Lattice Confined Fusion (LCF), Quantum Nucleonics, and Nanonuclear Physics

Abundant nuclear energy with no fuel-associated nuclear waste

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Bottom-line Up Front

- NASA GSFC announced Lattice-Confined Fusion (LCF)in 2020
  - Exceptionally promising, transformational, and very exciting result
  - Promise of solid-state fusion energy without lasers / big magnets
  - Just one problem: our attempt to duplicate this has not been successful
    - We have clear tritium-production data that does not indicate LCF in this system
    - In collaboration with our partners at the MU Research Reactor and Cyclotron

- TTU has invented a new form of LCF that works
  - Light-element, fission / fusion pathway, with reduction to practice
  - Reduction to practice, and provisional filings
    - Provisional patents to produce LiD and other needed materials to support scale-up
    - Irradiations planned at MURR to establish full reduction to practice
    - Earlier production of ^{18}F from LiOD irradiations for PET also provides reduction to practice
    - We are ready now to take this to the next level through commercialization

Rapid investment in novel nuclear science

- According to Matt Trevithick of DCVC, over $5,000M of private equity investment has occurred in novel nuclear science in the past two years
  - For example, Google and Chevron funded TAE Technologies ($p + {}^{11}B \rightarrow 3\alpha$) in Orange County, CA at $250M, part of a $1.2B funding round: https://www.cnbc.com/2022/07/19/google-chevron-invest-in-fusion-startup-tae-technologies.html
  - MIT and Commonwealth Fusion Systems demonstrated a revolutionary HTS 20 tesla magnet in September, 2021, with a huge impact on fusion: https://news.mit.edu/2021/MIT-CFS-major-advance-toward-fusion-energy-0908
  - Many other examples
  - The Biden - Harris Administration launched a bold vision for commercial fusion energy in March, 2022:
  - https://www.whitehouse.gov/ostp/news-updates/2022/03/15/fact-sheet-developing-a-bold-vision-for-commercial-fusion-energy/
  - Over $700M / year in current government investment in fusion, and expanding

‘Nano-Nuclear’ Science

Conventional nuclear reactors convert the kinetic energy of their fission fragments into heat inside the nuclear fuel, requiring heat energy conversion, and adequate primary cooling is essential for nuclear safety. TTU has proposed an alternative design (along with early-phase fuel development data) that avoids today’s nuclear safety concerns entirely, since no major thermal energy conversion pathway is involved.

- Conventional nuclear heat is converted to useful work / electricity, with a ‘thermal conversion (TC)’ efficiency of only about ~ 34%
- ‘Direct Conversion (DC)’ design converts charged particles from the nuclear reaction to electricity directly, with potential efficiencies of almost twice the TC efficiencies. This requires a new nanometer-scale nuclear fuel design.
- One possible generator is a ‘sub-critical’ particle-beam-pumped design for proportional, on-demand power delivery, and fast on / off. No run-away possibilities!
- Nanoparticle fuel confinement is an unresolved issue, but we have a NASA NIAC award with Positron Dynamics and LLNL to address this concern now

- Our new design and early-phase development data removes intrinsic limitations to the safety and efficiency of current-day nuclear energy through an innovative, sub-critical, sintered fuel containment design, and facilitates our new nuclear pathway based upon Lattice Confined Fission / Fusion (LCFF)

NASA GSFC Announces Fusion Shortcut

Published in <u>IEEE Spectrum</u> on February 27, 2022, based upon papers published in Phys. Rev. C in 2020

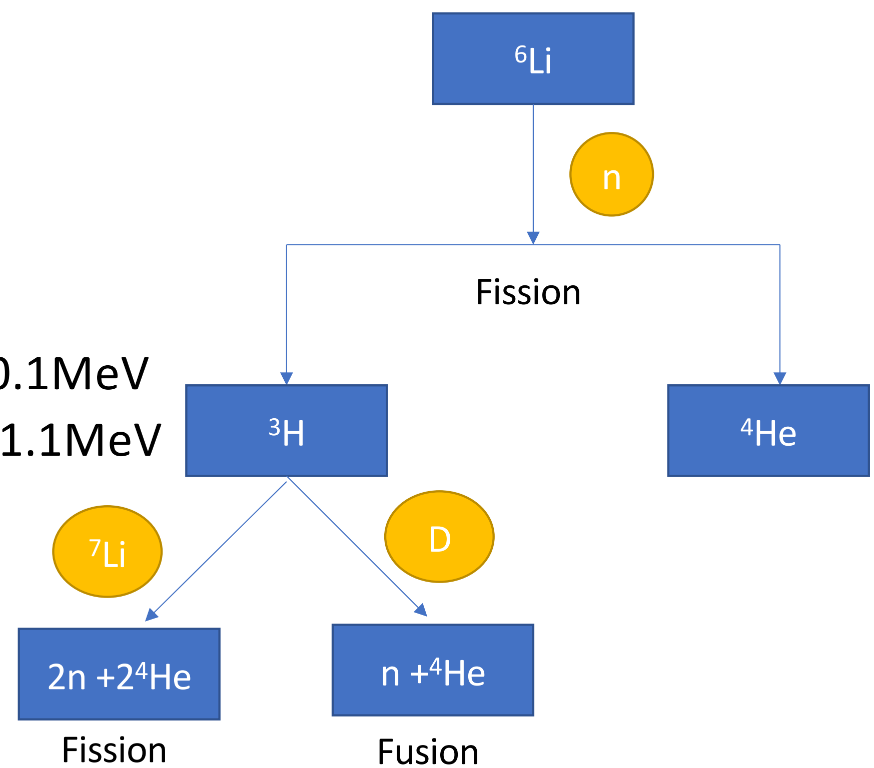
Stated that Lattice – Confined Fusion may replace the need for big laser systems and / or huge magnets to achieve future fusion energy

But our tritium diagnostics to date do not confirm these NASA experimental results

