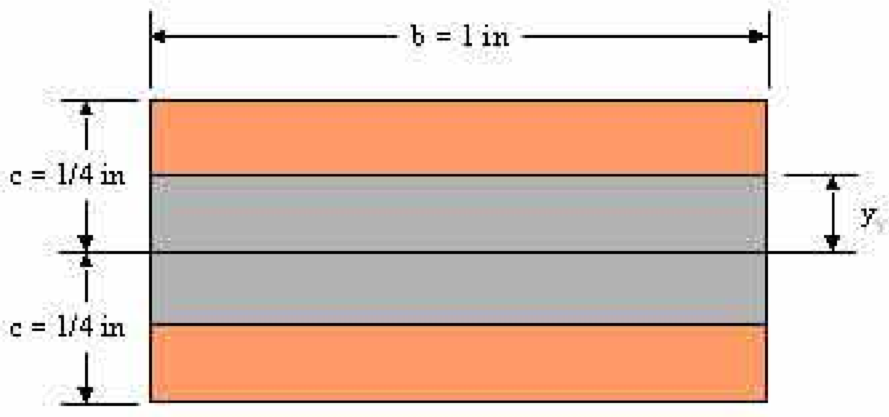
Residual Stress Calculations

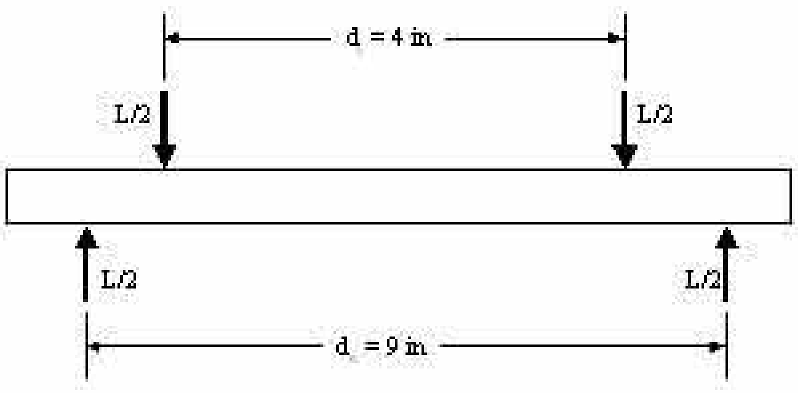
To obtain analytical results for residual stresses from bending, the principle of superposition is used. By superimposing the loaded stress curve with a theoretical unloading curve, we can develop an analytical result of residual stress. This process is illustrated in the example below.

Example



A member of uniform rectangular cross section 1 by 1/2 by 11 in is subjected to a a four point bend.

Assume that the member is made of an elastoplastic material with a yield strength of 47 ksi and a modulus of elasticity of 10600 ksi. The load force is 2125 lbf.



$E := 10600 \times 10^3 \cdot psi$ $\quad \sigma_Y := 47 \times 10^3 \cdot psi$ $\quad L := 2125 \cdot lbf$

The moment from this force at the center of the beam is calculated as follows:

$M := \frac{1}{4} \cdot L \cdot (d_o - d_i)$ $\quad M = 2656\, lbf \cdot in$

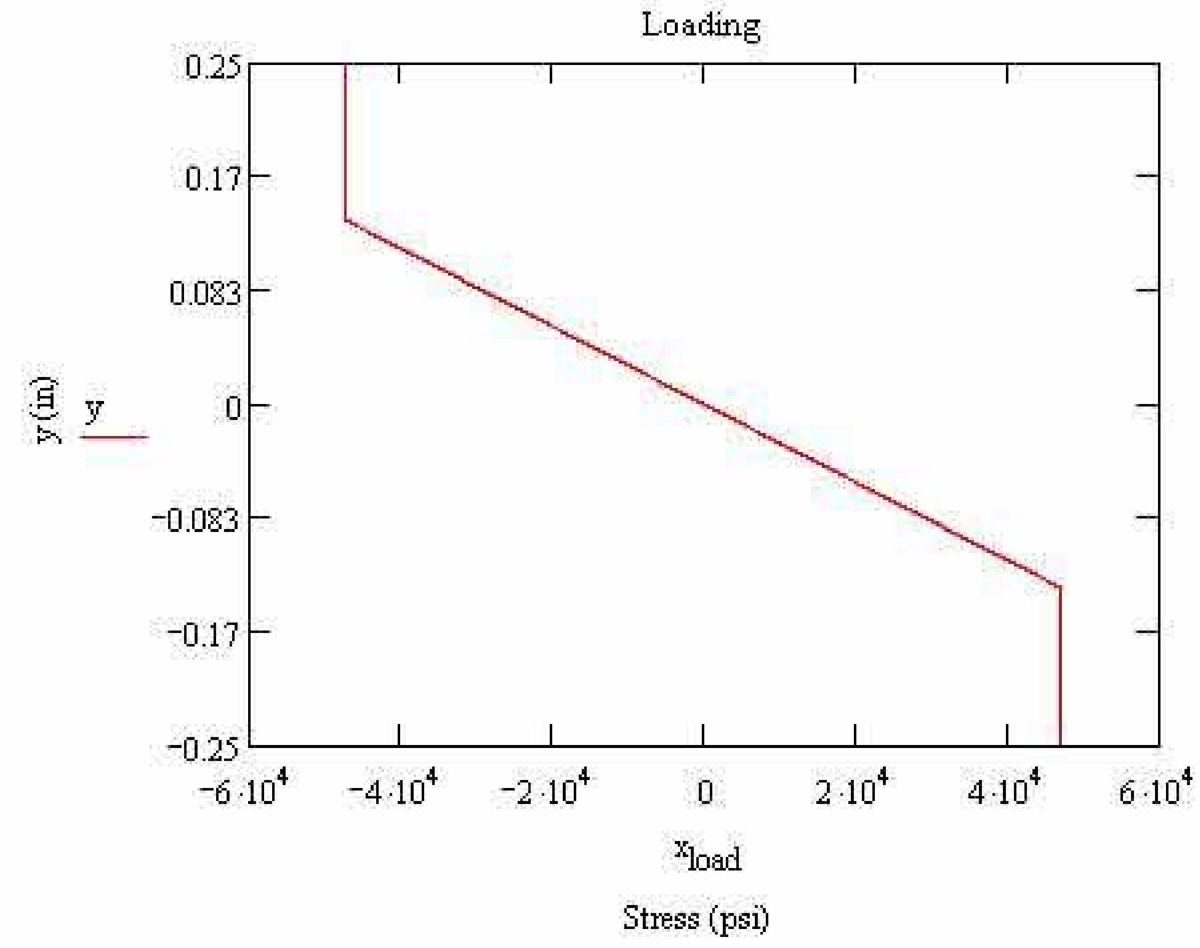
First we need to determine the thickness of the elastic core (y_Y), which also requires us to find the elastic moment (M_Y).

