THE QUARK

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A Very Simplified explanation of the Navier-Stokes equations and their history

By George Collier

Mathematics is the language of physics. It permits us to describe phenomena in a highly objective and unambiguous manner. It is obvious then that historically, vast amounts of research have been devoted to finding equations that can model the phenomena that we observe in the world around us. Today I am going to look at the equation that we use to model fluids, and how math and physics can be at odds with one another.

The unfortunate reality that all large structures on the surface of the earth must grapple with is that they are submerged in an ocean of oxygen that would love nothing more than to destroy them. All structures are burdened by turbulence and drag, and the larger the structure, the bigger the risk. In the 1800s people became aware that a better understanding of fluid dynamics would be needed to engineer ever larger structures.

During the early 1800s, French mathematician and physicist Augustin-Louis Cauchy had derived an equation to describe the momentum transfer across any system. Meanwhile a fellow French physicist and contemporary Claude-Louis Navier was busy building bridges across France. While doing so, Navier was able to derive a set of equations that could model any fluid accurately. Unfortunately, Navier ultimately had his reputation as an engineer ruined when he made critical errors during the construction of a bridge in Paris. He was ultimately accused of relying too heavily on math in his construction projects.

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About The Quark

The Quark is a monthly newsletter produced by the Public Relations Committee of the Texas Tech chapter of the Society of Physics Students (SPS). Our goal is to help new students become more familiar with the Physics Department and provide returning students more insight on aspects of the department they might not have been aware about.

If you have any questions about The Quark or SPS, you can email our Public Relations Officer Rob Chambers at Robert.Chambers@ttu.edu .

Evidently Navier was a better mathematician than engineer.

The equations that he derived are now known as the Navier-Stokes equations. The second part are a set of partial differential equations, and they are quite complex. Typically, finding solutions to differential equations ranges in difficulty from “hard” to “literally impossible.” The Navier-Stokes equations tend to lean more towards the impossible side of things. In some extreme cases, the equations could predict fluids moving at infinite velocities, or that water might randomly explode. Of course, this represents an obvious disconnect between physical reality and mathematics. To the best of my knowledge, no one has ever opened a bottle of water only to find that its chaotic behavior has caused it to explode out of the bottle at superluminal speeds.

NAVIER-STOKES EQUATION

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\frac{Du}{Dt}$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\frac{Dv}{Dt}$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\frac{Dw}{Dt}$$

Navier-Stokes Equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$$

Continuity

These equations may appear intimidating at first, but they are still based on the same physics laws taught in high school. The bottom equation is labeled “continuity”, and simply states that mass is conserved. In essence, the change in density over time and the change in the velocity of our fluid in each direction cancel out, thus conserving mass. So, our first equation is simply Newton’s first law adapted for fluids. As for the three equations above it, these equations are basically a fancy way of writing Newton’s second law. The three equations each describe the motion of a fluid in one dimension. On the right side of each equation, we see the time derivatives of the terms u, v, and w. Each of these terms describes the velocity of our fluid in one direction. So, these derivatives are the acceleration in each direction. The rho by which each derivative is being multiplied represents fluid density, which for our purposes represents mass. On the left side we see a far scarier set of symbols, but since we already have mass multiplied by acceleration on one side, we know that the left side represents the total force acting on the fluid. The terms multiplying rho by g describe the external forces acting on our fluid, while the derivative of p with respect to x, y, and z represents the pressure gradient across our fluid in each direction. Our final term, mu, represents viscosity. Together these terms describe the internal and external forces acting on the fluid as being equal to mass and acceleration. Amazingly, even with complicated equations like this one, they are still contingent on extremely simple laws of nature.

The Navier-Stokes equations are ultimately too complex to be practically useful in their current forms. Thankfully, physicists don't need to be mathematically exact. They have the luxury of being approximate. With this in mind, the Reynolds-averaged Navier-Stokes equations were derived. These take the average of the Navier-Stokes equations across time. You can think of it as trying to take the average of a bunch of small elements of a fluid rather than trying to simulate it as a continuous object. Doing so allows us to simplify the equations enough to use them for real world applications. So, while mathematicians struggle to figure out the deeper meaning to these equations, physicists have been busy making use of them.