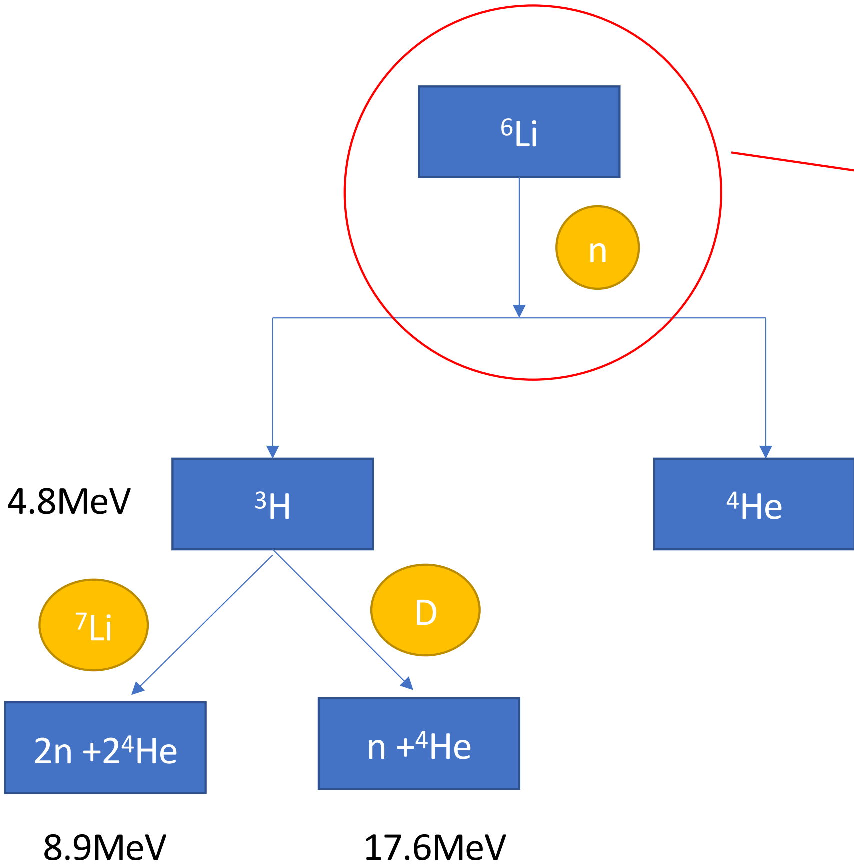
Another form of collisional fusion in LiD

^{6}Li natural abundance is 7.6%, ^{7}Li is 92.4%, and both are stable

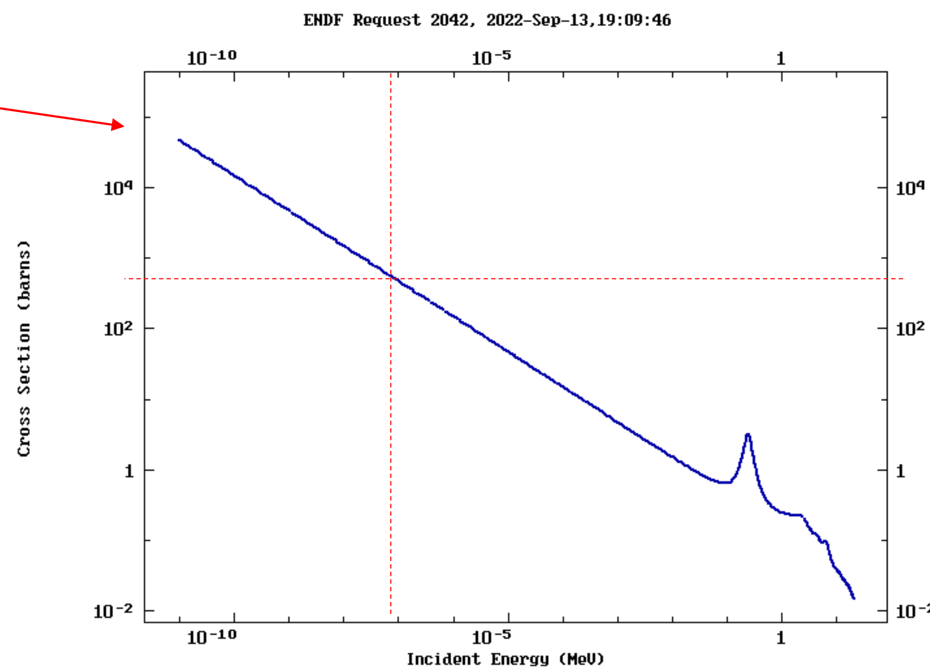
- The basis for a new commercial nuclear cycle!
- Primary: ^{6}Li (n,t) α, Q = 4.8 MeV,
  - σ ~ 400 b for thermal n
  - Triton (2.7 MeV) induces secondary reactions
  - alpha (2.1 MeV) produces heat
- Secondary reactions:
  - 1) fusion: D(t,n)α, Q = 17.6 MeV, σ ~ 10 b @ 0.1MeV
  - 2) fission: ^{7}Li(t,2n)2α, Q = 8.8 MeV, σ ~ 1b @ 1.1MeV
  - May be configured so that each reaction is roughly equally likely



