 2 = 10$
3. $1 = r - 3$
4. $s - 6 = 0$
5. $-t + 11 = -2$
6. $19 = 7x - 2$
7. $s - \frac{1}{4} = \frac{-3}{2}$
8. $-12 + z = 2$
9. $16 = -4a$
10. $-a + 11 = 0$
11. $-6s - \frac{1}{5} = \frac{14}{5}$

12. $3 - \frac{a}{2} = 5$

13. $\frac{5}{6} = \frac{4}{3}v - \frac{5}{4}$

14. $-0.3b - 1 = 1.4$

15. $t + 0.25 = 0.75$

16. $-2 = \frac{z}{3}$

17. $\frac{1}{6}x = 4y$ for x

18. $\frac{y}{5} + \frac{1}{2} = \frac{5}{2}$

19. $5t - 1 = 3t + 3$

20. $-s + 11 = 5s - 7$

21. $4\left(\frac{y}{2} - 1\right) - 3 = -2y + 9$

22. $4x + 16 = 3y$ for x

23. $3a + \frac{2}{7} = -a + \frac{1}{14}$

24. $5z + 3 = -3(3z - 1) + 7$

25. $-12b = 6$

26. $t - 4r = 10s$ for r

27. $\frac{-1}{4}t = -10$

28. $3(a - b) = 4c$ for a

29. $-2\left(\frac{z}{2} - 1\right) + 4 = 3\left(z + \frac{2}{3}\right)$

30. $\left(\frac{-1}{4}\right)(2t + 8) = \left(\frac{1}{3}\right)(6t - 1)$

The following guidelines may be useful when solving application problems.

- Read the problem carefully and decide what must be found. Draw a picture when appropriate.
- Choose a variable to represent the unknown quantity and write it down.
- Translate the problem into an equation using the variable and the given information.
- Solve the equation.
- Compare the solution with the problem to see if the answer makes sense.

31. A 60 *centimeter* piece of twine is cut into 2 pieces. If one piece is $\frac{1}{2}$ as long as the other, find the length of each piece of twine.

32. One plus the product of four and a number is seven times the number. Find the number.

33. A rectangle has a perimeter of 60 cm. If the length is 10 cm greater than the width, find the length and width of the rectangle.

34. To convert from Celsius to Fahrenheit, use the formula $F = \frac{9}{5}C + 32$. Solve for C.

35. A \$10,000 car was marked down to \$8,200. What percent was the car marked down?

36. What percent of 120 is 78?

37. Jane paid \$55.25 for a calculator to use in her engineering class. It was on sale at 15% off the original price. What was the original price?

38. At the Westwood Market a 432 g can of kidney beans cost 56¢ and a 765 g can cost 95¢. Which is the better buy?

Linear Equations and Inequalities in One Variable

Solving linear inequalities is similar to solving linear equations, with one exception. When multiplying or dividing an inequality by a negative number, **ALWAYS** reverse the direction of the inequality symbol. Solve the following inequalities and graph the solution set on a number line.

39. $a + \frac{42}{5} > 15$

40. $-4x > 8$

41. $\frac{3y}{4} < -6$

Solve the following inequalities.

42. $3x + 2 \leq 7x - 10$

43. $7a + \frac{1}{2} > 3a + \frac{2}{3}$

44. $0.9y - 0.63 \leq 0.4y + 2.70$

45. $2(z + 4) \geq 3z - 7$

46. $\left(\frac{1}{2}\right)(a - 3) < \left(\frac{3}{2}\right)(2a + 6)$

47. $-5(3x - 4) - (7x + 1) > 19$

48. $\frac{1}{3}(x + 2) \leq \frac{1}{2}(x + 1)$

49. $7 - (3y + 5) > 3 - (y + 8)$

50. $\frac{x}{4} \geq -7$

51. $17 - 3m > 5$

Linear Equations and Inequalities in Two Variables

A solution to a linear equation in two variables is an ordered pair (x, y) that makes the equation a true statement.

1. Determine which of the ordered pairs are solutions to $y = \frac{1}{2}x + 1$.

   $(2,1)$ $(0,1)$ $(-4, -1)$ $(1,2)$

Complete each ordered pair so that it is a solution to the given equation.

2. $x - 4 = 2y$; $(0,\_\_)$ $(\_\_,-1)$ $(-3,\_\_)$

3. $2x + y = 3$; $(-1,\_\_)$ $\left(\_\_,\frac{1}{2}\right)$ $(1.7,\_\_)$

The x-intercept is the point where the graph crosses the x-axis and is written $(x, 0)$. The **y-intercept** is the point where the graph crosses the y-axis, and is written $(0, y)$. Find the x and y-intercepts of each of the following linear equations.

	x-intercept	y-intercept
4. $x - 3y = 6$	________	________
5. $x = -2$	________	________
6. $\frac{x}{2} + y = 1$	________	________
7. $y = 4$	________	________
8. $3x - 4y = 12$	________	________
$y = x$	________	________

Graph the following lines using the x and y-intercepts.

10. $x = 3$ 11. $\frac{x}{2} + y = 1$ 12. $3x - 4y = 12$

Slope is a measure of the slant or steepness of the line. Slope is defined to be the "rise" of the line divided by the "run" of the line. Slope is defined mathematically as

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where} \quad (x_1, y_1) \quad \text{and} \quad (x_2, y_2) \quad \text{are points on the line.}$$

Find the slope of the line containing each pair of points.

13. $(4, -3)$ and $(-1, -3)$ 14. $(7, -2)$ and $(-3, 2)$

The slope and y-intercept can be found by manipulating the equation of the line into the form $y = mx + b$, called the **slope-intercept form of a linear equation**, where m is the slope and b is the y-intercept. Find the slope and y-intercept of each equation.

15. $y = \frac{1}{3}x + 2$ 16. $y = -3x$

17. $y + 3 = 0$

18. $3x + 4y = 8$

Use the slope and y-intercept to graph each line.

19. $y = -3x$

20. $2x - y = 4$

21. $5x = 3y + 12$

Summary of Techniques for Finding an Equation of a Line		
When given:	Use:	Or Use:
A. slope and y-intercept	$y = mx + b$	
B. slope and any point	$y - y_1 = m(x - x_1)$	$y = mx + b$
C. two points	$m = \frac{y_2 - y_1}{x_2 - x_1}$ and $y - y_1 = m(x - x_1)$	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
D. a parallel line and a point	slopes of parallel lines are equal (then see B)	
E. a perpendicular line and a point	slopes of perpendicular lines have a product of -1 (then see B)	

Linear Equations and Inequalities in Two Variables

There are two common forms used to write equations of lines. One is the slope-intercept form, and the second is **standard form** which is written $Ax+By=C$. Find an equation of each line described below. Give each answer in slope-intercept form and again in standard form with integer coefficients.

22. the line containing $(1,4)$ and $(-2,-5)$

23. the line with slope of 0 and containing $(0,-2)$

24. the line perpendicular to $y=2x+3$ and containing $(2,5)$

25. the line parallel to $x+y=1$ and containing $(2,2)$

26. the line with slope of $\frac{3}{10}$ and containing $(4,3)$

27. the line containing $(3,-1)$ and $(-3,-5)$

A solution to a linear inequality is an ordered pair that makes the inequality true.

28. Determine which of the ordered pairs are solutions to $x - 3y \geq 1$.

$(4,1)$ $(2,1)$ $(1,0)$

The following guidelines may be useful when graphing the solutions of a linear inequality in two variables.

- Determine the boundary line. Temporarily replace the inequality symbol with an equal sign and graph the resulting linear equation. If the inequality is $\leq$ or $\geq$ draw the line as a solid line. If the inequality is $<$ or $>$ draw the line as a dashed line.
- Test a point to determine which half-plane contains points that are solutions to the inequality. Shade in the correct half-plane.

Graph each inequality.

29. $y > \frac{1}{4}x - 2$

30. $2x - 3y < 6$

31. $y + 5 \leq 3$

Systems of Equations and Inequalities

A solution of a system of equations (or inequalities) is an ordered pair or triple that makes each equation (or inequality) in the system true.

1. Determine which of the ordered pairs are solutions to the system $\begin{cases} 3x - 5y = 9 \\ 2x + 4y = -5 \end{cases}$

$(0, -1.8)$ $\quad$ $(-5, 9)$ $\quad$ $\left(\frac{1}{2}, \frac{-3}{2}\right)$ $\quad$ $(1.1, -1.8)$

A system of linear equations may have zero, one, or many solutions depending on whether the graphs of the equations in the system intersect in zero points (parallel lines), one point (intersecting lines), or many points (collinear lines).

The following guidelines may be useful when solving a system of linear equations.

- The graphing method requires graphing both lines on the same axes to determine any point(s) of intersection. It is difficult and time consuming to do this accurately enough to determine the solution.
- The substitution method involves solving one equation for one of the variables, substituting this result into the second equation, then solving for the remaining variable. This is especially useful when one of the variables has a coefficient of 1.
- The addition (or elimination) method is the process of adding the equations together in such a way that one of the variables is eliminated. It may be necessary to multiply one or both equations by a constant before a variable can be eliminated.
- Cramer's rule is a method which uses determinants. For a review of determinants and Cramer's rule, consult an algebra textbook.

Solve each of the following systems of equations.

2. $2x + 4y = 14$
   $y = x + 4$

3. $-6x + y = 5$
   $4x - y = -1$

4. $4x - 2y = -7$
$-3x + 7y = 8$

5. $-3x + 6y = 15$
$x - 2y = -5$

6. $\frac{1}{4}x - y = 4$
$2x + y = 12$

7. $2x + 3y = 71$
$4x - y = 9$

8. $5x + 2y = 3$
$-10x - 4y = 6$

9. $2x - 3y = 9$
$5x - 7y = 12$

10. $x + 3 = 0$
$2x - 5y = -16$

11. $9x + 4y = -45$
$x + \frac{2}{3}y = -5$

12. $11x + 8y = -338$
$-13x - 9y = 389$

13. $0.67x + 0.3y = -0.425$
$0.33x + 1.5y = 5.425$

14. $x + y + z = 5$
$x - y + 2z = 11$
$x + 2y - z = -4$

15. $4x - y - z = 4$
$2x + y + z = -1$
$6x - 3y - 2z = 3$

16. The sum of two numbers is 180. The first number is 18 more than 5 times the second number. Find the numbers.

17. The perimeter of a garden is 145 *feet*. If the length is 5 *feet* more than the width, find the length and width of the garden.

18. There were 394 people in a theater. The total receipts were $1674. The admission price for adults is $4.50 and the admission price for children is $3.75. How many adults and how many children were in the theater?

19. A plane can fly to another airport 550 *miles* away in 2 *hours* with a tail wind. The return trip takes 3 *hours* because of the head wind. How hard is the wind blowing? (Round the answer to the nearest mile per hour.)

20. The total number deer, elk, and bison in a certain area is 4000. In a recent hunt, 6% of the bison were killed, 14% of the elk were killed and 18% of the deer were killed. The Fish, Wildlife and Parks department counted 588 dead animals at the check stations. If there are three times as many deer as bison, how many of each animal were living in the area before the hunt?

To solve a system of inequalities, graph each of the inequalities in the system on the same axes. The area of intersection illustrates the solution of the system. Solve the following systems of inequalities by graphing.

21. $x - 2y > 4$
$y < 5x - 1$

22. $y \geq 2x - 3$
$y < -3x + 2$

23. $x < -2y + 6$
$2x > 6y + 9$

Polynomials

The **definitions and laws of exponents** (where a and b are real numbers and n and m are integers) are listed below.

- $a^0 = 1$
- $a^1 = a$
- $a^m a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $a^{-n} = \dfrac{1}{a^n}$
- $\dfrac{1}{a^{-n}} = a^n$
- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Find each value of n.

1. $x^n x^7 = x^{11}$
2. $\dfrac{a^8}{a^n} = a^3$
3. $(y^5)^n = y^{-20}$
4. $x^n = 1$
5. $\dfrac{1}{t^n} = t^{-5}$
6. $\left(\dfrac{r}{t}\right)^n = \dfrac{t}{r}$
7. $r^6 r^n = r$
8. $(2x^4)^n = 8x^{12}$
9. $\dfrac{w}{w^n} = w^9$

Simplify each of the following.

10. $x^3 x^4$
11. $(p^5)^2$
12. $y^{-10} y^3$
13. $\dfrac{b^5}{b}$
14. h^0
15. $\dfrac{x^{-7}}{x^3}$
16. $(x^3 y^5)^6$
17. $(9z^{12})^3$
18. $\left(\dfrac{x^5}{y^2}\right)^7$
19. $(3r^3 t^8)(5rt^{-5})$
20. $\left(\dfrac{x}{z}\right)^{-1}$
21. $\dfrac{r^2 s^3 t^4}{r s^{-1} t^5}$

22. $(a^3b^{-6})^2(ab^2)^{-3}$

23. $\left(\frac{xy}{7yz}\right)^{-4}$

24. $\frac{(8cd^5)^4}{(4c^3d)^6}$

25. $(9m^5n^{-3})^0$

26. $\frac{(5x^{-9}y^8)^{-2}}{(4xy)^{-5}}$

27. $(12a^{-7}b^5)^2(6a^2b^3)^{-10}$

In order for a number to be in **scientific notation**, it must be written in the form

$a \times 10^n$ where $1 \leq a < 10$ and n is any integer.

Compute each of the following and write the answer in scientific notation.

28. $8(2.3 \times 10^7)$

29. $\frac{5.44 \times 10^4}{3.4 \times 10^7}$

30. $(4.66 \times 10^7)(2.9 \times 10^{-13})$

31. $\frac{1.12 \times 10^{-3}}{5.6}$

32. Pluto is 4.644×10^9 *miles* from earth. Write this number in standard notation.

33. If 1 *mole* of water contains 6.023×10^{23} *molecules*, how many molecules are in 100 *moles* of water?

Polynomials

Evaluate each polynomial if $x = \frac{-1}{3}$, $y = 4$, and $z = -6$.

34. $y + z^2$

35. $3x + z - y^2 - z^3$

36. $18x^2 + y^5 + 7z$

37. $\frac{3}{2}xy^2z$

38. $x^3 - 4y$

39. $y^3 - 8y^2 + y - 12$

40. The polynomial $L = 0.5n^2 - 0.5n$ can be used to compute the number of lines that can be drawn through n points. How many lines can be drawn through 12 points?

The following guidelines may be useful when working with polynomials.

- Add or subtract polynomials by combining like terms.
- Multiply monomials by multiplying the numerical coefficients together, and using the laws of exponents for the variables.
- Multiply binomials by using the distributive property, or the FOIL method. Recall FOIL stands for "First - Outer - Inner - Last".
- Multiply larger polynomials using the distributive property.
- Remember, $(x+y)^2 = (x+y)(x+y)$, which is **NOT** the same as $x^2 + y^2$.
- Divide a polynomial by a monomial using fractions. That is, write $\frac{a+b}{c}$ as $\frac{a}{c} + \frac{b}{c}$, and simplify.
- Divide a polynomial by a binomial using a process similar to long division of numbers.