Nonetheless, it would be wrong to make fun of mathematicians for being interested in something that is "good enough" for physics. In the end, physics owes its ability to model incredibly difficult phenomena to the hard work of mathematicians who spend years trying to rigorously solve problems rather than simply guesstimating on their solutions.

Professor Spotlight: Dr. Thomas Kupfer

By Victor Bradley

Dr. Thomas Kupfer is an Astrophysicist raised in rural Southern Germany. After completing his primary education, he began work towards a mechanical engineering trade certification (a common feature of German education), where he realized he was fascinated with how things fundamentally worked. He returned and finished secondary education and fell in love with his physics classes. From there, he decided that physics was the career path he wanted to pursue. He went on to attend the Friedrich Alexander University Erlangen-Nuremberg in Germany, where he was given an opportunity to join an astrophysics-based research group. While completing his undergraduate degree he was also able to spend four months in Northern Ireland on a research project at the Armagh observatory. He quickly realized that astrophysics was a perfect match and chose his major as Astro-, Particle and Nuclear physics. From there he went on to complete his Ph.D in physics at the Radboud University Nijmegen in the Netherlands. After completing his degree, he was offered a post-doctoral position at the California Institute of Technology, and then another post-doctoral position at the University of California at Santa Barbara in the Kavli Institute for Theoretical Physics. Dr. Kupfer was then given the chance to join Texas Tech as a Professor in 2020.

Dr. Kupfer's main area of research involves studying objects in space that change in very short periods of time, often in just a single night, to understand the more 'extreme' side of astrophysics. These include strongly correlated binary star systems, and very rapidly pulsating stars. Objects such as these are often sources of some of the most interesting phenomena in astrophysics, such as: gravitational waves, type Ia supernovae, strong tidal forces, etc. One thing that Dr. Kupfer finds particularly beautiful about physics as a whole is how everything 'fits together', saying "It works so well together, If the gravitational potential was not divided by R but R2, if the electromagnetic field was not how it is, if one unit was wrong it would completely break down".

Dr. Kupfer is currently teaching Astrophysics I this semester but is also currently the advisor for the Society of Physics Students, a position he feels is very important. He emphasizes the importance of a sense of community amongst physics students and "to have other students to learn together, to work together but also to have fun together." He advises students to take breaks when necessary, citing times in his career when he was simply not performing at his best due to burnout and underscores the importance of getting along well with your potential advisors. He believes that regardless of how brilliant a professor may be, if you do not get along well you will have a hard time putting together a successful project. He encourages students to talk to their professors as much as possible, despite it at times being intimidating. He likes sharing a German saying, "We all cook with water", which means despite being at different stages of their careers, students can still relate to senior members of the faculty and they can relate to students.

Outside of the classroom Dr. Kupfer enjoys playing soccer, volunteering for the American Red Cross, and used to play many brass instruments. He is also an avid football fan and is looking forward to the return to normalcy so he may finally be able to attend a Texas Tech game.

Student Spotlight: Cheslee Hibler

By Samuel Cano

Growing up in Saudi Arabia, Cheslee Hibler quite literally experienced a different childhood than the typical American. Her father worked in the oil industry and moved Cheslee and her family there when she was six years old. She remained there until age seventeen when she graduated from Dhahran Academy. There were some cultural differences of course, for example, in order to watch a movie her family would often take a day trip to the neighboring country Bahrain since movie theaters were illegal in Saudi Arabia until recently. She did enjoy a nice caveat of her father's job that had a huge impact on her childhood: a travel allowance. Her family traveled every

opportunity they could and Cheslee was able to visit around thirty-five countries throughout her childhood. Most evenings growing up, Cheslee would take walks around the compound, where the clear skies would provide a vivid view of the stars and planets among the glow of the Milky Way. Her first introduction to physics came shortly after moving to an Aramco Residential Camp during her junior year of high school when she found herself with access to a real library for the first time. In her free time, she couldn't help but read the entire Physics and Astronomy section. From this point on it was pretty clear that she had found her path. After graduating from Dhahran Academy, Texas Tech University seemed like the obvious choice since her sister, a Ph.D. candidate at the TTU Health Science Center, was already here and because of the Astrophysics concentration.