Step 1: n + ^{6}Li → ^{3}H + ^{4}He + 4.78 MeV

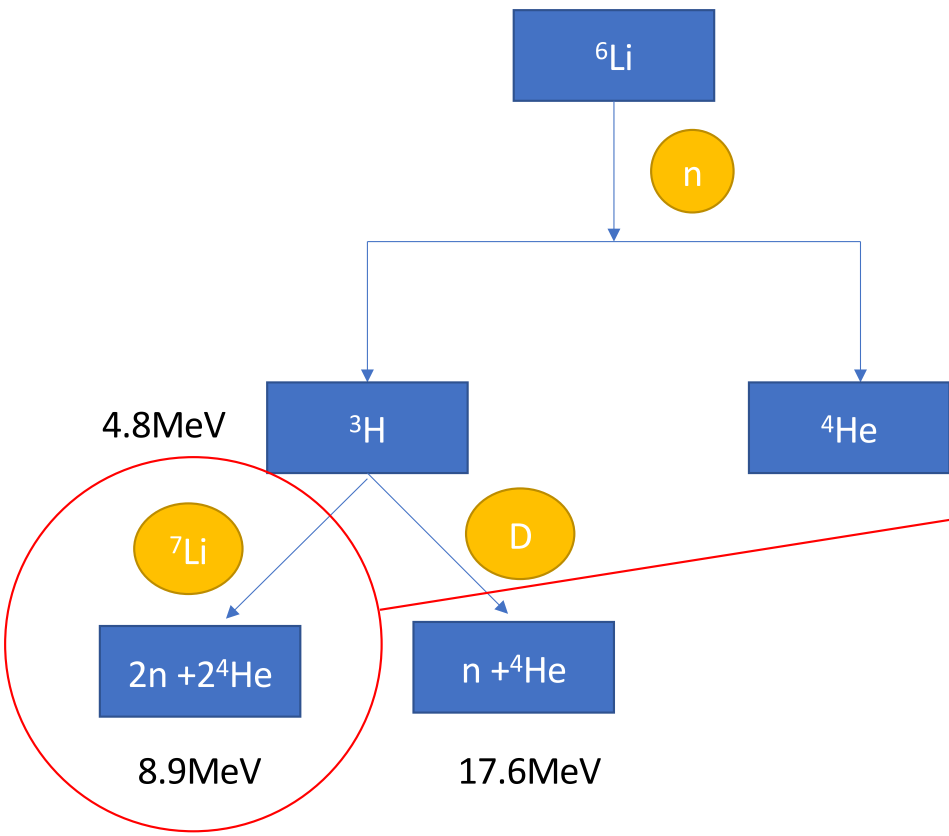


LI-6(N,T)HE-4,SIG MT105 QM=4.78365e+6 eV

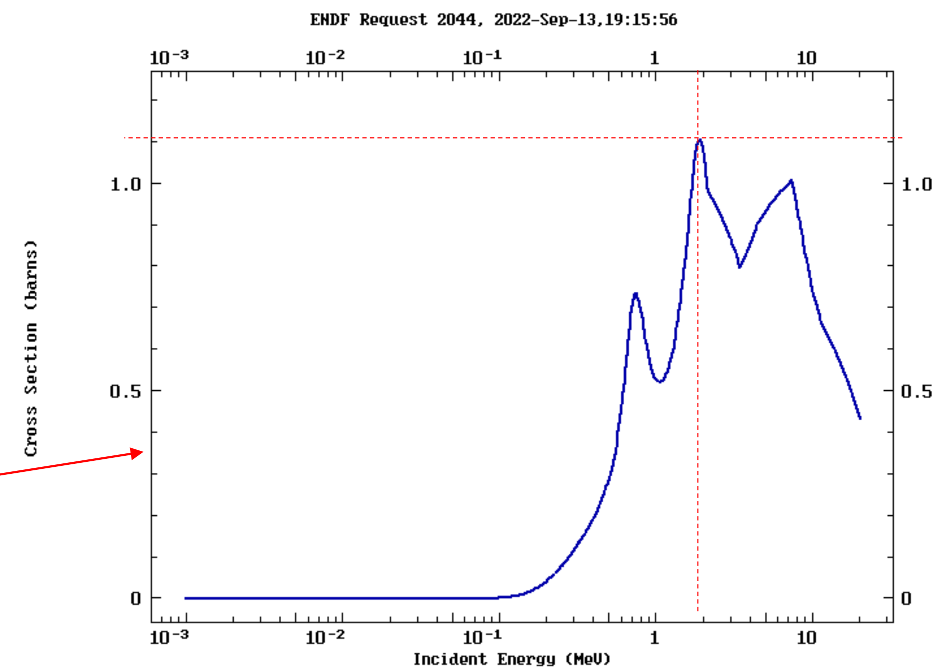


σ ~ 400 barns @ 26 meV

Step 2: $^{3}H + {}^{7}Li \rightarrow + 2\ {}^{4}He + 2\ n + 8.87\ MeV$

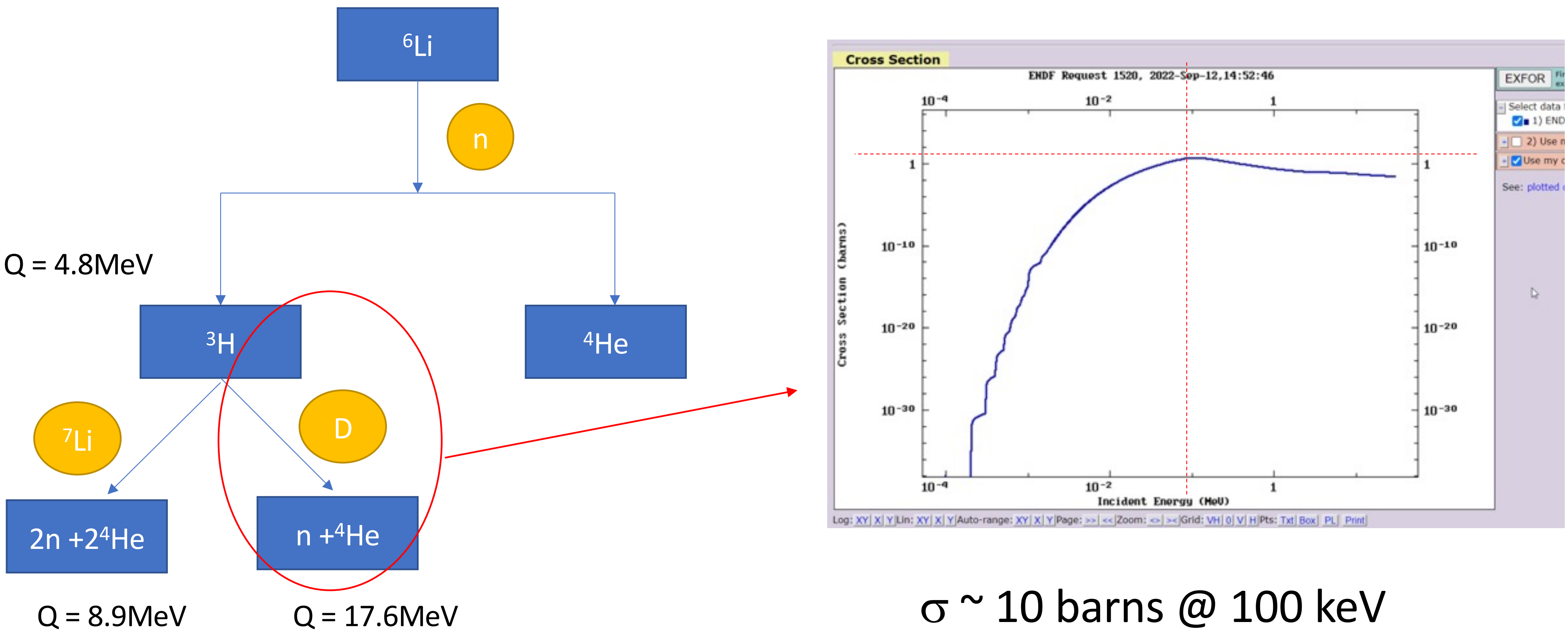


LI-7(T,2N+A)HE-4,SIG MT24 QM=8.86548e+6
QI=8.86548e+6 eV



σ ~ 1.1 barn @ 1.9 MeV

Step 3: $^{3}H + D \rightarrow n + {}^{4}He + 17.1$ MeV



σ ~ 10 barns @ 100 keV

A new light-element fission / fusion cycle

- <u>No</u> nuclear waste from the fuel!
  - If all tritons are consumed in secondary reactions
  - If not, then triton collection / re-cycling
- Can a super-critical nuclear chain reaction be sustained?
  - Requires the nuclear cycle to generate a gain in thermal neutron number
  - Also requires a low-density core that may be brought critical, such as the current developments at Positron Dynamics and LLNL
  - If not, then a sub-critical device may still show excellent energy gain
- Better mass → energy conversion than ^{235}U fission
  - Q/mc^2: = 0.091% for ^{235}U fission; = 0.18% for this LiD cycle assuming equal secondary branch probabilities

Lithium production is already at a large scale

- Global Li production: 2×10^8 kg in 2021, up from 5.6×10^7 kg in 2010
  - 2.8×10^{10} kg of Li has been mined world-wide
  - World Li reserves currently estimated at 2×10^8 kg
- Each electric passenger car contains ~ 8 kg of Li, about 1,150 moles
  - Just 11% of the Li in one car battery may produce 4.2×10^7 kWh of energy
  - Enough energy to fully charge 184,000 cars, resulting in > 45 million driving miles (this assumes a 35% nuclear thermal to electrical energy conversion)
  - Deuterium is in plentiful supply, but not yet at this industrial scale of production