Perform the indicated operation:

41. $(12y^2 - 9y - 15) + (22y^2 + y - 7)$

42. $(8r^2s^5)(-3rs^7)$

43. $(25p^3 - 50p^2 + 15p) \div (5p)$

44. $(z^2 + z + 1)(z - 1)$

45. $(m^3n + 5mn - mn^2 - 3m^2n^2) + (6m^2n^2 - mn + 4mn^2 - m^3n)$

46. $(b^2 + 2b - 1)^2$

47. $(2u^3 - 5u^2 + u + 3) \div (2u + 1)$

48. $(8x^3y^2 - x^2y^3 - 12xy) \div (4xy)$

49. $(2s - 1)(3s - 5)$

50. $(0.2r^2s + 3.1rs - 1.4rs^2) - (-1.9r^2s - 4.2rs + 0.8rs^2)$

51. $\left(\frac{1}{3}m^5n^3\right)\left(\frac{2}{5}mn^2\right)$

52. $(4z+5)(4z-5)$

53. $(y^2-3y+7) \ - \ (2y^2+5y+8) \ - \ (-5y^2-y+2)$

54. $(8a^4-a) \div (2a-1)$

55. $(1+2y)^2$

56. $(x^2+3)(2x^2-7)$

57. $(2m^2-5m-1)(3m^2+m+2)$

58. $(10x^2-6x-1) \ + \ (4x^2+x+11) \ - \ (x^2-5x-9) \ + \ (2x^2+3x-7)$

Basic Factoring

To **factor** an expression means to write it as a product which is equivalent to the original expression. An expression is **completely factored** when none of the factors can be factored any further.

The following guidelines may be useful when factoring.

- Always check for a common factor (a factor which occurs in every term).
- Determine the number of terms.
- If there are two terms, check for a **difference of two squares** which factors as $A^2 - B^2 = (A - B)(A + B)$.
- Remember: the sum of two squares cannot be factored, except for a common factor.
- If there are three terms, a trinomial, try the **trial and error method** to determine the factors. Check the factoring by re-multiplying using the distributive property, or the FOIL method.
- Make sure all factors are completely factored.

Factor completely.

1. $5n^2 + 55n$

2. $a^2 - 11a + 24$

3. $c^2 - 225$

4. $m^2n^2 - 7mn + 12$

5. $4x^2 + 49$

6. $3y^4 - 13y^3 + 12y^2$

7. $36y^3 - 16y$

8. $2t^3 + 28t^2 + 96t$

9. $t^8 - 256$

10. $x^3 - x^2y - 6xy^2$

11. $42 - 49a + 7a^2$

12. $27r^3 - 48rt^2$

13. $m^2 + 8mn + 12n^2$

14. $6c^2 - 7c - 10$

15. $ab^2 - 9ab + 20a$

16. $8x^2 - 30xy + 7y^2$

17. $-20x^2 + 2xy + 6y^2$

18. $\frac{1}{64}m^2 - \frac{1}{9}n^2$

19. $4a^2 - 28ab + 48b^2$

20. $(x-4)^2 - (y+2)^2$

Factoring is a method of solving equations with degree greater than one. In general, set one side of the equation equal to zero, factor the polynomial side of the equation and use the **zero product principle**. The zero product principle states that if a and b are real numbers, and $a \cdot b = 0$. then $a = 0$ or $b = 0$. Solve each of the following.

21. $2t^2 - 14t = 0$

22. $x^2 - 4x - 5 = 0$

23. $b^2 - 49 = 0$

24. $5t^2 + 9t = 2$

25. $c^3 = 7c^2 + 44c$

26. $4y^2 = 81$

27. $a^2 - ab - 6b^2 = 0$ for a

28. $(2x - 3)(x + 4) = -9$

29. $4m^3 - 3m^2 = m$

30. $y(3y + 13) = 10$

31. The square of a number minus twice the number is 48. Find the number.

32. The product of two consecutive integers is 132. Find the integers.

33. The length of a rectangular courtyard is 4 times the width. If the area of the courtyard is 324 *square feet*, find the dimensions of the courtyard.

34. The area of a triangle is 42 *square inches*. If the base is 5 *inches* more than the height, find the base and height of the triangle.

Rational Expressions and Equations

In a rational expression, restrictions on the variable(s) are necessary in order to prevent division by zero. To determine the necessary restrictions, set the denominator equal to zero and solve. State which values of the variable must be excluded in the following.

1. $\dfrac{2a+9}{(a+5)(a-17)}$

2. $\dfrac{r+5}{r^2+2r-15}$

3. $\dfrac{4m-1}{3m^2-6m}$

4. $\dfrac{8ab^2}{2a^2b+10ab}$

5. $\dfrac{5s}{2s^2-11s-6}$

6. $\dfrac{3b-9}{9b^2-16}$

A rational expression is **simplified** if the numerator and denominator have no common factors. The following guidelines may be useful when performing operations with rational expressions.

- When simplifying rational expressions, completely factor both the numerator and denominator, then cancel all factors common to both.
- Multiplication does not require a common denominator. Completely factor each polynomial, multiply numerators together, multiply denominators together, then simplify.
- Division involves multiplying by the reciprocal of the divisor, then simplifying.
- Addition or subtraction requires a common denominator. Write each expression with a common denominator, then write the sum or difference of the numerators over the common denominator and simplify.

Perform the indicated operation. Simplify as much as possible. (Assume that all values of the variable that cause division by zero are excluded.)

7. $\dfrac{k-4}{k^2+5k+6} \cdot \dfrac{k^2+8k+12}{k^2-10k+24}$

8. $\dfrac{12z^2+24z}{z^2+4z+4} - \dfrac{5z+10}{z^2+4z+4}$

9. $\dfrac{6}{h} + \dfrac{5}{h-3}$

10. $\dfrac{4wz+5}{10wz} - \dfrac{6wz-7}{-10wz}$

11. $\dfrac{t-2}{t^2-1} \cdot \dfrac{2-2t}{t^2-t-2}$

12. $\dfrac{3mn-6m}{5n^2} \div \dfrac{6m^2n-12m^2}{5n}$

13. $\dfrac{12r^2-14rs-6s^2}{3r^2-2rs-s^2} \cdot \dfrac{4r^2-4s^2}{24r-36s}$

14. $\dfrac{2m+1}{3m^2-7m+2} + \dfrac{1}{m-2}$

15. $\dfrac{2z}{z+2} - \dfrac{2z^2+2z+1}{z^2+3z+2}$

16. $\dfrac{2x-1}{x^2-4} - \dfrac{3}{x+2} + \dfrac{1}{2-x}$

17. $\dfrac{x^2-1}{x^2y^3} \div \dfrac{x^2+2x+1}{x^2y^2}$

18. $\dfrac{a^2+2ab+b^2}{b^2-a^2} \cdot \dfrac{a^2-b^2}{b^2+ba}$

19. $\dfrac{c}{c^2+5c+6} - \dfrac{2}{c^2+3c+2}$

20. $\dfrac{1}{x}\left(\dfrac{1}{a+x} - \dfrac{1}{a}\right)$

21. $\dfrac{3x^2-2x-1}{x^2-4} \cdot \dfrac{3x^2-12x+12}{3x^2+7x+2}$

22. $\dfrac{a^2+2a-3}{3a^2+24a+48} \div \dfrac{2a^2-2}{3a^2-48}$

The following guidelines may be useful when solving rational equations.

- Factor all denominators and determine the restrictions on the variable.
- Determine the least common denominator (LCD).
- Eliminate the fractions by multiplying both sides of the equation by the LCD.
- Solve the resulting equation.
- Compare the answer(s) to the restrictions. This step is essential.

Solve the following rational equations.

23. $\dfrac{3s^2}{s-4} = \dfrac{27}{s-4}$

24. $\dfrac{2}{x-3} = \dfrac{3}{x+3}$

25. $\dfrac{1}{v-4} = \dfrac{3}{v+4} - \dfrac{2}{v^2-16}$

26. $2 + \dfrac{7}{x} = \dfrac{4}{x^2}$

27. $\dfrac{r+3}{r} - \dfrac{r+4}{r+5} = \dfrac{15}{r^2+5r}$

28. $\dfrac{22}{2k^2-9k-5} - \dfrac{3}{2k+1} = \dfrac{2}{k-5}$

29. $\dfrac{7}{u-5} = \dfrac{u^2-10}{u^2-u-20} - 1$

30. $\dfrac{5}{z-4} - \dfrac{3}{z-1} = \dfrac{z+11}{z^2-5z+4}$

31. $\frac{1}{a} = b - \frac{1}{c}$ for c

32. $\frac{1}{a} = b - \frac{1}{c}$ for a

33. $\frac{1}{W} = \frac{1}{L} + \frac{1}{X}$ for L

34. $R = \frac{2p}{p+r}$ for p

35. $r = \frac{m}{a(b+c)}$ for b

36. $R = \frac{gs}{g-s}$ for g

A **ratio** is the quotient of two quantities. Setting two ratios equal to each other creates a **proportion**. If the proportion contains a variable, it is a rational equation. Solve the following problems involving ratio and proportion.

37. The ratio of how much a certain business spends on equipment to how much it earns is 1:8. How much does that certain business earn if it spends $23,549 on equipment?

38. The formula for diluting a certain chemical with water is 9 parts of water to 4 parts chemical. How much water should be added to dilute 16 *cc* of chemical?

The following guidelines may be useful when working with variation.

- When y **varies directly** as x, the equation of variation is $y = kx$ where the constant of variation is k (some positive number).
- When y **varies inversely** as x, the equation of variation is $y = \frac{k}{x}$ where the constant of variation is k (some positive number).

Find the constant of variation and an equation of variation for each of the following.

39. b varies directly as a and $b = 18$ when $a = 3$.

40. x varies inversely as z and $x = 0.3$ when $z = 9$.

Solve the following variation problems.

41. If s varies directly as r and $s = \frac{1}{2}$ when $r = \frac{1}{6}$, find s when $r = 2$.

42. If t varies inversely as m and $t = \frac{2}{3}$ when $m = 3$, find t when $m = \frac{1}{4}$.

43. The wavelength w of a wave varies inversely as the frequency f. A wave with a frequency of 600 *kilohertz* has a length of 550 *meters*. What is the wavelength of a wave with a frequency of 800 *kilohertz*?

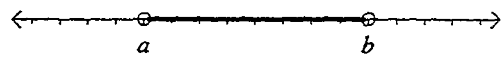
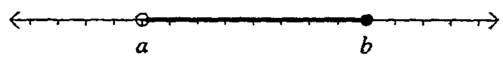
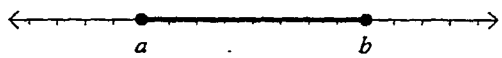
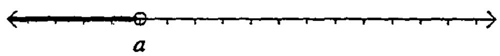
44. The amount of money earned on a job varies directly as the number of hours worked. If Ray earned $63.75 for 15 *hours* of work, how many hours does he have to work to earn $85.00?

COLLEGE ALGEBRA MATERIAL

The problems included in pages 33 – 50 are generally discussed in a College Algebra course. The College Algebra course at Texas Tech University is Math 1320. Mastering this material is required in order to meet the prerequisites for entering Math 1321 (Trigonometry).

Expanding the Basics

A table illustrating the relationship between inequality notation, a number line graph and interval notation is given below.

Inequality Notation	Number Line Graph	Interval Notation
$a < x < b$		(a, b)
$a \leq x < b$		$[a, b)$
$a < x \leq b$		$(a, b]$
$a \leq x \leq b$		$[a, b]$
$x > a$		(a, ∞)
$x < a$		$(-\infty, a)$
$x \geq a$		$[a, \infty)$
$x \leq a$		$(-\infty, a]$

The **intersection** ($\cap$) of two sets is a set consisting of all the elements that are in both sets. Mathematically the word AND means intersection. The **union** ($\cup$) of two sets is a set containing all the elements of the two sets. Mathematically the word OR means union. Consequently, the expression $a < x$ AND $x < b$ is equivalent to $a < x \cap x < b$. Similarly $a < x$ OR $x > b$ is equivalent to $a < x \cup x > b$. Solve the following inequalities, write the solution using interval notation, and graph the solution set on a number line.

1. $x \leq -3$

2. $x > 5$

3. $-1 \leq x \leq 6$

4. $x < 4$ OR $x > 7$

5. $x \leq -3$ AND $x > -6$

Expanding the Basics

Describe each of the following sets using inequality notation.

6. $[-1, \infty)$

7. $(-\pi, 3]$

8. $(-\infty, 0) \cup (1, 4)$

9. $(-4, 2] \cap [0, 4]$

The following guidelines may be useful when working with absolute values.

- $|x - a| = b$ is equivalent to $x - a = b$ OR $x - a = -b$
- $|x - a| > b$ is equivalent to $x - a > b$ OR $x - a < -b$
- $|x - a| < b$ is equivalent to $x - a < b$ AND $x - a > -b$ which can also be written as $-b < x - a < b$

Solve the following absolute value equations and inequalities.

10. $|x + 15| = 22$

11. $2|a - 3| = -6$

12. $|2m - 6| \geq 5$

13. $|3 - 5y| < 1$

14. $|19 - 14n| - 5 > 4$

15. $|8x - 16| - 17 \leq 15$

16. $\left|\frac{1}{9}(t + 1)\right| < 9$

17. $\left|\frac{-1}{2}(z - 3)\right| \geq 4$

Conjunctions are combined inequalities such as $a < x < b$. When a variable occurs in more than one part, it is sometimes necessary to split conjunctions apart as $a < x$ AND $x < b$ in order to solve them. Solve the following.

18. $-5 < 2a - 1 < 3$

19. $-27 \leq 7c - 6 < c - 12$

20. $7 < 2 - 3x < 26$

21. $-2x + 3 \leq 6x + 1 \leq 7$

The following expanded list of guidelines may be useful when completely factoring polynomials.

- Factor out the greatest common factor.
- When factoring a binomial, check for:
  - the **difference of squares**, which factors as
$$A^2 - B^2 = (A + B)(A - B)$$
  - the **sum or difference of cubes**, which factors as
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$
$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$
- When factoring a trinomial, check for two binomial factors.
- When factoring a polynomial with 4 terms, try **factoring by grouping**.
  - Either form two groups of 2 terms each and look for a common factor, or
  - form a group of 3 terms as a perfect square trinomial, then look for a difference of squares.
- Check to see if any factor with more than one term can be factored further. This is necessary to ensure the polynomial is completely factored.

Completely factor each of the following.

22. $8 - t^3$

23. $64r^3 + 27$

24. $a^6 + 0.34a^3 + 0.0289$

25. $3(r-1)^2 - 28(r-1) - 20$

26. $18a^{-2} - 7a^{-1} - 1$

27. $4a^{12} - 4b^6$

28. $9x^2 + 12x + 4 - 9y^2$

29. $y^2 - x^2 + 10x - 25$

30. $1 - 64t^{-2}$

31. $(x^2 - y)^3 + y^3$

32. $(m^3 + 2m^2)(m^2 - 6m + 5) + (2m^2 - 2m)(2m^2 + m - 6)$

Expanding on Rationals

This section revisits rational expressions and incorporates the additional factoring from the previous section. State restrictions on the variables and simplify.

1. $\dfrac{8a^3-1}{8a^2+2a-3}$

2. $\dfrac{r^2-t^2}{r^2-rt+5r-5t}$

3. $\dfrac{2h^2+7h+6}{3h^3+2h^2-12h-8}$

Simplify the following. Assume that all values of the variable which cause division by zero have been excluded.