At TTU, Cheslee immersed herself in the Society of Physics Students (SPS) over the course of her freshman year after Dr. Glab took her to the lounge to meet members during orientation. She became increasingly involved by attending meetings, semester trips, and volunteering whenever she could. She continued to do so, becoming Secretary her junior year, and now the President of SPS in her last year. Being President during the COVID-19 pandemic has created a lot of unique challenges for an organization that prides itself on bringing social and networking events for young scientists. Going completely online for over a year now, SPS has continued to provide students with opportunities to hang out virtually over discord and is excited to get new members when things settle back down. She has also been involved acting as a Lab Assistant for the introductory astronomy labs and helping to set up telescopes at the Preston Gott Observatory.

Outside of physics, Cheslee enjoys cooking with her sister, watching movies, and hanging out with her friends, who happen to be physics majors. One hobby she particularly enjoys is axe throwing, and there are actually a few cool places in Lubbock to do it. Another one of her big interests is sharks, and although she wouldn't call herself a fanatic she has amassed a collection of plush sharks over the years which are all named after famous scientists. And her favorite shark, the Epaulette Shark, with black spots on its pectoral fin resembling military epaulettes, has graced her zoom profile picture this past year. Beyond that, Cheslee still loves taking walks at night and seeing the same night sky that inspired her so many years ago.

Cheslee's favorite class was Observational Astronomy and because of that, her favorite astronomical object is NGC-457, also known as the Owl Cluster, because that's what she got to learn so much about in the course. For incoming or newer physics students her advice, first of all, is to join SPS for networking and to relax with like-minded people. Secondly, she really emphasizes that students learn more about their professors and reach out to the ones who work on things they're interested in. Having a positive relationship with a professor helps out in so many ways, but especially with finding research opportunities, having someone to go to for advice, and to learn from someone who has gone through this process.

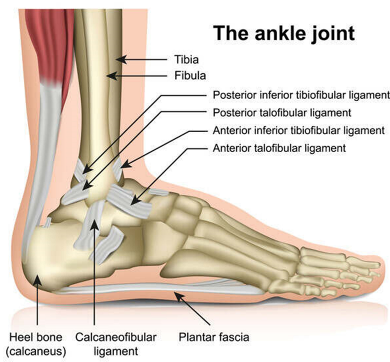
At the end of this semester, Cheslee will graduate with a double degree in Mathematics and Physics with a concentration in Astrophysics. From here she hopes to continue her education and pursue a Master's in Physics. After that, she is most interested in working at an observatory because of the great experiences she's had researching and helping out at the observatory.

Rational Running

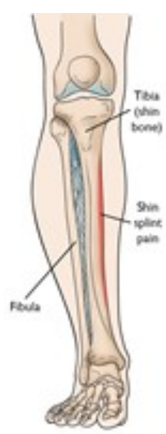
By Akash Maheshwari

Anyone who has ever tried running knows that the activity includes repetitive significant forces on one's joints, legs, and feet. In fact, every time a person lands while running, a force 3.5 times their bodyweight is applied to their leg and foot. As a result, even slight alterations in one's strides can result in injury, pain, and prolonged soreness. Therefore, proper running attire and footwear plays an important role in reducing the likelihood of any running-related injury.