- When incorporated with EV battery reprocessing plants, this new cycle promises to produce abundant energy through local microgrids, to support expansion of the EV industry to scale without major new grid investments

Introduction

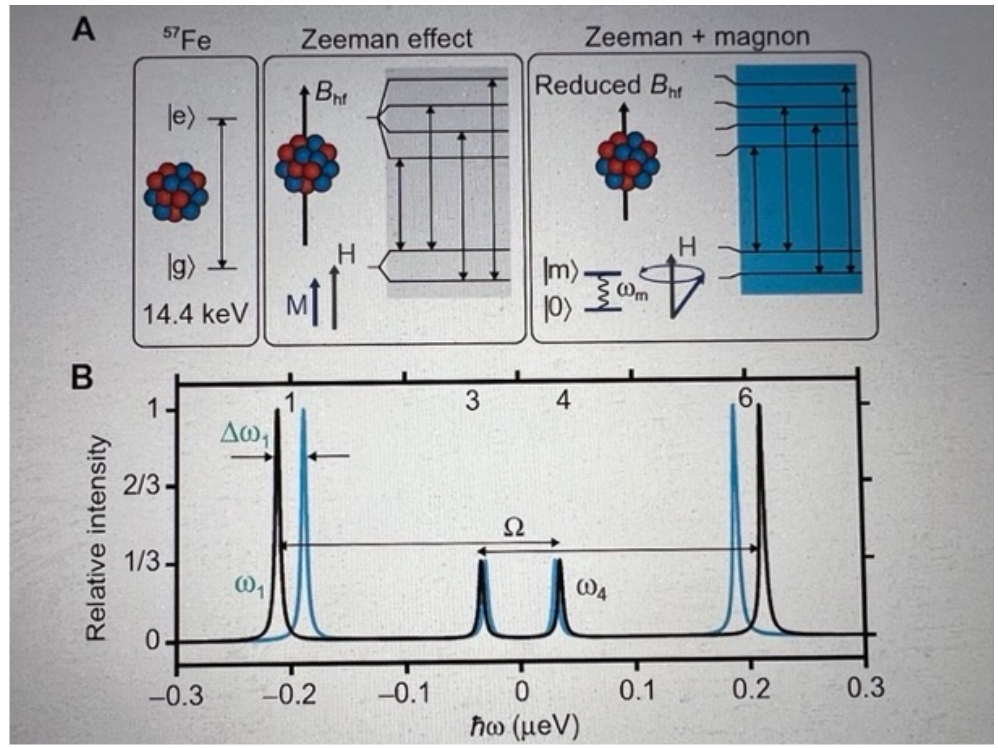
- Question: Can lattice dynamics influence nuclear dynamics in metals?
  - Their time and energy scales are extremely different
    - Lattice: $\tau \sim$ 0.1 ps (10^{-13} s), $\Delta E \sim 1$ eV
    - Nuclear: $\tau \sim$ 10 zs (10^{-20} s), $\Delta E \sim 10$ MeV
  - Yes, since fields and quantum coherence in one system influences the other
    - Hyperfine interactions, such as nuclear Zeeman couplings
      - Unpaired electrons produce huge magnetic induction at the nucleus, estimated at > 100 T
      - For example, the sd and sf hybridizations as Z increases
    - The Mossbauer Effect (1961)
      - When the quantum state of the lattice doesn't change, then decaying nuclei don't recoil independently
      - Mossbauer Spectroscopy to $\Delta\nu/\nu \sim 10^{-13}$, an excellent probe of the nuclear Zeeman Effect ($H_{Fe} \sim 28$T)
      - Sources of $^{57}Fe^*$ include ^{57}Co decay, and direct x-ray illumination
- There were many examples of new methods of photon control of quantum nuclear dynamics at PQE'22, usually at x-ray energies, and primarily in Germany (Max Plank Institutes, Universities, and at DESY)

Nuclear Quantum Dynamics: Mossbauer Effect

Lattice excitations, circularly polarized magnons, in a permalloy film, change the magnetic field at the ^{57}Fe nucleus from 28 T to 25T, resulting in shifted energy transitions shown in blue. Hence, this lattice excitation modifies quantum coherence in ^{47}Fe nuclei.