4. $\dfrac{x^a y^{a+b}}{(3xy)^a}$

5. $(x^{-1}+y^{-1})^{-1}$

6. $\dfrac{7a^2+18a+8}{3a^3-24}-\dfrac{a+8}{3a^2+6a+12}$

7. $\dfrac{t^{-1}+3t^{-2}}{t^2+7t+12}$

8. $\dfrac{c^3+27}{4c+12}\div\dfrac{c^2-3c+9}{12c^2-8}$

9. $\dfrac{1-2t^{-1}}{1-4t^{-2}}$

10. $\dfrac{-2}{9-(b+2)^2} - \dfrac{3}{2b^2+11b+5}$

11. $\dfrac{r^2+4r+4-s^2}{2r^2-rs} \cdot \dfrac{4r^2-s^2}{2r+2s+4}$

12. $\dfrac{\dfrac{t^2}{3s} - \dfrac{s}{3}}{\dfrac{t}{2s} + \dfrac{1}{2}}$

13. $\dfrac{8h^2-18k^2}{h^4-k^4} \div \dfrac{2h^2+2hk+2k^2}{h^3-k^3}$

14. $x^{-1} - \dfrac{1}{x^2+x} \div \dfrac{2x+1}{2x^2+3x+1}$

15. $\dfrac{2}{3} + \dfrac{4x^2+12x+9-y^2}{x^2+xy-3x-3y} \cdot \dfrac{x+y}{6x+9-3y}$

16. $\dfrac{\dfrac{2}{k-2}+1}{\dfrac{2}{k+2}-1}$

17. $(2y-1)(y^2-9)^{-1} - 3(y+3)^{-1} + (3-y)^{-1}$

18. $\dfrac{1-m}{6m-2m^2}-\dfrac{16m-20}{16m^2-2m^3-30m}+\dfrac{m+1}{8m-m^2-15}$

19. $\dfrac{4r^2-t^2}{r^2-t^2-2t-1}\div\dfrac{2r^2+5rt-3t^2}{r^2t-rt^2-rt}\cdot\dfrac{r^2+6rt+9t^2}{2r^2t+rt^2}$

20. $\dfrac{4p+12}{p^3+p^2-25p-25}\div\dfrac{p^2+6p+9}{2p-10}+\dfrac{2p-6}{3p-1}\cdot\dfrac{3p^2-p}{p^3+p^2-9p-9}$

21. $(b^2+5b+6)\cdot\dfrac{b^2-4b+4}{b^2(b+3)-4(b+3)}\div(b^3-8)$

Solve the following rational equations.

22. $(a^2 - 2a + 1)^{-1} + (a - 1)^{-1} = 2$

23. $(r)^{-2} + (qt)^{-1} = 3t$ for q

24. $t - \dfrac{2t}{v+3} = 4$ for v

25. $\dfrac{f-1}{23f - 14f^2 - 3} = \dfrac{1}{3f - 2f^2} + \dfrac{3}{7f^2 - f}$

26. $\dfrac{-2}{4x^2 - 1} = \dfrac{3 - 2x}{8x^3 + 1} - \dfrac{4x}{(4x^2 - 1)(4x^2 - 2x + 1)}$

27. $\dfrac{10r}{2r^3 - r^2 - 8r + 4} + \dfrac{3r}{r^2 - 4} = \dfrac{3 - 2r}{2r^2 - 5r + 2}$

28. $\dfrac{ab(3 + c) + (3a + b)(ac + b)}{a^2b^2} = 5$ for c

29. The bath tub can fill in 8 *minutes* and drain in 12 *minutes.* If the drain is left open when the tap is turned on, how long will it be before the tub begins to overflow?

30. A small plane travels 240 *miles* with the wind in the same time it would take to fly 190 *miles* against the wind. If the wind has a constant rate of 15 *miles per hour*, what is the speed of the plane in still air?

31. Lynn travelled 60 *km* on her bicycle in 2 *hours*. She travelled at 35 *km per hour* except when going up a steep mountain pass where her speed dropped to 20 *km per hour*. How many *kilometers* did she travel at 35 *km per hour*? (Round to the nearest tenth.)

32. A landscape architect is making a scale drawing of a triangular piece of property using similar triangles. The shortest side of the property is 47.9 *meters* and the longest side is 93.4 *meters*. Find the shortest side of the scale drawing to the nearest tenth if the longest side of that triangle is 13.5 *cm*. (Similar triangles have exactly the same angle measures, and corresponding sides are proportional.)

Expanding on Rationals

When solving a rational inequality do not multiply both sides by an expression containing a variable. Since the variable could represent zero, a positive number, or a negative number, it is unknown which direction the inequality should go. The following guidelines may be useful when solving rational inequalities.

- Add or subtract to get one side of the inequality equal to zero.
- Write the nonzero side of the inequality as a single rational expression.
- If the rational expression is to be positive (greater than zero), the solution is the interval(s) where the numerator and denominator have the same sign.
- If the rational expression is to be negative (less than zero), the solution is the interval(s) where the numerator and denominator have opposites signs.

Solve the following inequalities.

33. $\frac{12-x}{9} < 0$

34. $\frac{-7}{4m-5} > 0$

35. $\frac{2x-9}{-3} \geq 4$

36. $\frac{3a+7}{a+4} \geq 0$

37. $\frac{1}{9-4y} \leq 0$

38. $\frac{6}{3p+1} > -3$

39. $\frac{x-3}{x+5} > 8$

40. $\frac{3-2b}{4b+1} < 0$

41. $\frac{w+3}{w-1} \leq \frac{1}{2}$

Roots and Radicals

The **nth root** of an expression, a, can be written as $\sqrt[n]{a}$ where n is the **index**, $\sqrt{\ }$ is the **radical** sign, and a is the **radicand**. When writing square roots the index is usually omitted.

If the index is an even number, then the radical represents an **even root**. When finding the even root of an expression the radical sign always indicates the principle root, or the positive root. The negative root is indicated by $-\sqrt{\ }$.

Determine the following. Use a calculator when needed. If the answer is irrational, round it to the nearest hundredth.

1. $\sqrt{8}$
2. $-\sqrt{0.0001}$
3. $\sqrt{\frac{196}{400}}$
4. $\sqrt{-81}$
5. $\sqrt[6]{117649}$
6. $-\sqrt[4]{-20736}$
7. $\sqrt[3]{-274.625}$
8. $\sqrt[9]{0.1256}$
9. $\sqrt[3]{\frac{27}{125}}$

When variables are present in the radicand of even roots, absolute value symbols may be necessary to ensure that the principle root is positive. For example, $\sqrt{x^2} = |x|$.

If the index is an odd number, then the radical represents an **odd root**. An odd root has only one real number value and absolute value symbols are not used. For example, $\sqrt[3]{x^3} = x$.

In problems 10-63, all radicals represent real numbers.

Roots and Radicals

Simplify the following radical expressions and round irrational answers to the nearest hundredth. Absolute value signs may be needed in the answer.

10. $\sqrt{1.44x^2}$

11. $\sqrt{57a^2}$

12. $\sqrt{(a+b)^2}$

13. $\sqrt{(xy)^2}$

14. $\sqrt[3]{a^9}$

15. $\sqrt[4]{x^8y^{12}}$

16. $\sqrt{(4x-5)^2}$

17. $-\sqrt[3]{\dfrac{8a^9b^{15}}{a^6b^9}}$

18. $\dfrac{\sqrt[5]{a^{25}}}{\sqrt[3]{a^{15}}}$

A **rational exponent** is an exponent of the form $\frac{m}{n}$. If a is a real number, m and n are integers and $n > 0$, then

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m \qquad \text{or} \qquad a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

Write the following expressions using rational exponents and simplify.

19. $\sqrt[4]{cd^3}$

20. $\sqrt{3ab}$

21. $\sqrt[6]{\dfrac{x^{12}y^{24}}{729}}$

22. $\sqrt[5]{\dfrac{a^{22}b^7c^{10}}{32a^7b^2}}$

23. $\dfrac{\sqrt{5x}}{x}$

24. $\dfrac{\sqrt[3]{8a}}{\sqrt{ab}}$

Write the following expressions using a single radical.

25. $a^{\frac{2}{3}}$

26. $(2xy)^{\frac{3}{4}}$

27. $a^{\frac{2}{3}}b^{\frac{3}{4}}$

28. $r^{\frac{1}{4}}s^{\frac{1}{5}}t^{\frac{1}{6}}$

29. $\dfrac{a^{\frac{7}{12}}}{b^{\frac{-1}{6}}}$

30. $\dfrac{z^{\frac{4}{5}}}{x^{\frac{1}{3}}}$

The following guidelines may be useful when simplifying radical expressions.

- Remove all factors which are perfect powers from the radicand.
- Do not leave a product or quotient of radicals.
- Use only positive integer exponents.
- Cancel all factors common to the numerator and denominator.

Perform the indicated operations and simplify the following radical expressions.

31. $\sqrt{4x^4y^2}$

32. $\sqrt[4]{(3xy)^4}$

33. $\sqrt[5]{(a-b)^8}$

34. $\sqrt{5a^2+60a+180}$

35. $\sqrt[4]{\dfrac{64x^6y^{-7}z^9}{4x^{-2}yz^{-3}}}$

36. $\sqrt[3]{8x}\,\sqrt[3]{3x}$

37. $\sqrt[3]{x^3y^2}\,\sqrt[3]{xy^5}$

38. $\dfrac{\sqrt{196x^3}}{\sqrt{2x^2}}$

39. $\sqrt[4]{a^6b^4}$

40. $\sqrt{5mn^7}\sqrt{80m^4n}$

41. $\dfrac{\sqrt{720x^9y^7}}{\sqrt{5xy^3}}$

42. $\sqrt[5]{36}\,\sqrt[5]{12}\,\sqrt[5]{18}$

43. $\dfrac{\sqrt{600}}{\sqrt{8}}$

44. $2\sqrt{5}-3\sqrt{20}-4\sqrt{45}$

45. $\dfrac{\sqrt[3]{p^3-27}}{\sqrt[3]{p-3}}$

46. $3\sqrt{2a}-\dfrac{\sqrt{8a}}{3}-\sqrt{72a}$

47. $\sqrt[3]{56x^2}\,\sqrt[3]{49x^2}$

48. $\sqrt[3]{ab}\,\sqrt[6]{ab^2}$

49. $\sqrt{a}(\sqrt{16a^3}-\sqrt{25a})$

50. $(\sqrt{2}+\sqrt{3})(\sqrt{2}+4\sqrt{3})$

51. $(2\sqrt{k}-3)^2$

Roots and Radicals

When solving radical equations, it is often necessary to use the **principle of powers.** The principle of powers states that if x and y are real numbers, n is an integer, and $x = y$ then $x^n = y^n$. The following guidelines may be useful when solving radical equations.

- Isolate the radical.
- Use the principle of powers to eliminate the radical.
- Solve the resulting equation.
- Always check the answers because using the principle of powers may result in extraneous answers.

Solve the following equations.

52. $\sqrt{b+3} = b+1$

53. $\sqrt{4x+9} = 9-x$

54. $\sqrt{2x+3} = \sqrt{3x-5}$

55. $\sqrt{3z+12} = z-2$

56. $\sqrt{x^2+2x+9} = x+3$

57. $\sqrt[3]{a+1}+1=0$

58. $\sqrt{y+4}+\sqrt{y+7} = 3$

59. $m^3 = 54n$ for m

60. $A = \sqrt{\dfrac{AB^2}{C}}$ for B

61. $x^2+(y+5)^2 = 25$ for y

A drinking cup is in the shape of a right circular cone. If the height is twice the radius, what is the height when the volume is 7 in^3? Round the answer to the nearest hundredth. The formula for the volume of a right circular cone is $V = \frac{\pi r^2 h}{3}$.

63. Use the formula for the distance between two points, $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, and the Pythagorean Theorem, $a^2 + b^2 = c^2$, to determine if a triangle with vertices that occur at the points $A(-3, 1)$, $B(1, 3)$, and $C(0, 5)$ is a right triangle.

The **complex numbers** are a set of numbers that use the **imaginary unit**, i. The imaginary unit is defined as $i = \sqrt{-1}$ or equivalently $i^2 = -1$. The **conjugate** of a complex number $a + bi$ is defined as $a - bi$. The following guidelines may be useful when performing operations using complex numbers.

- Add or subtract corresponding real and imaginary parts.
- Multiply using the distributive property or the FOIL method.
- Divide by multiplying the numerator and denominator by the conjugate of the denominator. Simplify.

Perform the indicated operation.

64. $(1 + i) + (3 - 4i)$

65. $(6 - 7i) - (8 - 9i)$

66. $\sqrt{-9}\sqrt{-4}$

67. $(-6i)(-4i)$

68. $2i(5 + 3i)$

69. $(4 + 3i)(4 - 3i)$

70. $(7 + 6i)^2$

71. $\dfrac{-7i}{-2 + 8i}$

72. $\dfrac{5 - 9i}{3 + 2i}$

73. $\sqrt{-529}$

74. i^{240}

75. i^{235}

Solving Non-Linear Equations and Inequalities

A **quadratic equation** is an equation of the form $Ax^2 + Bx + C = 0$, where $A \neq 0$. The following guidelines may be useful when solving a quadratic equation.

- If there is no linear term, i.e. if $B = 0$, the equation can be solved by isolating the x^2 term on one side and taking the square root of both sides.
- If there is a linear term present, factoring may work. Set one side of the equation equal to zero and try to factor the nonzero side. Use the principle of zero products.
- The **quadratic formula** can always be used to solve a quadratic equation. If the equation is in the form $Ax^2 + Bx + C = 0$, where $A \neq 0$, then

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

Some rational equations result in a quadratic equation when fractions are cleared. Other equations which are not quadratic, but are quadratic in form, can also be solved by the above techniques. Solve the following equations.

1. $c^2 = 121$

2. $8d^2 + 20d = 0$

3. $4s^2 - 3s - 5 = 0$

4. $9 + 12y = -4y^2$

5. $6b^2 + 20 = 0$

6. $2q^2 - 3q = \frac{1}{8}$

7. $\sqrt{11}q + q^2 = 0$

8. $6r^2 + 139r + 90 = 0$

9. $z^2 + \frac{5}{4}z - \frac{3}{2} = 0$

10. $\frac{x^2}{2} - 4x = -9$

11. $\frac{1}{y} - \frac{2y}{y+1} = 0$

12. $27p^4 + 3p^2 = 2$

13. $6q - 13\sqrt{q} = -6$

14. $15(3u^2 - 7)^2 + 11(3u^2 - 7) + 2 = 0$

15. $2x^3 + 9x^2 = 5x$

16. $\frac{12}{c^2 - 4} - \frac{c-1}{c^2 + 5c + 6} = \frac{18}{c^2 + c - 6}$

17. $A = \frac{4}{3}\pi r^3$ for r

18. $\frac{l}{x} = \frac{x}{s}$ for x

19. $h = -16t^2 + 10t$ for t

20. $.3t^2 = s^2 - q$ for s

21. The time required to travel from Bozeman to Butte, a distance of 180 *kilometers*, was 1 *hour* more than the time required to make the return trip. The rate going to Butte was 15 *kilometers per hour* slower than the rate returning. Find the rate on the return trip.

22. A picnic area, 40 *meters* wide by 60 *meters* long is to be doubled in area by extending each side an equal amount. By how much should each side be extended?

The following guidelines may be useful when solving quadratic inequalities.

- Add or subtract to get one side of the inequality equal to zero.
- Factor the nonzero side.
- If the expression is to be positive, the solution is the interval(s) where the factors are all positive OR there is an even number of negative factors.
- If the expression is to be negative, the solution is the interval(s) where there is an odd number of negative factors.

Solve the following inequalities.

23. $2x^2 - 5x - 3 < 0$

24. $(7 - 2x)(x + 4) > 0$

25. $a^2 \geq 25a$

26. $25 \geq z^2$

27. $2(4n^2 + 3n) > -1$

28. $x^3 + 7x^2 - 8x \leq 0$

PRE-CALCULUS MATERIAL

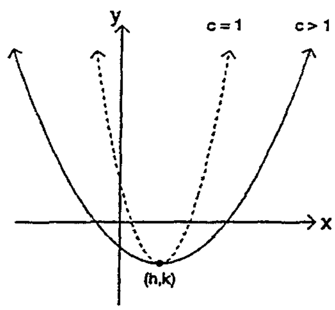
The problems included in pages 52 – 73 are generally discussed in pre-calculus courses (i.e, prerequisites for Math 1351). The section on Trigonometry contains prerequisite material for Math 1350 (Analytical Geometry).