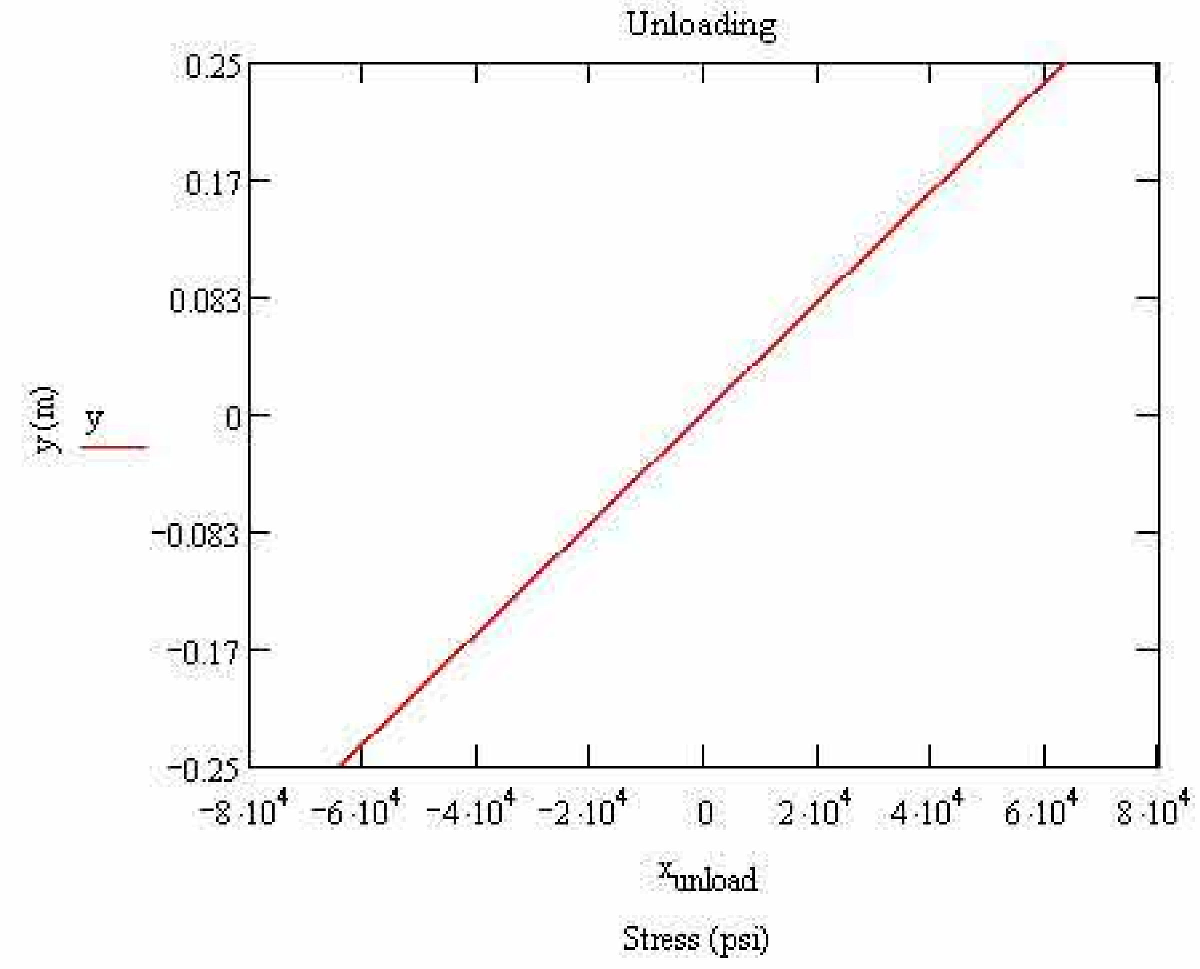
$M_Y := \frac{I}{c} \cdot \sigma_Y$ $\quad M_Y = 1958\, lbf \cdot in$

$y_Y := c \cdot \sqrt{3 \cdot \left(1 - \frac{2 \cdot M}{3 \cdot M_Y}\right)}$ $\quad y_Y = 0.134\, in$

Now we determine the maximum stress (σ_m), which will become the unloading curve.

$\sigma_m := \frac{M \cdot c}{I}$ $\quad \sigma_m = 6.375 \times 10^4\, psi$





Finally, by adding the previous two graphs together, we obtain the residual stress in the material as a function of the vertical location.