See also K.P. Heeg, *et al.*, "Coherent X-ray – optical control of nuclear excitons" in Nature 590, pp.401 – 418 (2021). They state that these results may "… unlock coherent optical control for nuclei, and pave the way for nuclear Ramsey spectroscopy and spin-echo-like techniques, which should not only advance nuclear quantum optics, but also help to realize X-ray clocks and frequency standards."

A new suggestion: Can this also influence novel nuclear reactions, and thereby impact future nuclear energy science? Not certain yet, but possibly yes…



"Coherent control of collective nuclear quantum states via transient magnons", Fig. 1

(Bocklage *et al.*, Sci. Adv. 7 : eabc3991 29 Jan 2021)

Nuclear Dynamics, and Novel Nuclear Reactions

- Does the nuclear polarization, nuclear excitation, and nuclear quantum population inversion in some nuclei increase their cross-sections for nuclear reactions? If so, then intense X-ray / gamma irradiation may make fusion easier to achieve
  - Analogy to atomic physics: Excited dimer molecules, like XeCl, Rydberg atom clusters, etc., only form when at least one atom is in its excited state. Are there nuclei that fuse more readily when at least one is in an excited nuclear state?
  - But nuclear dynamics are based upon the strong and electro-weak nuclear force, not EM, but symmetry changes associated with nuclear excitations still apply.

- Recently, a team centered at NASA Glenn SFC measured novel d+d fusion in metals at low deuteron (d) projectile energy, but under high-flux gamma radiation. This has a theoretical basis …
  - Theory: V. Pines, *et al*., "Nuclear Fusion Reactions in Deuterated Metals", NASA TP-20205001617, and Phys. Rev. C <u>101</u>, 044609 (2020)
  - Simulation: T.L. Benyo and L.P. Forsley, "MCNP Fusion Modeling of Electron-Screened Ions", 2021 MCNP® User Symposium, Los Alamos National Laboratory
  - Experiment: B. Steinetz, *et al*., "Novel nuclear reactions observed in bremsstrahlung-irradiated deuterated metals", NASA TP-20205001616, and Phys. Rev. C <u>101</u>, 044610 (2020)

Electron Screening in d+D fusion at low energy

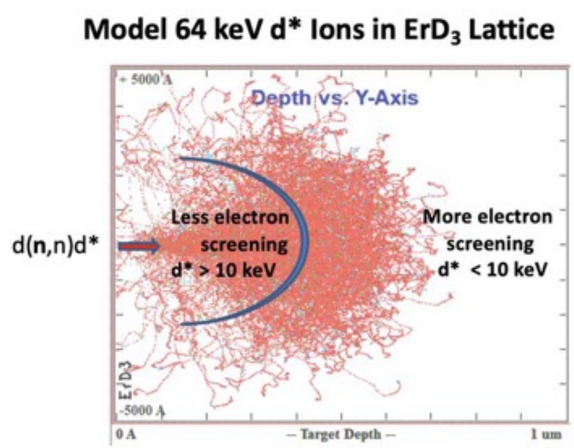
- V. Pines, *et al*., predict big enhancements in fusion, specifically in $D(d,n)^3He$ and $D(d,p)T$ reactions, due to EM-enhanced screening
  - As two fully-ionized positive ions, one with atomic number Z1 and the other with Z2, approach each other with CM kinetic energy E, electrons are drawn in between them as they approach, and lessen the electrostatic repulsion, and hence the coulomb barrier to fusion. This provides a screening potential $-U_e$ that typically is about the Fermi energy of the e^- in the host metal
- Screening potential $-U_e$ measured by accelerator d+D collisions in various materials with $E >> U_e$:
  - ~ 25 eV in gaseous D_2 (Greife *et al*., Z. Phys. A <u>351</u> 107 (1995)
  - ~ 50 eV in deuterated insulators, semiconductors, & noble metals (Cu, Ag, Au)
  - ~180 eV in Be, to ~800 eV in Pd -- Why? This is well above the metal's Fermi energy!
  - Measurements from many independent groups (Raiola, Strieder, Czerski, Kasagi, Google collaboration, others) confirm this
- V. Pines *et al.* show that intense EM radiation (γ or X-ray) in metals increases U_e due to induced plasma screening and Compton-effect electron screening
  - In conventional fusion, the d-beam energy E is much larger than this electron screening potential U_e, but for $E \sim U_e$ things are predicted to become <u>very</u> interesting…

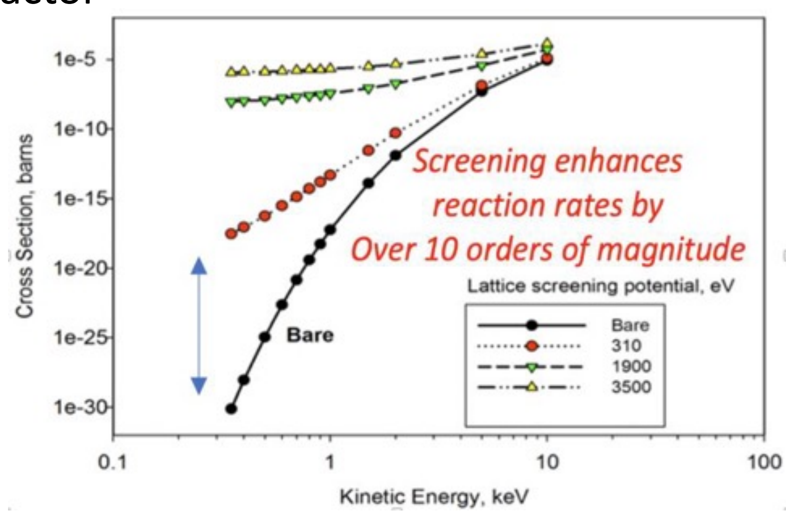
Huge Cross-Section Enhancements at Low E

These predictions are from V. Pines, *et al.,* NASA TP-20205001617, and Phys. Rev. C 101, 044609 (2020)