Conic Sections

The **conic sections** and the **standard forms** of their equations are shown below. A great deal can be determined about the graph when the equations are written in standard form. The figures below illustrate this relationship.

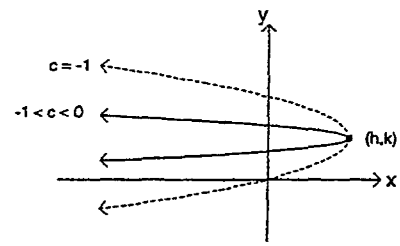
Vertical Parabola

$$c(y-k)=(x-h)^2$$



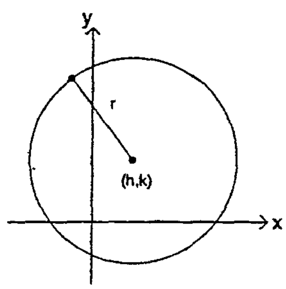
Horizontal Parabola

$$c(x-h)=(y-k)^2$$



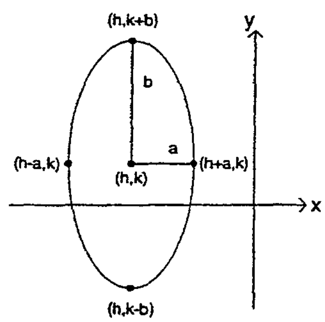
Circle

$$(x-h)^2+(y-k)^2=r^2$$



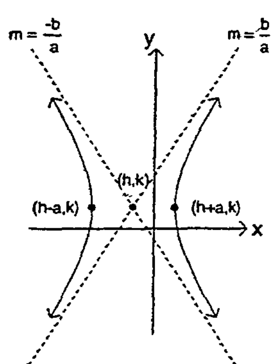
Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



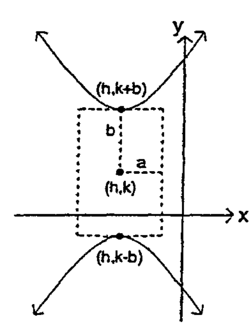
Horizontal Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$



Vertical Hyperbola

$$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$$



Sketch the graphs of the following conic sections.

1. $(x-2)^2 = y+4$

2. $(x-1)^2 + (y-7)^2 = 4$

3. $\dfrac{(x+2)^2}{9} + (y-4)^2 = 1$

4. $\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

5. $x+4 = 2(y-6)^2$

6. $\dfrac{(y-2)^2}{9} + \dfrac{(x-3)^2}{16} = 1$

7. $(y+3)^2+(x-4)^2=25$

8. $-(y-4)=(x+3)^2$

Complete the square to write the following equations in standard form.

9. $9x^2-18x+25y^2-250y+409=0$

10. $x^2+y^2-8x+16y+55=0$

11. $3631+896x-54y=9y^2-64x^2$

12. $2y^2-x-28y+104=0$

13. $126x-16y-4y^2-117=9x^2+16$

14. $0=10x-1-6y-x^2$

15. Write the equation in standard form for the horizontal parabola with vertex at the point $(-2, -9)$ which passes through the point $(5, -2)$.

16. Give the equation in standard form for the conic section with a radius of 5 and a center at the point $(-2, 5)$.

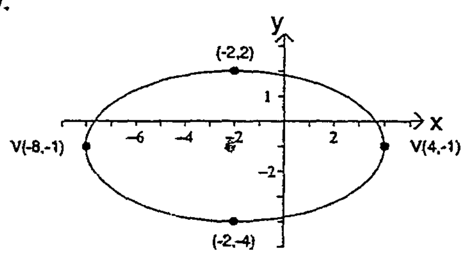
17. What is the length of the minor axis for the vertical ellipse with a vertex at $(-2, 5)$ and an endpoint of the minor axis at $(2, 0)$.

18. Find the vertices of the conic section given by $\dfrac{(x-4)^2}{25} - \dfrac{(y+3)^2}{4} = 1$.

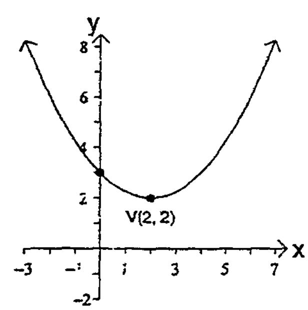
$\sqrt{29}$

Write the equation in standard form for the conic section represented by the graph.

19.



20.



Functions and Graphs

A **function** is a set of ordered pairs, (x, y), which assigns to every x-value one and only one y-value. The set of all x-values is called the **domain** of the function, and the set of all y-values is called the **range** of the function.

Functions are usually denoted by lower case letters such as f, g, h, etc. Given a function f, and an element of the domain, x, an element of the range is denoted by $f(x)$. Determine the domain and range for each of the following functions.

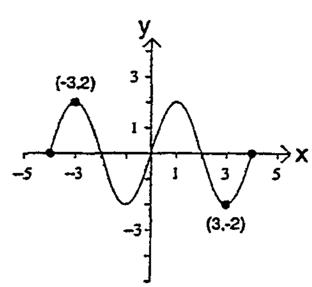
1. $f(x) = 1 - x^2$
2. $y = \dfrac{2}{x-1}$
3. $g(p) = \dfrac{2p-5}{p+1}$
4. $y = |x| - 2$
5. $f(x) = \dfrac{1}{x^2 + 6x + 9}$
6. $h(x) = \sqrt{\dfrac{2}{x+5}}$

A function is said to be **positive** over the interval (a, b) if $f(x) > 0$ for all x in (a, b). Likewise a function is **negative** over the interval (a, b) if $f(x) < 0$ for all x in (a, b).

A function is **increasing** over the interval (a, b) if the graph rises as it progresses from left to right. A function is **decreasing** over the interval (a, b) if the graph falls as it progresses from left to right.

Using the graph, determine the domain and range of the represented functions. Also determine intervals where the function is increasing, decreasing, positive and negative.

7.



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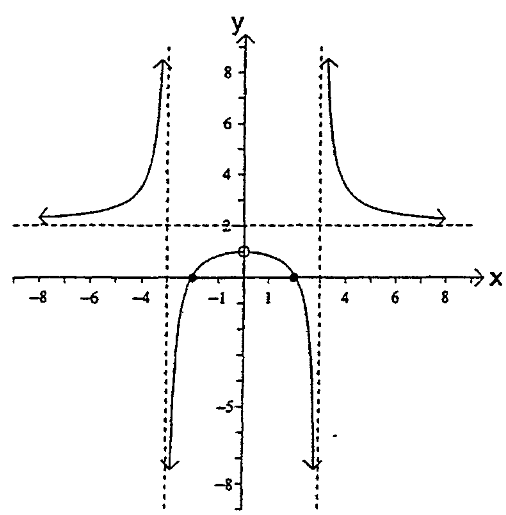


Given $f(x) = 3x^2 - 5$, $g(r) = 3r - 7$ and $h(t) = \sqrt{1-t}$, find the following.

9. $f(a)$

10. $h(0)$

11. $g(-5)$

12. $f(x+a)$

13. $h(3p+2)$

14. $\dfrac{f(x+a) - f(x)}{a}$

15. $\dfrac{h(x+p)}{g(3)}$

16. $\dfrac{f(x) - f(a)}{x-a}$

17. $g(x+1)$

The **composition of functions** f and g is denoted by $(f \circ g)(x)$ or $(g \circ f)(x)$. These expressions are defined as $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$. Given $f(x) = 3x^2 - 5$, $g(r) = 3r - 7$ and $h(t) = \sqrt{1-t}$, find the following.

18. $(h \circ g)(-3)$

19. $(g \circ h)(-3)$

20. $(h \circ f)(x)$

21. $(f \circ h)(t-3)$

Two functions f and g are **inverse functions** if $(f \circ g)(x) = f(g(x)) = x$ for all x in the domain of g, AND $(g \circ f)(x) = g(f(x)) = x$ for all x in the domain of f. The following guidelines may be useful when algebraically determining the inverse of a function.

- If a function is given using function notation like $f(x)$, replace the function notation with y.
- Solve this equation for x (the domain element).
- Interchange the variables x and y.
- Replace y with a new name for the inverse function. If the original function was given as $f(x)$, the inverse function is labeled $f^{-1}(x)$.

Determine the equation which represents the inverse for each of the given functions.

22. $a(x) = \dfrac{3x+5}{2} - 7$

23. $b(x) = \sqrt{\dfrac{3x-1}{2}}$, for $x \geq \dfrac{1}{3}$

24. $c(x) = (x-1)^2$, for $x \geq 1$

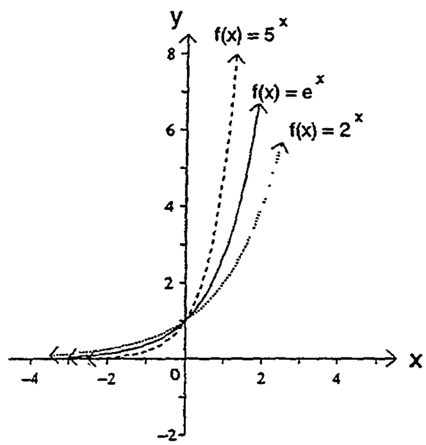
25. $d(x) = (x+7)^3 - 1$

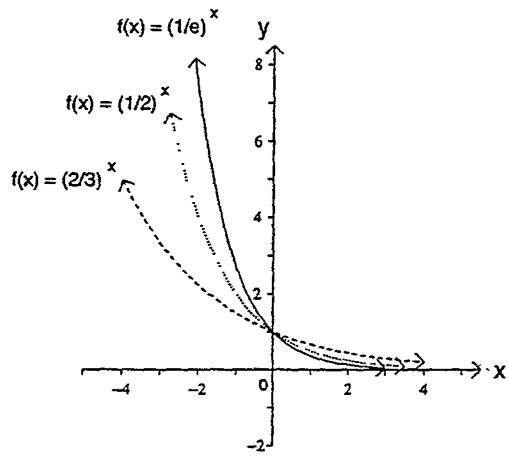
26. $f(x) = \dfrac{4x+9}{x-3}$, for $x \neq 3$

27. $g(x) = \sqrt{2-x}$, for ≤ 2

Exponential and Logarithm Functions

When a function contains a variable in the exponent, it is called an **exponential function.** The exponential function with **base** a is defined as $f(x) = a^x$ where $a > 0$ and $a \neq 1$. When the base of an exponential function is $e \approx 2.71828$, the function is called the **natural exponential function.** The graphs below illustrate the general shape of the exponential function with various bases.





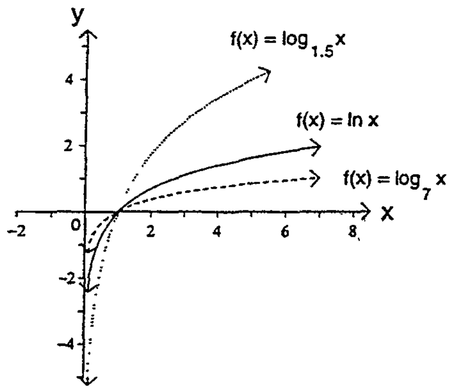
Sketch the graph of the following.

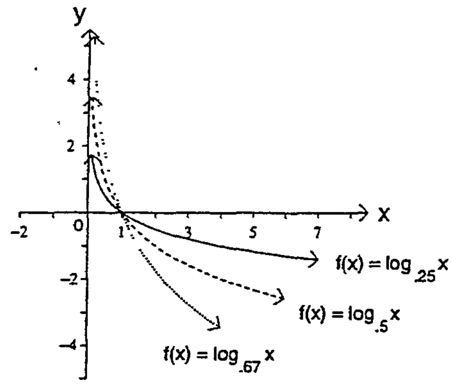
1. $f(x) = \left(\frac{\pi}{2}\right)^x$
2. $f(x) = 10^x$
3. $f(x) = e^{-x}$

A **logarithm function** with **base** a is the inverse of the exponential function with base a, and is written $f(x) = \log_a x$ where $a > 0$, $a \neq 1$ and $x > 0$. The logarithm function with base 10 is called the **common logarithm** and is written $f(x) = \log x$. The logarithm function with base e is called the **natural logarithm** and is written $f(x) = \ln x$.

Exponential and Logarithm Functions

The graphs below illustrate the general shape of the logarithm function with various bases.





Sketch the graph of the following.

4. $f(x) = \log x$ 5. $f(x) = 2 + \log_2 x$ 6. $f(x) = \ln(x - 1)$

Since logarithms and exponentials with the same base are inverse functions, changing from one to the other utilizes the following fact. For $y > 0$, the **exponential form,** $y = a^x$, is equivalent to the **logarithmic form,** $x = \log_a y$. Write the following in logarithmic form.

7. $3^7 = 2187$ 8. $2^{-4} = \dfrac{1}{16}$

9. $\sqrt{49} = 7$ 10. $e^4 = 54.6$

Write the following in exponential form.

11. $\log 2.78 = .44$

12. $\log_2 9.73 = 3.28$

13. $\ln e = 1$

14. $\log_{25} 5 = \frac{1}{2}$

The **properties and laws of logarithms** (where N, M, and t are real numbers and $a,\ b > 0$ and $a,\ b \neq 1$) are listed below.

- $\log_a a = 1$
- $\log_a 1 = 0$
- $\log_a M = \frac{\log_b M}{\log_b a}$
- $\log_a N^t = t \log_a N$
- $\log_a(NM) = \log_a N + \log_a M$
- $\log_a \frac{N}{M} = \log_a N - \log_a M$

Use the properties of logarithms to expand the following.

15. $\log_a(x^2 y)$

16. $\log \sqrt{\frac{xy}{z}}$

17. $\ln(5\sqrt{A})$

18. $\ln \frac{(3x^2 y)^4}{\sqrt{z}}$

Write each of the following as a single logarithm.

19. $\log_a x + \log_a(2x)$

20. $\log \frac{x^2}{y} + \log(5x)$

21. $\ln(x^2 + x - 2) - \ln(x - 1)$

22. $4\log(2x) - (\log x + \log(3x^2))$

Expand the following, then write the answer using only natural logarithms.

23. $\log_2(4t^2)$

24. $\log \dfrac{1}{q}$

25. $\log \dfrac{(x-3)^2}{10}$

26. $\log_3[x^4(x+1)]$

The following guidelines may be useful when solving exponential or logarithmic equations. For real numbers r and t with $a,\ b > 0$ and $a,\ b \neq 0$:

- $a^r = a^t$ implies $r = t$
- $a^r = b^r$ implies $a = b$
- $\log_a r = \log_a t$ implies $r = t$ where $r,\ t > 0$

When solving logarithmic equations, it is sometimes helpful to change from logarithmic form to exponential form. Solve the following equations.

27. $216^x = 6$

28. $\left(\dfrac{3}{4}\right)^x = \dfrac{16}{9}$

29. $2^{3x-1} - 8 = 0$

30. $16^{4-x} = 2$

31. $2^{\frac{x}{3}} + \dfrac{7}{4} = 2$

32. $3^{x+1} = \dfrac{1}{\sqrt{27}}$

33. $\log_3(3x) = -2$

34. $\log_x 64 = 3$

35. $\log_8 x = \frac{1}{3}$

36. $\log(6x) - \log(x+1) = \log 3$

37. $\ln x + 4\ln 2 = \ln \frac{4}{x}$

38. $\log_3(2x+3) + \log_3(x-1) = 1$

Solve the following equations for the indicated variable.