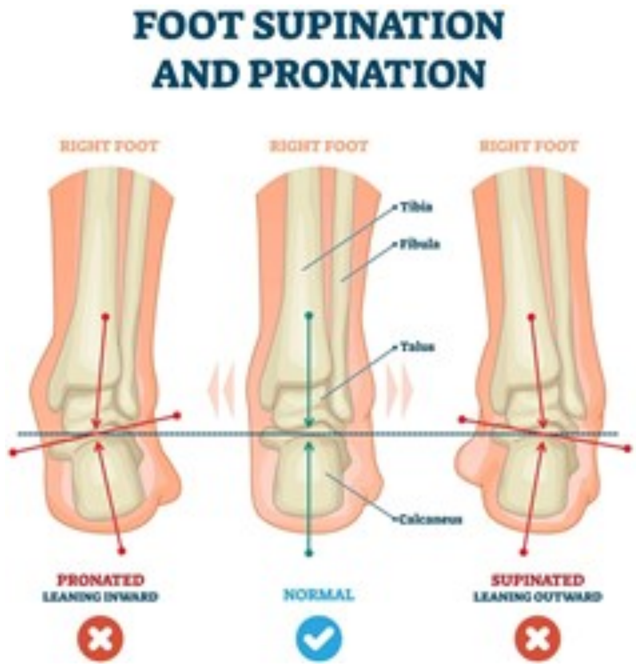
One common word used in describing running shoes is stability. What exactly is meant by stability shoes and how exactly do they help runners prevent injuries? Before this question can be answered, it is important to review some basic human anatomy. The heel bone (pictured below) is called the calcaneus.



The two bones that make up the lower leg are called the tibia (the larger bone that is located centrally) and the fibula (the smaller bone that is located distally).

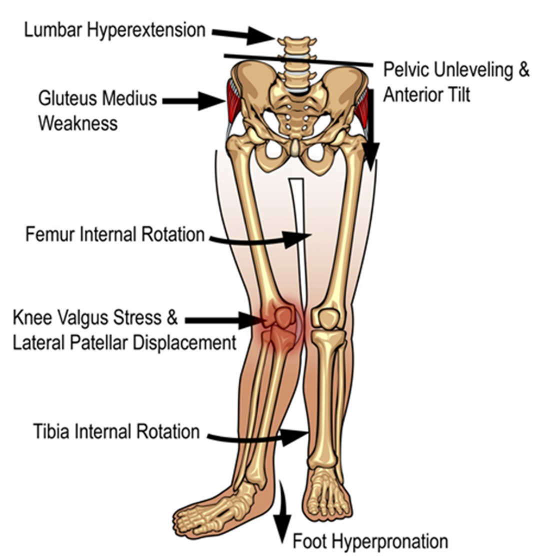


The position that the calcaneus has relative to the position of the tibia and fibula is significant. If the calcaneus centrally leans inward (if there is calcaneal eversion), then an individually pronates. If the calcaneus leans outward (if there is calcaneal shifting), then an individually supinates. See the image below for clarification.



The arrows in the above picture illustrate that slight idiosyncrasies in a person's stride can lead to a misbalance of forces. This misbalance of forces can directly affect other areas and joints in the body. For instance, the joints in the legs and knees can be placed under increased stress due to an imbalance of forces, which can lead to injuries over time. Some sources suggest that individuals can develop hip pain and back pain from excessive pronation or supination while running. For instance, consider the potential effects of overpronation in the image below.

With this understanding, we may now begin to truly comprehend what stability running shoes do. Stability shoes provide additional support for individuals that overpronate or oversupinate. In this way, stability shoes attempt to correct the imbalance of forces that may occur so that forces are more balanced and better distributed. The image below demonstrates one footwear technology that provides additional support to runners who overpronate or oversupinate in their stride.



Stability shoe technology therefore attempts to adjust the placement of the calcaneus relative to the tibia and fibula so that an individual has a more neutral stride. A more neutral stride will have a better balance of forces. As a result, the likelihood of injuries in the form of stress fractures, plantar fasciitis, shin splints, back pain, joint pain, and other general pain is greatly reduced when a runner chooses proper running shoes. It is interesting to note that only about 30% of the population has a neutral stride. The remaining population either overpronates (95% of this group) or oversupinates (about 5% of this group). Consequently, it is essential to consider your stride and calcaneal position when shopping for running shoes. If you are interested in determining if you overpronate or oversupinate, consider visiting these links and taking a diagnostic test:

Now you know how stability shoes work and how to run rationally!

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