- When $E < U_e$, screening effects become much more substantial in metals, d+D fusion becomes easier, and Oppenheimer –Phillips (OP) reactions with the host lattice ions are predicted to produce even higher energy neutrons than the fusion neutrons
- Neutron collisions with D are much more effective at accelerating D than are collisions with light ions (e-,e+) or heavier ions (d^+,t^+,p)
- Gammas and X-rays above 2.23 MeV produce neutrons by D photodissociation, and resulting 145 keV neutrons collide with D to produce 64 keV d that trigger nuclear fusion reactions
- $\sigma_{exp}(E) = f(E)\ \sigma_{bare}(E)$, where f(E) = screening enhancement factor

Simulations (MCNP6.2 code): T.L. Benyo and L.P. Forsley, “MCNP Fusion Modeling of Electron-Screened Ions”, 2021 MCNP® User Symposium, Los Alamos National Laboratory →





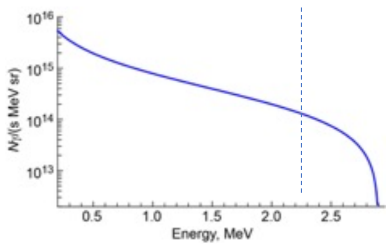
Technical Strategy: 'Bootstrap' d+D fusion

- Usually E >> U_e, the effect of screening is to lower the Coulomb Barrier to fusion. In this limit, it is equivalent to boosting the CM energy to E + U_e
- But for E ~ U_e, the cross-section enhancement for fusion becomes much larger than the unscreened (free d-ion fusion) case, but the fusion rate is still exceptionally small at this very low E (typically less than 800 eV).
  - Google collaboration: at 12 keV in PdD, U_e ~ 1 keV, but still a very small fusion rate
  - Prof. Jeremy Munday, UCD in this Google collaboration spoe on this at PQE'22
- From Pines, *et al*., intense gammas produce plasma electrons and Compton electrons that increase U_e, and make large-angle d+D scattering more probable
  - This increases the fusion cross section for E ~ U_e by many orders of magnitude, often by a factor of 10^{10} or more.
  - Resulting fusion neutrons (E = 2.4 MeV) collide with D, produce d, and trigger d+D fusion
  - In ErD_3 and TiD_2, the D is dense enough (~$10^{23}/cm^3$) to possibly (??) trigger a fusion chain reaction
- So, can we increase U_e enough by intense X-ray / gamma exposure so that, at very low deuteron energy, we obtain an adequate fusion rate to trigger enough fusion reactions to exceed break-even energy release? Maybe someday to actually sustain a fusion chain reaction indefinitely? Probably not, but this is the hopeful, stretch goal.

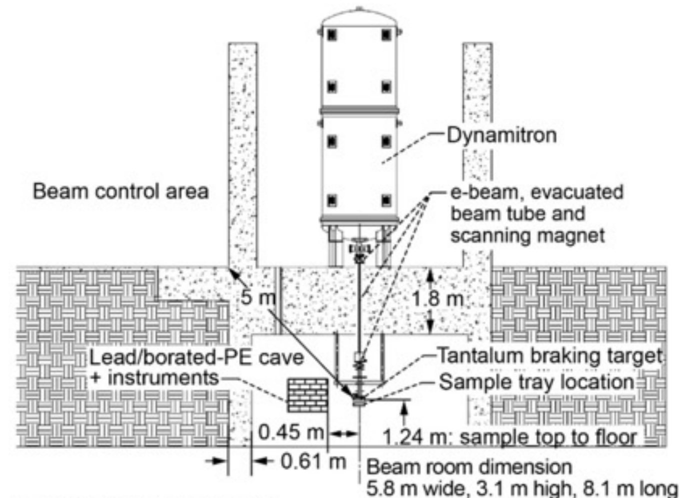
Experimental Confirmation(?)

- B. Steinetz, et al., NASA TP-20205001616, and Phys. Rev. C <u>101</u>, 044610 (2020)
- An e-beam was used to produce bremsstrahlung X-rays up to 2.9 MeV that then irradiated both ErD_3 & TiD_2 samples, producing photo-neutrons, <u>and</u> energetic 2.4 MeV and ~5 MeV neutrons from d+D fusion and OP lattice reactions, respectively.

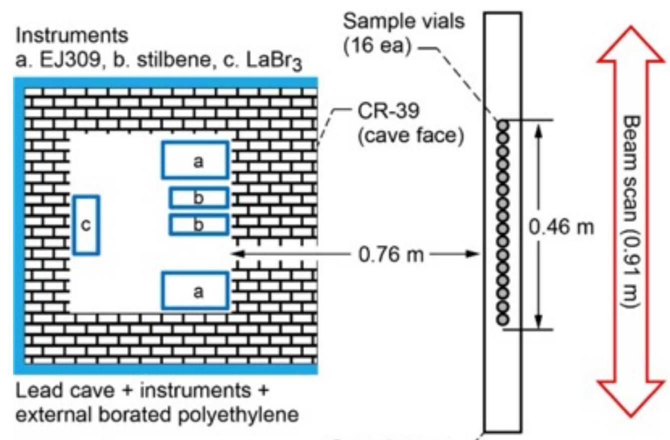
Electron beam energy: 0.45 to 3.0 MeV and current: 10 to 30 mA into Ta target:



Produces gammas up to 2.9 MeV. At 0.45 mA per sample, ~ 2.9×10^{12} gammas per s per sr. Photo-neutrons D(γ,n)p produced above 2.25 MeV, at E_n = 0.145 MeV



(a) Cross-sectional view.