39. $r = \log(8b)$ for b

40. $t = 5 - 2^{3a}$ for a

41. $q + 7 = e^{(r-5)}$ for r

42. $5x - \ln \frac{d}{7} = 3$ for d

The formula for continuously compounded interest is $A = Pe^{rt}$, where A is the future value, P is the amount invested, r is the interest rate, and t is the time in years. Use this formula to solve the following problems.

43. How much money needs to be deposited into an account today in order to have \$75,000 in 5 years if the interest is compounded continuously at a rate of 8.9%?

44. How long will it take for \$1500 to double if it is deposited into an account where the interest is compounded continuously at a rate of 6% per year? Round to the nearest tenth.

The formula for population growth is $P = P_0 e^{rt}$, where P is the projected population, P_0 is the present population, r is the rate of growth per year, and t is the number of years of growth. Use this formula in the following problems.

45. What will the population of a city be in 20 years if it has a population today of 235,000 and has an annual growth rate of 0.56%? What will the population be in 10 years? What was the population 5 years ago?

46. How long will it take a town to grow from 450 people to 1000 people if the annual growth rate is 2.34%? Round to the nearest year.

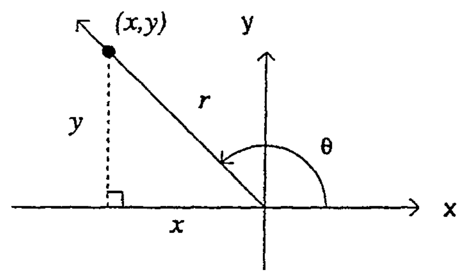
Trigonometry

The definitions of the **trigonometric functions** can be stated in two ways. One way is to locate an angle in standard position in the coordinate system and label a point, (x, y), on the terminal side of the angle. Use the Pythagorean Theorem to find $r = \sqrt{x^2 + y^2}$, then the trigonometric functions are defined as follows.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$\sec\theta = \frac{r}{x} \qquad \csc\theta = \frac{r}{y}$$

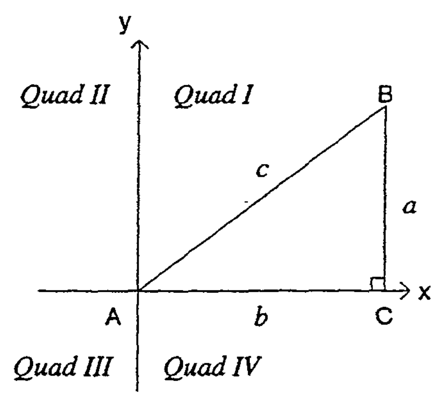


The trigonometric functions can also be defined in terms of the side of a right triangle which is "opposite" the angle, "adjacent" to the angle, and the "hypotenuse". This is illustrated using angle A from the triangle shown below in quadrant I.

$$\sin A = \frac{opposite}{hypotenuse} \qquad \cos A = \frac{adjacent}{hypotenuse}$$

$$\tan A = \frac{opposite}{adjacent} \qquad \cot A = \frac{adjacent}{opposite}$$

$$\sec A = \frac{hypotenuse}{adjacent} \qquad \csc A = \frac{hypotenuse}{opposite}$$



1. If $(4, -3)$ is a point on the terminal side of θ find $\sin\theta$, $\cos\theta$ and $\tan\theta$.

2. If $(-5,-12)$ is a point on the terminal side of θ, find $\sin\theta$, $\cos\theta$ and $\tan\theta$.

3. If $\sin\beta = \dfrac{12}{13}$ and $\cos\beta < 0$ find $\cos\beta$, $\tan\beta$, $\cot\beta$, $\sec\beta$ and $\csc\beta$.

4. If $\sin\alpha > 0$ and $\tan\alpha < 0$ then α lies in what quadrant?

An angle can be measured in **radians** as well as **degrees**. The relationship between radians and degrees is: 2π radians $= 360^o$. If an angle is written without a degree symbol, then it is assumed to be measured in radians. Convert the following.

5. $90^o =$ _____ radians

6. $135^o =$_____ radians

7. $240^o =$ _____ radians

8. $270^o =$_____ radians

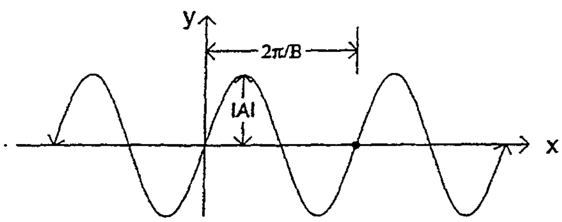
9. $\dfrac{5\pi}{4} =$ _____^{o}

10. $\dfrac{\pi}{6} =$ _____^{o}

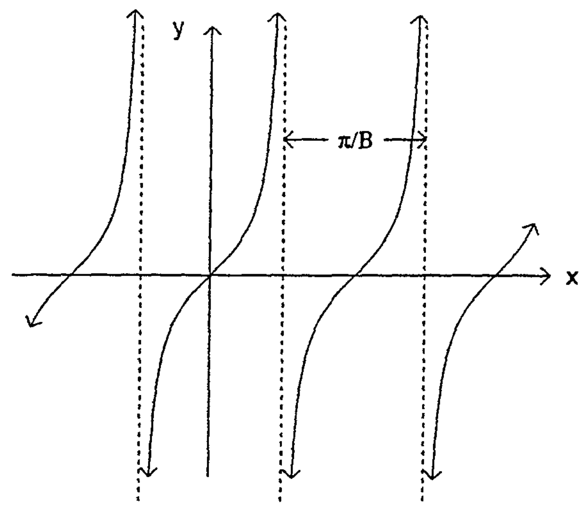
11. $\dfrac{7\pi}{3} =$ _____^{o}

12. $\dfrac{5\pi}{12} =$ _____^{o}

The general shapes of three of the trigonometric functions are shown below.



$y = A\sin Bx$



$y = A\tan Bx$

$y = A\cos Bx$

The **amplitude**, $|A|$, of the sine and cosine curves is the distance between the curve and the x-axis at the maximum or minimum points. Amplitude does not apply to the tangent curve. The **period** of the curve is the interval required before the curve begins to repeat. The period of the sine and cosine curves is $\frac{2\pi}{B}$. The period of the tangent curve is $\frac{\pi}{B}$. For each of the following state the amplitude (if it applies) and period, and then sketch the graph.

13. $y = 4\sin \pi x$

14. $y = -\tan \frac{x}{3}$

15. $y = 2\cos 2x$

The **inverse trigonometric functions** are defined as follows:

- $y = \arcsin x \iff \sin y = x$ where $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$
- $y = \arccos x \iff \cos y = x$ where $0 \leq y \leq \pi$
- $y = \arctan x \iff \tan y = x$ where $\frac{-\pi}{2} < y < \frac{\pi}{2}$

16. If $y = \arctan x$ and $x < 0$ then y lies in what quadrant?

17. If $y = \arccos x$ and $x < 0$ then y lies in what quadrant?

Find each of the following.

18. $\arcsin \frac{-1}{2}$

19. $\arctan(-1)$

20. $\arccos \frac{1}{2}$

21. $\arctan \sqrt{3}$

22. $\cos\left(\arcsin \frac{5}{13}\right)$

23. $\tan(\arccos x)$

24. $\sin\left(\arctan \frac{-4}{3}\right)$

25. $\cos(\arctan x)$

26. $\tan\left(\arcsin \frac{-7}{25}\right)$

Some of the basic **trigonometric identities** are given below:

$\csc x = \dfrac{1}{\sin x}$ $\qquad$ $\sec x = \dfrac{1}{\cos x}$ $\qquad$ $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1}{\cot x}$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\tan^2 x + 1 = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \csc^2 x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ $\qquad$ $\cos 2x = \cos^2 x - \sin^2 x$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ $\qquad$ $\sin 2x = 2 \sin x \cos x$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ $\qquad$ $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

Verify the following using the basic identities.

27. $\csc \alpha = \cot \alpha \ \sec \alpha$

28. $\sec \beta - \cos \beta = \tan \beta \ \sin \beta$

29. $(\tan \theta + \cot \theta) \sin^2 \theta = \tan \theta$

30. $\dfrac{1}{1 + \sin \phi} + \dfrac{1}{1 - \sin \phi} = 2 \sec^2 \phi$

31. If A is in Quadrant III with $\cos A = \frac{-5}{13}$, and B is in Quadrant II with $\cos B = \frac{-1}{\sqrt{5}}$, find the following.

a. $\cos(A + B)$ $\qquad$ b. $\sin(A - B)$ $\qquad$ c. $\tan(A - B)$

Find the following.

32. $\cos\left(\theta + \frac{3\pi}{2}\right)$

33. $\cos(2 \arcsin x)$

34. $\sin\left(\arccos\frac{3}{5} + \arctan\frac{-7}{24}\right)$

35. $\tan(45^o - \theta)$

Solving trigonometric equations involves familiar algebraic techniques such as the addition and multiplication principles, and factoring, as well as using identities to help simplify the equation. Solve for x where $0 \leq x < 2\pi$.

36. $\tan^2 x = \tan x$

37. $2\cos^2 x + 3\cos x - 2 = 0$

38. $\tan 2x = 1$

39. $\sin^2 x + \cos x + 1 = 0$

Solve for θ where $0^o \leq \theta < 360^o$.

40. $2\sin\theta\ \cos\theta - \sqrt{3}\cos\theta = 0$

41. $\cos 3\theta = 0$

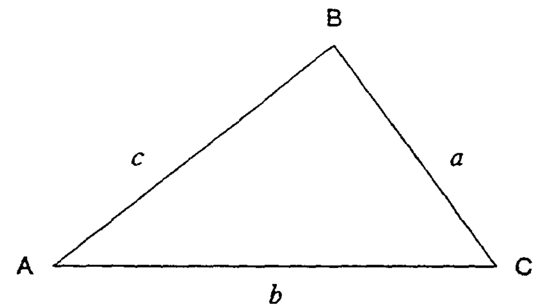
42. $5\sec\theta = 4 + 2\tan^2\theta$

43. $\sin 2\theta = \sin\theta$

Solving a triangle means to find the unknown side(s) and angle(s). If the triangle is a right triangle, then the Pythagorean Theorem or definitions of the trigonometric functions can be used. If the triangle is **oblique** (not a right triangle) then we must use either the law of sines or the law of cosines to solve the triangle.

Law of Sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of Cosines:
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Solve the following triangles.

44. $C = 90^o$, $a = 27$ and $A = 18^o$

45. $C = 90^o$, $b = 36$ and $c = 45$

46. $C = \dfrac{\pi}{2}$, $c = 19.23$ and $A = 0.401$

47. $C = \dfrac{\pi}{2}$, $a = 2305$ and $B = 0.5934$

48. $B = 52.5^o$, $C = 110.0^o$ and $a = 13.7$

49. $C = 26^o$, $c = 12$ and $A = 41^o$

50. $a = 110,\ b = 85$ and $c = 96$

51. $A = 112^o, b = 431$ and $c = 815$

52. When the angle of elevation of the sun is 75^o, the shadow cast by a tree is 13 ft. How tall is the tree?

53. A box weighing 1300 *pounds* is sitting on a plane inclined 12^o from the horizontal. What force is needed to move the box up the plane (disregarding friction)?

54. Two trains leave the station together. The tracks make an angle of 115^o with each other. If the first train is traveling at 60 mph, and the second train is traveling at 85 mph, how far apart are the trains (on a straight line) in 3 *hours*?

55. A force of 87 *pounds* and a force of 65 *pounds* meet at an angle of 105^o. Find the magnitude of the resultant (sum) force and the angle the resultant makes with the 87 *pound* force.

Use the following formulas to solve problems 56-59. In all cases, θ is the central angle measured in radians, r is the radius of the circle, and t is a unit of time.

Arc length, s: $$s = r\theta$$

Area of a sector, A: $$A = \frac{\theta r^2}{2}$$

Linear velocity, v: $$v = \frac{s}{t} \quad \text{or} \quad v = r\omega$$

Angular velocity, ω: $$\omega = \frac{\theta}{t} \quad \text{or} \quad \omega = \frac{v}{r}$$

Find each of the following to three significant digits.

56. Find the arc length subtended by a central angle of 140^o with radius of 1.6 *feet*.

57. Find the radius of a circle if a central angle of 20^o subtends an arc of 4000 *feet*.

58. Find the area of a circular sector with radius 3.4 *inches* and a central angle of 97^o.

59. Find the linear velocity of a point on the rim of a wheel if the wheel makes 2 *revolutions per minute* and has a radius of 25 *feet*.

ANSWERS

The answers for problems from all sections are included in pages 75 – 86.

ANSWERS

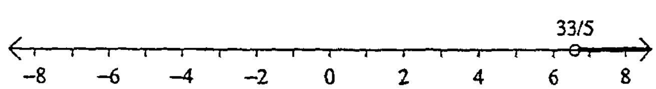
BASIC FACTS — pg. 1 - 4

1. $2 > -7$	2. $-100 < -87$	3. $\frac{2}{3} < \frac{3}{4}$	4. 5
5. $\frac{1}{7}$	6. 0	7. 0.375, 37.5%	8. $\frac{27}{10}$, 270%
9. $\frac{39}{1000}$, 0.039	10. 10.458	11. $0.14	12. -87
13. 326	14. $\frac{1}{12}$	15. -10.2	16. -301
17. undefined	18. -16	19. -46	20. $\frac{1}{2}$
21. 0.37	22. $\frac{4}{9}$	23. 39.0625	24. -12
25. 49	26. $\frac{35}{18}$	27. 10	28. 37
29. 27	30. 4	31. 16.1	32. -11.15
33. -22.25	34. $24\frac{1}{4}$ *ft.*	35. $5.88\ cm^2$	36. $\frac{-2}{9}x + \frac{8}{5}y$
37. $a^2 - 4$	38. $2 - 3y$	39. $\frac{10}{3}x - 5y$	40. $\frac{40}{3}m - 2 + \frac{48}{5}\lvert n\rvert$
41. $7.15r - 4.7t$			

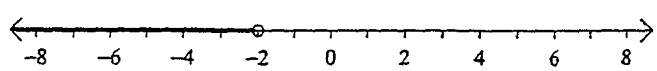
LINEAR EQUATIONS & INEQUALITIES IN 1 VARIABLE — pg. 5 - 8

1. 0.63	2. $x = 8$	3. $r = 4$	4. $s = 6$
5. $t = 13$	6. $x = 3$	7. $s = \frac{-5}{4}$	8. $z = 14$
9. $a = -4$	10. $a = 11$	11. $s = \frac{-1}{2}$	12. $a = -4$
13. $v = \frac{25}{16}$	14. $b = -8$	15. $t = 0.5$	16. $z = -6$
17. $x = 24y$	18. $y = 10$	19. $t = 2$	20. $s = 3$
21. $y = 4$	22. $x = \frac{3y-16}{4}$	23. $a = \frac{-3}{56}$	24. $z = \frac{1}{2}$
25. $b = \frac{-1}{2}$	26. $r = \frac{t-10s}{4}$	27. $t = 40$	28. $a = \frac{4c+3b}{3}$
29. $z = 1$	30. $t = \frac{-2}{3}$	31. 40 *cm*, 20 *cm*	32. $\frac{1}{3}$
33. $L = 20\ cm,\ W = 10\ cm$		34. $C = \frac{5}{9}(F - 32)$	35. 18%
36. 65%	37. $65.00	38. The 765 *g* can is a better buy.	