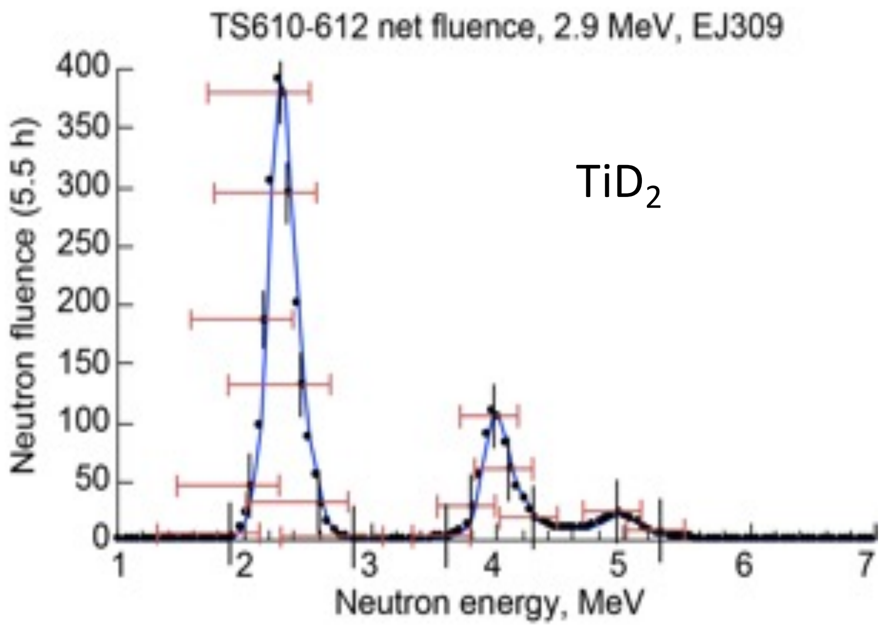


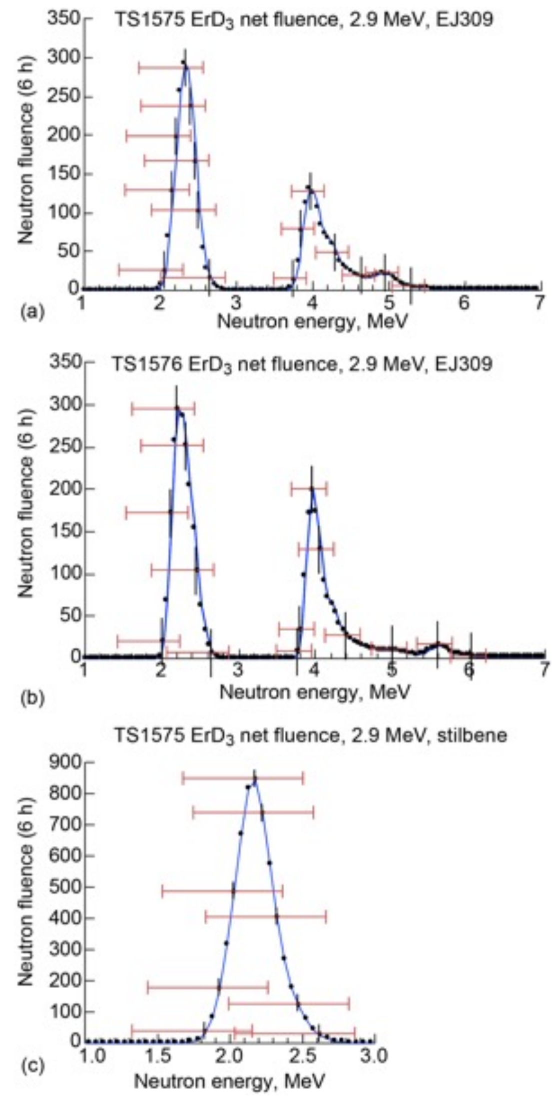
(b) Top view.

Experimental Confirmation? (Con’t)

B. Steinetz, et al., NASA TP-20205001616, and Phys. Rev. C 101, 044610 (2020)

Only 7 out of 10^6 neutrons produced are detected, due to the extensive shielding necessary to prevent RF interference with the detectors that are located in the lead cave near the accelerator and Ta bremsstrahlung target. At this extreme shielding, have neutrons from astrophysical origin really been ruled out?





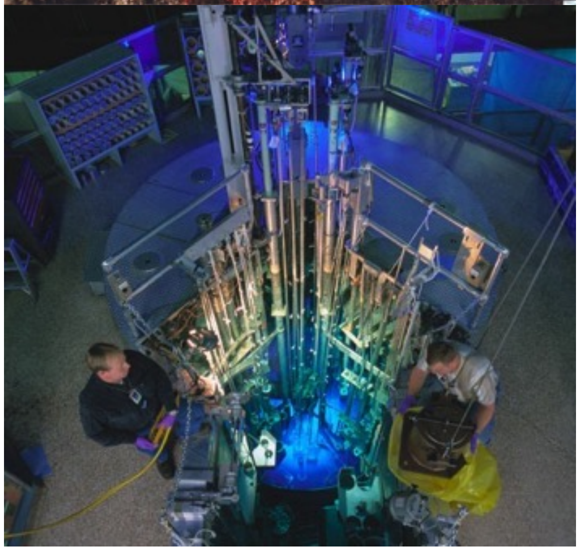
Steinetz, *et al:* Analysis and Significance

- According to a careful analysis of their data, Steinetz *et al*. conclude that their e-beam linear accelerator produces:
  - Gammas that photo-disassociate D to produce photo-neutrons at 145 keV
  - These photo-neutrons then scatter elastically with D, with σ = 3 b, to produce d at 64 keV
  - These d then collide with D to produce the two branches of d+D fusion, with σ = 17 mb (V. Pines, *et al*.)
  - The resulting 2.4 MeV fusion neutrons collide to produce even higher-energy d, with σ = 2.3 b
  - These higher-energy d collide with D to produce secondary fusion with σ = 0.1 b
  - Bonus: this also causes some d to accelerate toward Er or Ti, producing OP reactions.
- Not counting the 'bonus', if this is accurate, then a 64 keV d is releasing <u>at least</u> 6.5 MeV of kinetic energy in their experiment, and this possibly could be engineered to be greater still
  - So this is an energy gain of at least 100, initiated using electromagnetically-induced screening
  - <u>Initiated with a very-low energy d (64 keV) that may be easily accelerated to this energy</u>
- This science deserves close scrutiny, and replication in other experimental modalities