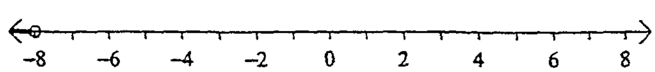
39. $a > \frac{33}{5}$



40. $x < -2$



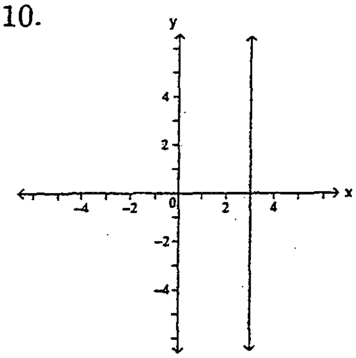
41. $y < -8$



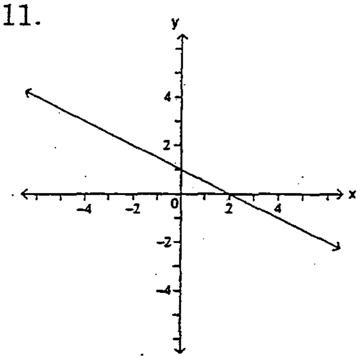
42. $x \geq 3$	43. $a > \frac{1}{24}$	44. $y \leq 6.66$	45. $z \leq 15$
46. $a > \frac{-21}{5}$	47. $x < 0$	48. $x \geq 1$	49. $y < \frac{7}{2}$
50. $x \geq -28$	51. $m < 4$		

LINEAR EQUATIONS & INEQUALITIES IN 2 VARIABLES — pg. 9 - 13

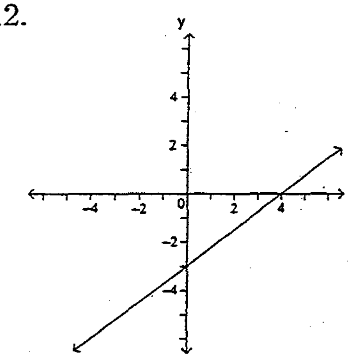
1. $(0,1)\ (-4,-1)$
2. $(0,-2)\ (2,-1)\ (-3,\frac{-7}{2})$
3. $(-1,5)\ (\frac{5}{4},\frac{1}{2})\ (1.7,-0.4)$
4. $(6,0)\ (0,-2)$
5. $(-2,0)$ none
6. $(2,0)\ (0,1)$
7. none $(0,4)$
8. $(4,0)\ (0,-3)$
9. $(0,0)\ (0,0)$
10.



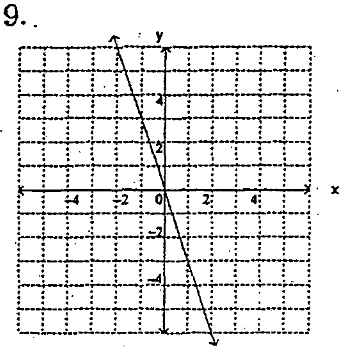
11.



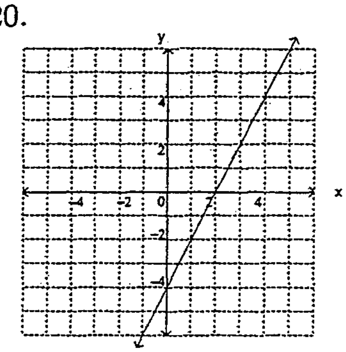
12.



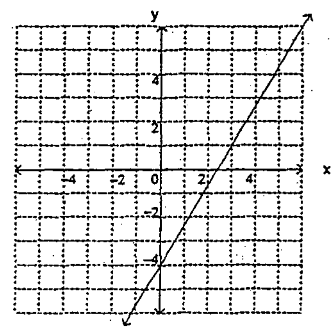
13. 0
14. $\frac{-2}{5}$
15. $m=\frac{1}{3},\ (0,2)$
16. $m=-3,\ (0,0)$
17. $m=0,\ (0,-3)$
18. $m=\frac{-3}{4},\ (0,2)$
19.



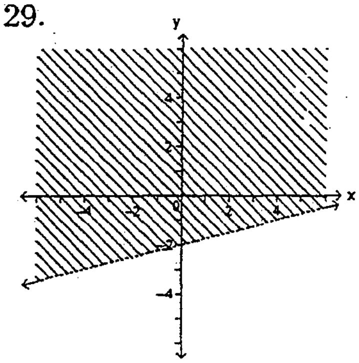
20.



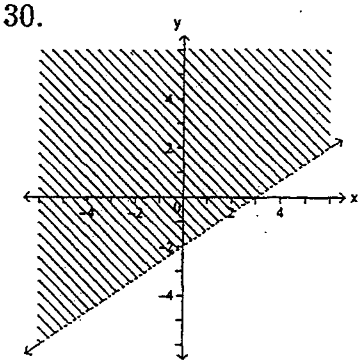
21.



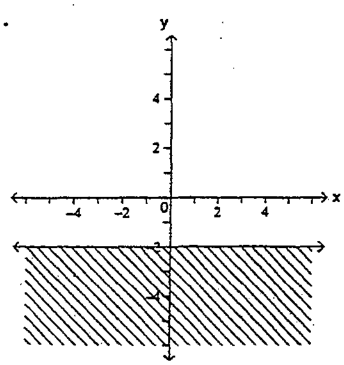
22. $y=3x+1$; $3x-y=-1$
23. $y=-2$; $y=-2$
24. $y=\frac{-1}{2}x+6$; $x+2y=12$
25. $y=-x+4$; $x+y=4$
26. $y=\frac{3}{10}x+\frac{9}{5}$; $-3x+10y=18$
27. $y=\frac{2}{3}x-3$; $2x-3y=9$
28. $(4,1)\ (1,0)$
29.



30.



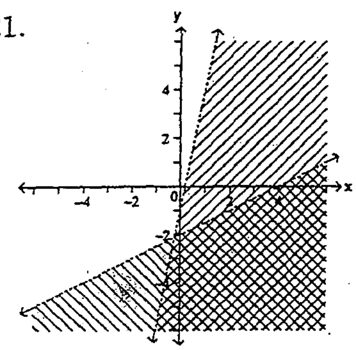
31.

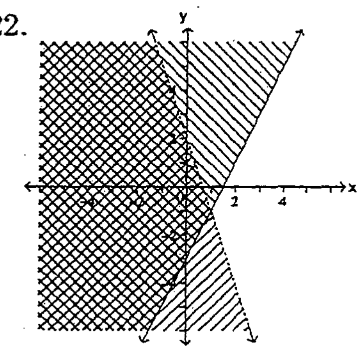


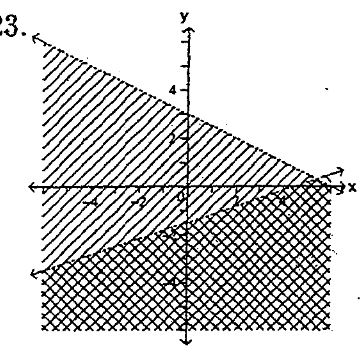
SYSTEMS OF EQUATIONS AND INEQUALITIES — pg. 14 - 17

1. $(\frac{1}{2}, \frac{-3}{2})$	2. $(\frac{-1}{3}, \frac{11}{3})$	3. $(-2, -7)$	4. $(\frac{-3}{2}, \frac{1}{2})$
5. many	6. $(\frac{64}{9}, \frac{-20}{9})$	7. $(7, 19)$	8. none
9. $(-27, -21)$	10. $(-3, 2)$	11. $(-5, 0)$	12. $(-14, -23)$
13. $(-2.5, 4.1\overline{6})$	14. $(2, -1, 4)$	15. $(\frac{1}{2}, 4, -6)$	

16. 1st number = 153, 2nd number = 27
17. length = 38.75 *ft.*, width = 33.75 *ft.*
18. 262 adults, 132 children
19. wind = 46 *mph*
20. There were 2100 deer, 1200 elk, and 700 bison.

21.


22.


23.


POLYNOMIALS — pg. 18 - 22

1. $n = 4$	2. $n = 5$	3. $n = -4$	4. $n = 0$
5. $n = 5$	6. $n = -1$	7. $n = -5$	8. $n = 3$
9. $n = -8$	10. x^7	11. p^{10}	12. y^{-7}
13. b^4	14. 1	15. x^{-10}	16. $x^{18}y^{30}$
17. $729z^{36}$	18. $\frac{x^{35}}{y^{14}}$	19. $15r^4t^3$	20. $\frac{z}{x}$
21. $\frac{rs^4}{t}$	22. a^3b^{-18}	23. $\frac{2401z^4}{x^4}$	24. $c^{-14}d^{14}$
25. 1	26. $\frac{1024x^{23}}{25y^{11}}$	27. $\frac{a^{-34}b^{-20}}{419904}$	28. 1.84×10^8
29. 1.6×10^{-3}	30. 1.3514×10^{-5}	31. 2×10^{-4}	32. 4644000000
33. 6.023×10^{25}	34. 40	35. 193	36. 984
37. 48	38. $\frac{-433}{27}$	39. -72	40. 66

41. $34y^2 - 8y - 22$	42. $-24r^3s^{12}$	43. $5p^2 - 10p + 3$
44. $z^3 - 1$		45. $4mn + 3mn^2 + 3m^2n^2$
46. $b^4 + 4b^3 + 2b^2 - 4b + 1$		47. $u^2 - 3u + 2 + \frac{1}{2u+1}$
48. $2x^2y - \frac{xy^2}{4} - 3$	49. $6s^2 - 13s + 5$	50. $2.1r^2s + 7.3rs - 2.2rs^2$
51. $\frac{2}{15}m^6n^5$	52. $16z^2 - 25$	53. $4y^2 - 7y - 3$
54. $4a^3 + 2a^2 + a$	55. $1 + 4y + 4y^2$	56. $2x^4 - x^2 - 21$
57. $6m^4 - 13m^3 - 4m^2 - 11m - 2$		58. $15x^2 + 3x + 12$

BASIC FACTORING — pg. 23 - 26

1. $5n(n+11)$
2. $(a-3)(a-8)$
3. $(c-15)(c+15)$
4. $(mn-3)(mn-4)$
5. not factorable
6. $y^2(3y-4)(y-3)$
7. $4y(3y+2)(3y-2)$
8. $2t(t+8)(t+6)$
9. $(t-2)(t+2)(t^2+4)(t^4+16)$
10. $x(x-3y)(x+2y)$
11. $7(6-a)(1-a)$
12. $3r(3r+4t)(3r-4t)$
13. $(m+6n)(m+2n)$
14. $(c-2)(6c+5)$
15. $a(b-4)(b-5)$
16. $(4x-y)(2x-7y)$
17. $-2(5x-3y)(2x+y)$
18. $(\frac{1}{8}m+\frac{1}{3}n)(\frac{1}{8}m-\frac{1}{3}n)$
19. $4(a-3b)(a-4b)$
20. $(x+y-2)(x-y-6)$
21. $t=0$ or $t=7$
22. $x=5$ or $x=-1$
23. $b=7$ or $b=-7$
24. $t=-2$ or $t=\frac{1}{5}$
25. $c=0$ or $c=11$ or $c=-4$
26. $y=\frac{9}{2}$ or $y=\frac{-9}{2}$
27. $a=3b$ or $a=-2b$
28. $x=\frac{1}{2}$ or $x=-3$
29. $m=\frac{-1}{4}$ or $m=0$ or $m=1$
30. $y=-5$ or $y=\frac{2}{3}$
31. 8 or -6
32. 11 and 12 or -11 and -12
33. $W=9\ ft.$, $L=36\ ft.$
34. $h=7\ in.$, $b=12\ in.$

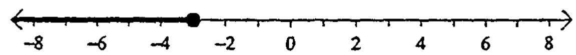
RATIONAL EXPRESSIONS AND EQUATIONS — pg. 27 - 31

1. $a\neq -5,\ a\neq 17$
2. $r\neq -5,\ r\neq 3$
3. $m\neq 0,\ m\neq 2$
4. $a\neq 0,\ b\neq 0,\ a\neq -5$
5. $s\neq \frac{-1}{2},\ s\neq 6$
6. $b\neq \frac{-4}{3},\ b\neq \frac{4}{3}$
7. $\frac{k+6}{(k+3)(k-6)}$
8. $\frac{12z-5}{z+2}$
9. $\frac{11h-18}{h(h-3)}$
10. $\frac{5wz-1}{5wz}$
11. $\frac{-2}{(t+1)^2}$
12. $\frac{1}{2mn}$
13. $\frac{2(r+s)}{3}$
14. $\frac{5m}{(3m-1)(m-2)}$
15. $\frac{-1}{z^2+3z+2}$
16. $\frac{-2x+3}{x^2-4}$
17. $\frac{x-1}{y(x+1)}$
18. $\frac{-a-b}{b}$
19. $\frac{c-3}{(c+3)(c+1)}$
20. $\frac{-1}{a(a+x)}$
21. $\frac{3(x-1)(x-2)}{(x+2)^2}$
22. $\frac{(a+3)(a-4)}{2(a+4)(a+1)}$
23. $s=\pm 3$
24. $x=15$
25. $v=9$
26. $x=\frac{1}{2},\ x=-4$
27. no solution
28. no solution
29. $u=-3$
30. no solution
31. $c=\frac{-a}{1-ab}$
32. $a=\frac{c}{bc-1}$
33. $L=\frac{WX}{X-W}$

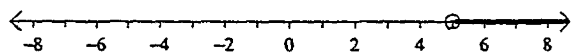
34. $p = \frac{-Rr}{R-2}$ 35. $b = \frac{m}{ar} - c$ 36. $g = \frac{Rs}{R-s}$
37. $188392 38. $36\ cm^3$ 39. $k = 6,\ b = 6a$
40. $k = 2.7,\ x = \frac{2.7}{z}$ 41. $s = 6$ 42. $t = 8$
43. 412.5 *meters* 44. 20 *hours*

EXPANDING THE BASICS — pg. 33 - 36

1. $(-\infty, -3]$

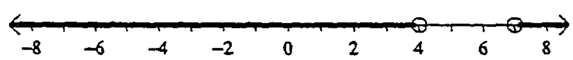


2. $(5, \infty)$

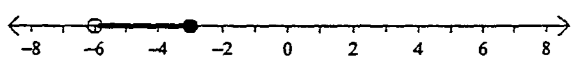


3. $[-1, 6]$

4. $(-\infty, 4) \cup (7, \infty)$



5. $(-6, -3]$



6. $a \geq -1$
7. $-\pi < m \leq 3$
8. $m < 0$ OR $1 < m < 4$
9. $0 \leq w \leq 2$
10. $x = 7,\ -37$
11. No solution
12. $(-\infty, \frac{1}{2}] \cup [\frac{11}{2}, \infty)$
13. $(\frac{2}{5}, \frac{4}{5})$
14. $(-\infty, \frac{5}{7}) \cup (2, \infty)$
15. $[-2, 6]$
16. $(-82, 80)$
17. $(-\infty, -5] \cup [11, \infty)$
18. $(-2, 2)$
19. $[-3, -1)$
20. $(-8, \frac{-5}{3})$
21. $[\frac{1}{4}, 1]$
22. $(2-t)(4+2t+t^2)$
23. $(4r+3)(16r^2-12r+9)$
24. $(a^3+0.17)^2$
25. $(3r-1)(r-11)$
26. $a^{-2}(9+a)(2-a)$
27. $4[(a^2-b)(a^4+a^2b+b^2)(a^2+b)(a^4-a^2b+b^2)]$
28. $(3x+2+3y)(3x+2-3y)$
29. $(y-x+5)(y+x-5)$
30. $t^{-2}(t-8)(t+8)$
31. $x^2(x^4-3x^2y+3y^2)$
32. $m(m-1)(m+2)^2(m-3)$

EXPANDING ON RATIONALS — pg. 37 - 42

1. $\frac{4a^2+2a+1}{4a+3},\ a \neq \frac{1}{2}, \frac{-3}{4}$
2. $\frac{r+t}{r+5},\ r \neq -5, t$
3. $\frac{2h+3}{(h-2)(3h+2)},\ h \neq \pm 2, \frac{-2}{3}$
4. $\frac{y^b}{3^a}$
5. $\frac{xy}{x+y}$
6. $\frac{2}{a-2}$
7. $\frac{1}{t^2(t+4)}$
8. $3c^2 - 2$
9. $\frac{t}{t+2}$
10. $\frac{-1}{(1-b)(2b+1)}$
11. $\frac{(r+2-s)(2r+s)}{2r}$
12. $\frac{2(t-s)}{3}$
13. $\frac{4h^2-9k^2}{(h^2+k^2)(h+k)}$
14. 0
15. $\frac{4x+y-3}{3(x-3)}$

16. $\frac{k+2}{2-k}$
17. $\frac{-2y+5}{(y-3)(y+3)}$
18. $\frac{-1}{2m}$
19. $\frac{r+3t}{r+t+1}$
20. $\frac{2(p+4)}{(p+3)(p+5)}$
21. $\frac{1}{b^2+2b+4}$
22. $a = \frac{1}{2}, 2$
23. $q = \frac{r^2}{3r^2t^2-t}$
24. $v = \frac{12-t}{t-4}$
25. $f = -2, 4$
26. no solution
27. $r = \frac{-3}{2}$
28. $c = \frac{5a^2b^2-6ab-b^2}{2ab+3a^2}$
29. The tub begins to overflow after 24 *min*.
30. The plane travels 129 *mph* in still air.
31. Lynn traveled 46.7 *km* at 35 *km per hour*.
32. The shortest side measures 6.9 *cm*.
33. $(12, \infty)$
34. $(-\infty, \frac{5}{4})$
35. $(-\infty, \frac{-3}{2}]$
36. $(-\infty, -4) \cup [\frac{-7}{3}, \infty)$
37. $(\frac{9}{4}, \infty)$
38. $(-\infty, -1) \cup (\frac{-1}{3}, \infty)$
39. $(\frac{-43}{7}, -5)$
40. $(-\infty, \frac{-1}{4}) \cup (\frac{3}{2}, \infty)$
41. $[-7, 1)$

ROOTS AND RADICALS — pg. 43 - 47

1. 2.83
2. -0.01
3. $\frac{7}{10}$
4. does not exist
5. 7
6. not defined
7. -6.5
8. 0.79
9. $\frac{3}{5}$
10. $1.2|x|$
11. $7.55|a|$
12. $|a+b|$
13. $|xy|$
14. a^3
15. $x^2|y^3|$
16. $|4x-5|$
17. $-2ab^2$
18. 1
19. $c^{\frac{1}{4}}d^{\frac{3}{4}}$
20. $(3ab)^{\frac{1}{2}}$
21. $\frac{x^2y^4}{3}$
22. $\frac{a^3bc^2}{2}$
23. $\frac{5^{\frac{1}{2}}}{x^{\frac{1}{2}}}$
24. $\frac{2}{a^{\frac{1}{6}}b^{\frac{1}{2}}}$
25. $\sqrt[3]{a^2}$
26. $\sqrt[4]{8x^3y^3}$
27. $\sqrt[12]{a^8b^9}$
28. $\sqrt[60]{r^{15}s^{12}t^{10}}$
29. $\sqrt[12]{a^7b^2}$
30. $\sqrt[15]{\frac{z^{12}}{x^5}}$
31. $2x^2|y|$
32. $3|xy|$
33. $(a-b)\sqrt[5]{(a-b)^3}$
34. $|a+6|\sqrt{5}$
35. $\frac{2x^2|z^3|}{y^2}$
36. $2\sqrt[3]{3x^2}$
37. $xy^2\sqrt[3]{xy}$
38. $7\sqrt{2x}$
39. $|ab|\sqrt{a}$
40. $20m^2n^4\sqrt{m}$
41. $12x^4y^2$
42. 6
43. $5\sqrt{3}$
44. $-16\sqrt{5}$
45. $\sqrt[3]{p^2+3p+9}$
46. $\frac{-11}{3}\sqrt{2a}$
47. $14x\sqrt[3]{x}$
48. $\sqrt[6]{a^3b^4}$
49. $4a^2-5a$
50. $14+5\sqrt{6}$
51. $4k-12\sqrt{k}+9$
52. $b = 1$
53. $x = 4$
54. $x = 8$
55. $z = 8$
56. $x = 0$
57. $a = -2$

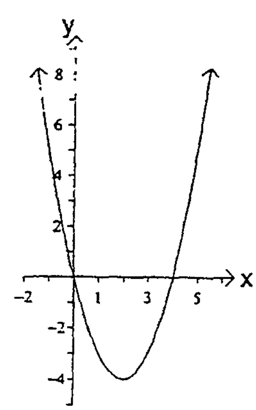
58. $y = -3$
59. $m = 3\sqrt[3]{2n}$
60. $B = \pm\sqrt{AC}$
61. $y = -5 \pm \sqrt{25 - x^2}$
62. The height is approximately 2.99 *in.*
63. $\overline{AB} = 2\sqrt{5}$, $\overline{BC} = \sqrt{5}$, $\overline{AC} = 5$, yes it is a right triangle.
64. $4 - 3i$
65. $-2 + 2i$
66. -6
67. -24
68. $-6 + 10i$
69. 25
70. $13 + 84i$
71. $\frac{-14}{17} + \frac{7}{34}i$
72. $\frac{-3}{13} - \frac{37}{13}i$
73. $23i$
74. 1
75. $-i$

SOLVING NON-LINEAR EQUATIONS AND INEQUALITIES — pg. 48 - 50

1. $c = \pm 11$
2. $d = \frac{-5}{2}, 0$
3. $s = \frac{3 \pm \sqrt{89}}{8}$
4. $y = \frac{-3}{2}$
5. $b = \pm i\sqrt{\frac{10}{3}}$
6. $q = \frac{3 \pm \sqrt{10}}{4}$
7. $q = -\sqrt{11}, 0$
8. $r = \frac{-45}{2}, \frac{-2}{3}$
9. $z = -2, \frac{3}{4}$
10. $x = 4 \pm i\sqrt{2}$
11. $y = \frac{-1}{2}, 1$
12. $p = \frac{\pm\sqrt{2}}{3}, \frac{\pm i}{\sqrt{3}}$
13. $q = \frac{4}{9}, \frac{9}{4}$
14. $u = \pm\sqrt{\frac{11}{5}}, \pm\frac{2\sqrt{5}}{3}$
15. $x = -5, 0, \frac{1}{2}$
16. $c = -1$
17. $r = \sqrt[3]{\frac{3A}{4\pi}}$
18. $x = \pm\sqrt{ls}$
19. $t = \frac{5 \pm \sqrt{25 - 16h}}{16}$
20. $s = \pm\sqrt{3t^2 + q}$
21. The return trip was made at a rate of 60 *kilometers per hour.*
22. Each side should be extended by 20 *meters.*
23. $(\frac{-1}{2}, 3)$
24. $(-4, \frac{7}{2})$
25. $(-\infty, 0] \cup [25, \infty)$
26. $[-5, 5]$
27. $(-\infty, \frac{-1}{2}) \cup (\frac{-1}{4}, \infty)$
28. $(-\infty, -8] \cup [0, 1]$

CONIC SECTIONS — pg. 52 - 55

1.



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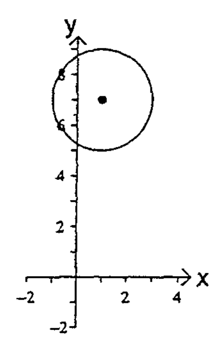
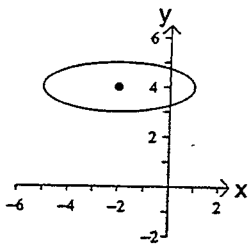


Answers

3.



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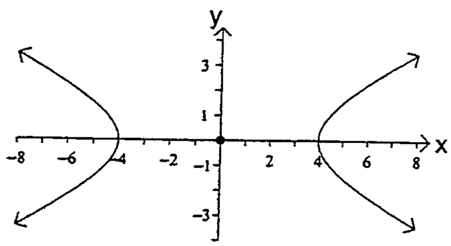

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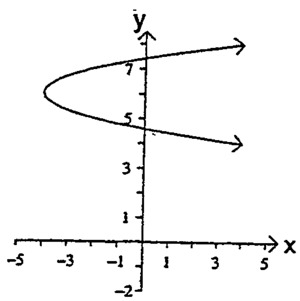

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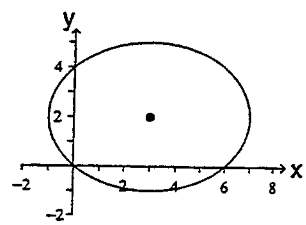
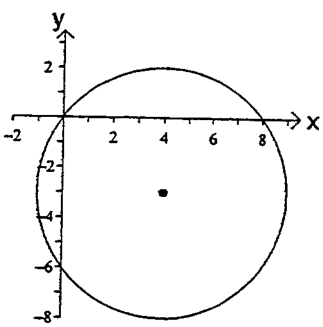


7.



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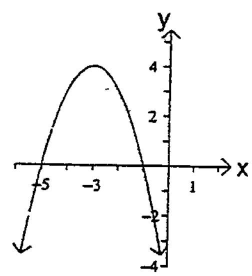


9. $\frac{(x-1)^2}{25} + \frac{(y-5)^2}{9} = 1$

10. $(x-4)^2 + (y+8)^2 = 25$

11. $\frac{(y+3)^2}{64} - \frac{(x+7)^2}{9} = 1$

12. $(y-7)^2 = \frac{1}{2}(x-6)$

13. $\frac{(x-7)^2}{36} + \frac{(y+2)^2}{81} = 1$

14. $-6(y-4) = (x-5)^2$

15. $(y+9)^2 = 7(x+2)$

16. $(x+2)^2 + (y-5)^2 = 25$

17. 8 units

18. $(-1,-3)$ and $(9,-3)$

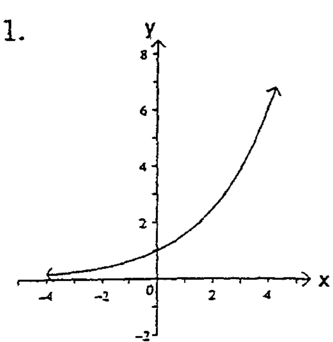
19. $\frac{(x+2)^2}{36} + \frac{(y+1)^2}{9} = 1$

20. $4(y-2) = (x-2)^2$

FUNCTIONS — pg. 56 - 58

1. $D = \{x | x \in \mathbb{R}\}$
   $R = \{y | y \in (-\infty, 1]\}$
2. $D = \{x | x \in \mathbb{R} \ \& \ x \neq 1\}$
   $R = \{y | y \in \mathbb{R} \ \& \ y \neq 0\}$
3. $D = \{p | p \in \mathbb{R} \ \& \ p \neq -1\}$
   $R = \{y | y \in \mathbb{R} \ \& \ y \neq 2\}$
4. $D = \{x | x \in \mathbb{R}\}$
   $R = \{y | y \in [-2, \infty)\}$
5. $D = \{x | x \in \mathbb{R} \ \& \ x \neq -3\}$
   $R = \{y | y > 0\}$
6. $D = \{x | x > -5\}$
   $R = \{y | y \in (0, \infty)\}$
7. $D = \{x | x \in [-4, 4]\}$
   $R = \{y | y \in [-2, 2]\}$
   Increasing:
   $x \in (-4, -3), (-1, 1) \ \& \ (3, 4)$
   Decreasing:
   $x \in (-3, -1) \ \& \ (1, 3)$
   Positive:
   $x \in (-4, -2) \ \& \ (0, 2)$
   Negative:
   $x \in (-2, 0) \ \& \ (2, 4)$
8. $D = \{x | x \in \mathbb{R} \ \& \ x \neq 0, \pm 3\}$
   $R = \{y | y \in (-\infty, 1) \text{ or } y \in (2, \infty)\}$
   Increasing:
   $x \in (-\infty, -3) \ \& \ (-3, 0)$
   Decreasing:
   $x \in (0, 3) \ \& \ (3, \infty)$
   Positive:
   $x \in (-\infty, -3), (-2, 0), (0, 2) \ \& \ (3, \infty)$
   Negative:
   $x \in (-3, -2) \ \& \ (2, 3)$
9. $f(a) = 3a^2 - 5$
10. $h(0) = 1$
11. $g(-5) = -22$
12. $f(x + a) = 3x^2 + 6xa + 3a^2 - 5$
13. $h(3p + 2) = \sqrt{-3p - 1}$
14. $\frac{f(x+a) - f(x)}{a} = 6x + 3a$
15. $\frac{h(x+p)}{g(3)} = \frac{\sqrt{1-x-p}}{2}$
16. $\frac{f(x) - f(a)}{x - a} = 3x + 3a$
17. $g(x + 1) = 3x - 4$
18. $(h \circ g)(-3) = \sqrt{17}$
19. $(g \circ h)(-3) = -1$
20. $(h \circ f)(x) = \sqrt{6 - 3x^2}$
21. $(f \circ h)(t - 3) = 7 - 3t$
22. $a^{-1}(x) = \frac{2x+9}{3}$
23. $b^{-1}(x) = \frac{2x^2+1}{3}$, for $x \geq 0$
24. $c^{-1}(x) = 1 + \sqrt{x}$, for $x \geq 0$
25. $d^{-1}(x) = \sqrt[3]{x + 1} - 7$
26. $f^{-1}(x) = \frac{3x+9}{x-4}$, for $x \neq 4$
27. $g^{-1}(x) = 2 - x^2$, for $x \geq 0$

EXPONENTIAL AND LOGARITHM FUNCTIONS — pg. 59 - 64

1.


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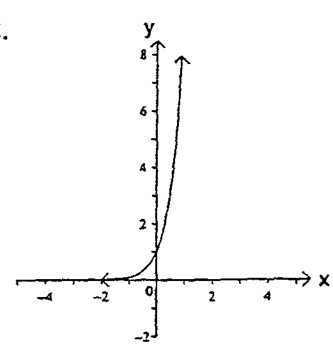
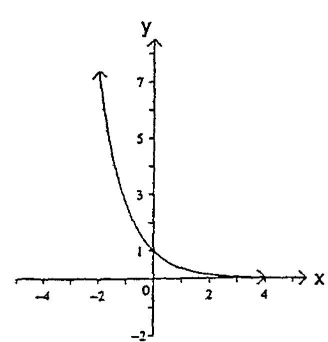
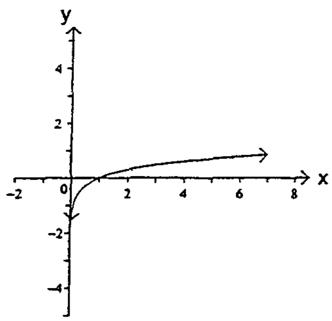


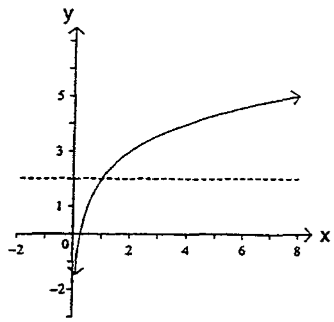
3.


Answers

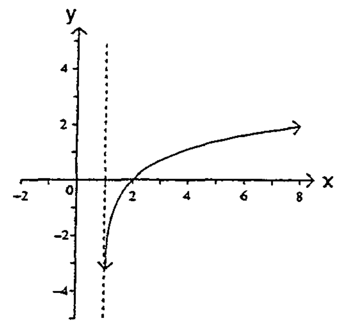
4.



5.



6.



7. $\log_3 2187 = 7$

8. $\log_2 \frac{1}{16} = -4$

9. $\log_{49} 7 = \frac{1}{2}$

10. $\ln 54.6 = 4$

11. $10^{0.44} = 2.78$

12. $2^{3.28} = 9.73$

13. $e^1 = e$

14. $5 = 25^{\frac{1}{2}}$

15. $2\log_a x + \log_a y$

16. $\frac{1}{2}(\log x + \log y - \log z)$

17. $\ln 5 + \frac{1}{2}\ln A$

18. $4\ln 3 + 8\ln x + 4\ln y - \frac{1}{2}\ln z$

19. $\log_a(2x^2)$

20. $\log \frac{5x^3}{y}$

21. $\ln(x+2)$

22. $\log \frac{16x}{3}$

23. $2 + \frac{2\ln t}{\ln 2}$

24. $-\frac{\ln q}{\ln 10}$

25. $\frac{2\ln(x-3)}{\ln 10} - 1$

26. $\frac{4\ln x + \ln(x+1)}{\ln 3}$

27. $x = \frac{1}{3}$

28. $x = -2$

29. $x = \frac{4}{3}$

30. $x = \frac{15}{4}$

31. $x = -6$

32. $x = \frac{-5}{2}$

33. $x = \frac{1}{27}$

34. $x = 4$

35. $x = 2$

36. $x = 1$

37. $x = \frac{1}{2}$

38. $x = \frac{3}{2}$

39. $b = \frac{10^r}{8}$

40. $a = \frac{\log_2(5-t)}{3}$

41. $r = 5 + \ln(q+7)$

42. $d = 7e^{(5x-3)}$

43. $48,061.82

44. 11.6 *yr.*

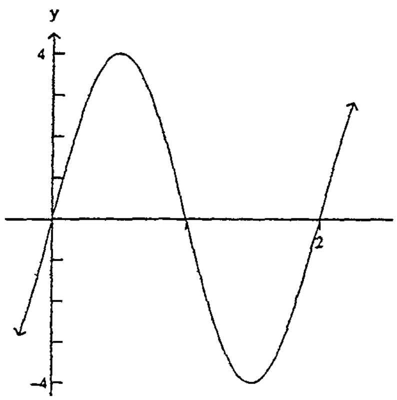
45. Pop. in 20 yrs will be 262,851.
Pop. in 10 yrs will be 248,535.
Pop. 5 yrs ago was 228,511.

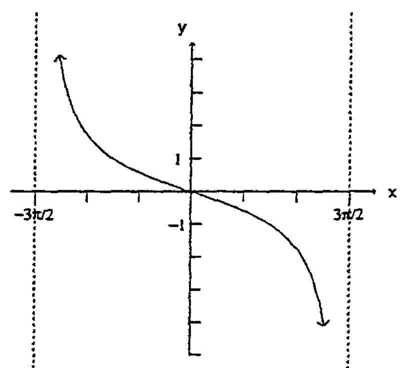
46. 34 *yr*

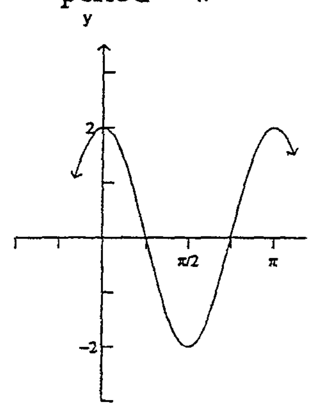
TRIGONOMETRY — pg. 65 - 73

1. $\sin\theta = \frac{-3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{-3}{4}$
2. $\sin\theta = \frac{-12}{13}$, $\cos\theta = \frac{-5}{13}$, $\tan\theta = \frac{12}{5}$
3. $\cos\beta = \frac{-5}{13}$, $\tan\beta = \frac{-12}{5}$, $\cot\beta = \frac{-5}{12}$, $\sec\beta = \frac{-13}{5}$, $\csc\beta = \frac{13}{12}$

4. II
5. $\frac{\pi}{2}$
6. $\frac{3\pi}{4}$
7. $\frac{4\pi}{3}$
8. $\frac{3\pi}{2}$
9. 225^o
10. 30^o
11. 420^o
12. 75^o
13. amplitude = 4
    period = 2
14. amplitude = NA
    period = 3π
15. amplitude = 2
    period = π







16. IV
17. II
18. $\frac{-\pi}{6}$
19. $\frac{-\pi}{4}$
20. $\frac{\pi}{3}$
21. $\frac{\pi}{3}$
22. $\frac{12}{13}$
23. $\frac{\sqrt{1-x^2}}{x}$
24. $\frac{-4}{5}$
25. $\frac{1}{\sqrt{1+x^2}}$
26. $\frac{-7}{24}$

Proof of the identities in problems 27 - 30 may vary.

27. $\csc\alpha = \cot\alpha\sec\alpha$

$$= \frac{\cos\alpha}{\sin\alpha}\frac{1}{\cos\alpha}$$
$$= \frac{1}{\sin\alpha}$$
$$= \csc\alpha$$

28. $\sec\beta - \cos\beta = \tan\beta\sin\beta$

$$= \frac{\sin\beta}{\cos\beta}\sin\beta$$
$$= \frac{\sin^2\beta}{\cos\beta}$$
$$= \frac{1-\cos^2\beta}{\cos\beta}$$
$$= \frac{1}{\cos\beta} - \frac{\cos^2\beta}{\cos\beta}$$
$$= \sec\beta - \cos\beta$$

Answers

29. $(\tan\theta + \cot\theta)\sin^2\theta = \tan\theta$

$$\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\sin^2\theta =$$

$$\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)\sin^2\theta =$$

$$\frac{(\sin^2\theta + \cos^2\theta)\sin\theta}{\cos\theta} =$$

$$\frac{\sin\theta}{\cos\theta} =$$

$$\tan\theta$$

30. $\frac{1}{1+\sin\phi} + \frac{1}{1-\sin\phi} = 2\sec^2\phi$

$$\frac{(1-\sin\phi)+(1+\sin\phi)}{(1+\sin\phi)(1-\sin\phi)} =$$

$$\frac{2}{1-\sin^2\phi} =$$

$$\frac{2}{\cos^2\phi} =$$

$$2\sec^2\phi$$

31a. $\frac{29}{13\sqrt{5}}$

31b. $\frac{22}{13\sqrt{5}}$

31c. $\frac{-22}{19}$

32. $\sin\theta$

33. $1 - 2x^2$

34. $\frac{3}{5}$

35. $\frac{1-\tan\theta}{1+\tan\theta}$

36. $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

37. $x = \frac{\pi}{3}, \frac{5\pi}{3}$

38. $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

39. $x = \pi$

40. $\theta = 60^o, 90^o, 120^o, 270^o$

41. $\theta = 30^o, 90^o, 150^o, 210^o, 270^o, 330^o$

42. $\theta = 60^o, 300^o$

43. $\theta = 0^o, 60^o, 180^o, 300^o$

44. $c = 87$, $b = 83$, $B = 72^o$

45. $A = 37^o$, $B = 53^o$, $a = 27$

46. $a = 7.51$, $b = 17.70$, $B = 1.170$

47. $A = 0.9774$, $b = 1555$, $c = 2780$

48. $A = 17.5^o$, $b = 36.1$, $c = 42.8$

49. $B = 113^o$, $a = 18$, $b = 25$

50. $A = 75^o$, $B = 48^o$, $C = 57^o$

51. $a = 1055$, $B = 22^o$, $C = 46^o$

52. 49 *ft.*

53. 270 *lb.*

54. 369 *miles*

55. 94 *lb.*, 42^o

56. 3.91 *ft.*

57. 11,500 *ft.*

58. 9.79 $in.^2$

59. 314 *ft./min.*

Sample Mathematics Placement Examination

Section I. Questions related to MATH 0301, *Essentials of Mathematics*. If they present any difficulty, you should review Algebra I topics before taking the placement examination if you hope to bypass MATH 0301.

1. $\frac{3}{5} + \frac{5}{3} - \frac{7}{5} \cdot \frac{1}{2} =$

   (a) 1/25 (b) 24/15 (c) 1/40 (d) 47/30 (e) $-1/2$

2. 10 is what percent of 0.5?.

   (a) 2000% (b) 200% (c) 5% (d) 10% (e) 0.5%

3. $\frac{-3+18}{-5} - 7 - (8 - 18) =$

   (a) -18 (b) 18 (c) 6 (d) -30 (e) 0

4. $9 - 2[1 - 2(2-5)^3] =$

   (a) 115 (b) 49 (c) -101 (d) -24 (e) -115

5. Evaluate $ab^2 - bc$ when $a = -4$, $b = 5$ and $c = 1/2$.

   (a) $-93/2$ (b) $-205/2$ (b) $155/2$ (d) $-101/2$ (e) $-105/2$

Section II. Questions related to MATH 0302, *Intermediate Algebra*. If they present any difficulty, you should review Algebra II topics before taking the placement examination if you hope to bypass MATH 0302.

6. Solve $8 - 6x < 7$ for x.

   (a) $x > 1/6$ (b) $x > 7/2$ (c) $x < 1/6$ (d) $x < 7/2$ (e) $x < -1/6$

7. Solve $-2(x+6) = 5x - (x+1)$ for x.

   (a) $x = 5/6$ (b) $x = -11/6$ (c) $x = -11/2$ (d) $x = 5/2$ (e) $x = 11/6$

8. $(2x^{-2}y^5)^3(3xy^5)^2 =$

   (a) $6x^{-4}y^{25}$ (b) $6x^{-2}y^{15}$ (c) $24x^{-6}y^{10}$ (d) $72x^4y^5$ (e) $72x^{-4}y^{25}$

9. An article usually sells for \$10.00 but is on sale at 20% off. If there is an 8% sales tax, the total cost to the customer is

   (a) \$8.00 (b) \$8.52 (c) \$8.64 (d) \$8.86 (e) \$8.80

10. $x^3(2x^{-2} + 4x) =$

    (a) $2x^{-6} + 4x^3$ (b) $2x^5 + 4x^4$ (c) $2x^{-5} + 4x^4$ (d) $2x + 4x^4$ (e) $6x$

Section III. Questions related to MATH 1320, *College Algebra*. If they present any difficulty, you should review College Algebra topics before taking the placement examination if you hope to bypass MATH 1320.

11. Solve $x^2 + 7 = 4x$ for x

    (a) $x = 2 \pm i\sqrt{3}$ (b) $x = -2 \pm i\sqrt{3}$ (c) $x = 2 \pm \sqrt{11}$ (d) $x = -2 \pm \sqrt{11}$ (e) $x = 2 \pm \sqrt{3}$

12. Solve $3xy^2 - 5yz = 4x$ for x.

    (a) $x = \frac{3y^2 - 5yz}{4}$ (b) $x = \frac{4 + 5yz}{3y^2}$ (c) $x = \frac{5z}{3y}$ (d) $x = \frac{-5yz}{y^2}$ (e) $x = \frac{5yz}{3y^2 - 4}$

13. If $f(x) = x^2 + 5x$ and $g(x) = \frac{1}{x+2}$, then $f(g(x)) =$

    (a) $\frac{x^2 + 5x}{x+2}$ (b) $\frac{1}{x^2 + 5x + 2}$ (c) $\frac{5x + 11}{(x+2)^2}$ (d) $x^2 + 5x + \frac{1}{x+2}$ (e) $\frac{1}{(x+2)^2} + 5x$

14. $\log(3x^{\frac{1}{5}}) =$

    (a) $(\log 3)\log(x^{\frac{1}{5}})$ (b) $\log 3 + \frac{\log x}{5}$ (c) $\log 3 + \log(\frac{x}{5})$ (d) $\frac{\log 3x}{5}$ (e) $3\log(x^{\frac{1}{5}})$

15. If $(1.06)^x = 2$, then $x =$

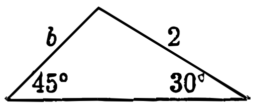
(a) $\log 2 - \log 1.06$ (b) $2^{1.06}$ (c) $\log 0.94$ (d) $\dfrac{\log 2}{\log 1.06}$ (e) 1.89

Section IV. Questions related to MATH 1321, *Trigonometry*. If they present any difficulty, you should review Trigonometry topics before taking the placement examination if you hope to bypass MATH 1321.

16. $5\pi/12$ radians =

(a) $(5/12)^\circ$ (b) 1.3° (c) 75° (d) 60° (e) 45°

17. Based on the pictured triangle, $b =$



(a) 2 (b) $\sqrt{2}$ (c) 1.5 (d) $2\sqrt{2}$ (e) $2/\sqrt{3}$

18. If $\sin\theta = 4/5$ and $\pi/2 < \theta < \pi$ then $\tan\theta =$

(a) $4/3$ (b) $-4/3$ (c) $3/4$ (d) $-3/4$ (e) $3/5$

19. $\cos(\text{Sin}^{-1} x) =$

(a) $\sqrt{1-x^2}$ (b) $\cos(90^\circ - x)$ (c) $1/x$ (d) 1 (e) x

20. $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta} =$

(a) 0 (b) $-2\csc\theta$ (c) $\sec^2\theta$ (d) $2\cos\theta\csc^2\theta$ (e) $2\sin\theta\sec^2\theta$

Section V. Questions related to MATH 1350, *Analytic Geometry*. If they present any difficulty, you should review Analytic Geometry topics before taking the placement examination if you hope to bypass MATH 1350.

21. The equation of the perpendicular bisector of the segment with endpoints $(7,-2)$ and $(1,6)$ is

(a) $3x - 4y = 4$ (b) $4x - 3y = 10$ (c) $4x + 3y = 22$ (d) $3x + 4y = 20$ (e) $4x + 3y = 14$

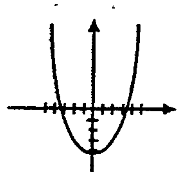
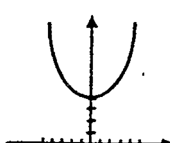
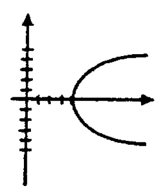
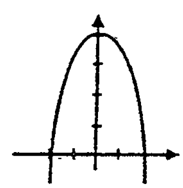
22. The distance between $(-2,-3)$ and $(6,1)$ is

(a) $2\sqrt{2}$ (b) $2\sqrt{5}$ (c) $4\sqrt{5}$ (d) $4\sqrt{2}$ (e) $\sqrt{48}$

23. An equation for the circle of radius 4 centered at $(3,-2)$ is

(a) $x^2+y^2+6x-4y=3$ (b) $x^2+y^2-6x+4y=-9$ (c) $x^2-y^2+6x+4y=11$
(d) $x^2-y^2-6x-4y=11$ (e) $x^2+y^2-6x+4y=3$

24. The graph of $y = x^2 + 4$ is

a)  b)  c)  d) 

25. An equation of the ellipse with foci at $(0,\pm 4)$ and a vertex at $(0,6)$ is

(a) $\dfrac{x^2}{36} - \dfrac{y^2}{20} = 1$ (b) $\dfrac{y^2}{20} - \dfrac{x^2}{36} = 1$ (c) $\dfrac{x^2}{36} + \dfrac{y^2}{20} = 1$ (d) $\dfrac{x^2}{20} + \dfrac{y^2}{36} = 1$ (e) $\dfrac{x^2}{20} - \dfrac{y^2}{36} = 1$

Solutions:

1.d	2.a	3.e	4.c	5.b	6.a	7.b	8.e	9.c	10.d	11.a	12.e	13.c
14.b	15.d	16.c	17.b	18.b	19.a	20.e	21.a	22.c	23.e	24.b	25.d	