Replication Attempt @ MU & TTU

- Instead of using an accelerator for irradiations, we use a nuclear reactor: MURR in Columbia, Missouri
  - Very high neutron flux region reactor: $(8.21 \pm 0.12) \times 10^{13}$ n/cm^2/s
  - 90% of these neutrons are thermal (E ~ 26 meV) and 10% are fast (E > 100 keV), and the neutron spectrum is well known
  - Gamma radiation level in MURR is smaller than in Steinetz, *et al.,* but reactor gamma flux / spectrum is uncertain
    - Gamma radiation level and spectrum has not been measured in MURR, and it is highly variable during the reactor cycle
    - Estimate is 10^{10} to 10^{11} n/s/cm^2, so ~ 1% of the measured gamma flux in Steinetz *et al.*
- All TiD_2 (experimental) and TiH_2 (control) samples are grown, measured, and analyzed at TTU
  - Irradiated in hermetically-sealed quartz vials in a flooded compartment near the reactor core
  - 50-hour high-n-flux exposures of 0.10 g of TiD_2 samples
- Our diagnostic is the level of T produced in our metallic samples, since it indicates fusion efficiency

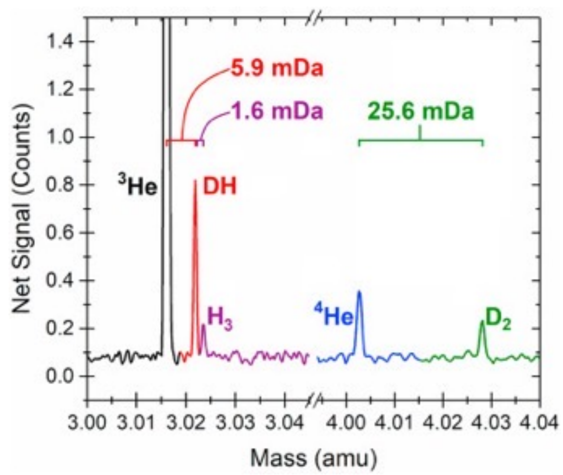
MURR

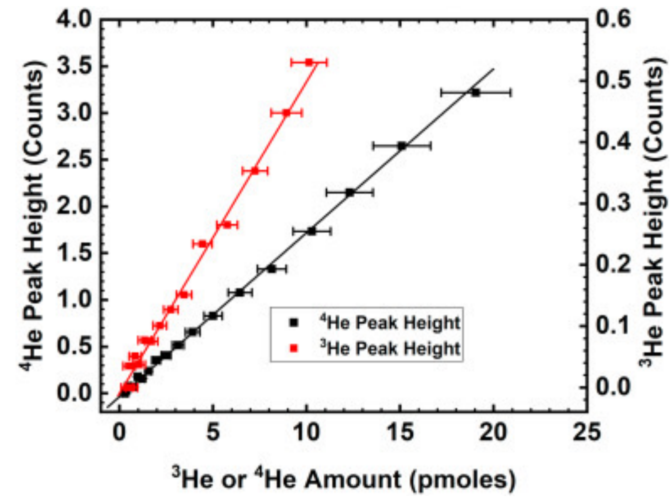


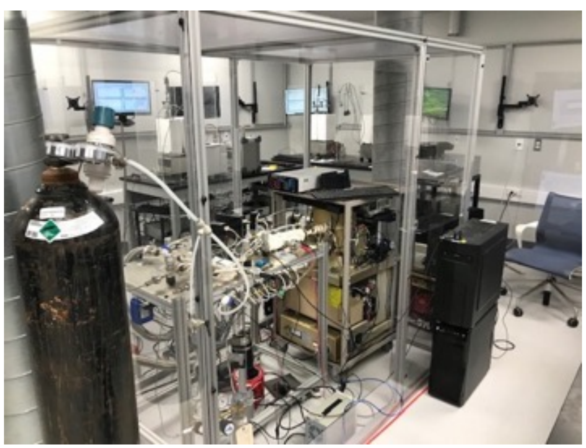
Custom FT-ICR Mass Spec – Performance (TTU)

Thorn *et al.*, Int. J. Mass Spec., full paper: https://doi.org/10.1016/j.ijms.2021.116574

We conduct real-time air invasion checks on all gas aliquots, and all equipment is surrounded by N_2 with < 0.005 ppm ^{4}He. We measure ^{3}He and ^{4}He with sub-pM accuracy, along with T to 10 fM accuracy. Continuous ^{1}n, X-ray, gamma monitoring of all experiments.







Perkin-Elmer Quantulus GCT 6220 scintillator system for tritium measurements →

Preliminary Results from MU and TTU

- Attempts have been made to replicate the results of Steinetz, *et al.*
- Do fast n collisions accelerate d to create d+D fusion reactions?
  - If D(d,n)^{3}He, then the ^{3}He will quickly burn up to tritium via ^{3}He(n,p)T
  - So no ^{3}He was measured (or expected) following the irradiation (confirmed)
    - Careful measurements using our group's custom FT-ICR mass spectrometers
    - See P. Thorn *et al*., https://doi.org/10.1016/j.ijms.2021.116574
  - Accelerated d also produce D(d,p)T reactions, and we measure the T quantitatively
- Excellent T extraction and radiation metrology have been developed at TTU
  - Quantitative measurements of D_2 extraction efficiency from TiD_2 irradiated samples
  - Perkin Elmer Quantulus Scintillator system measured T produced with 10^{-14} mole accuracy
- Modeling, using the LANL MCNP6.2 Monte Carlo Codes, by Dr. A. Gillespie
- Our initial tritium level measurements are well explained by D(n,γ)T alone
  - We do not yet see evidence of Steinetz, *et al.*'s results
  - *Is the lower gamma flux level in the reactor responsible for this difference?*

Further Steinetz *et al*. replication attempt

- Ongoing MCNP6 modeling will search for elastic D(n,n)d recoil and subsequent fusion cascades, such as $D(d,n)^3He$ and $D(d,p)^3H$
- TiD_2 and TiH_2 irradiations are being conducted in a well characterized cyclotron vault, with only fast neutrons
  - MU's GE 16.8 MeV 'PET Trace' Cyclotron
  - Fast neutron (> 0.4 MeV) flux ~ 5×10^9 n/cm^2/s
  - Very few thermal neutrons
  - ~ one month of of irradiation time
  - Neutron flux is measured and published by John Brockman's group: B.D. Jeffries, *et al.,* Appl. Rad. Isot. **154** (2019) 108892

‘Nano-Nuclear’ Science

- Conventional nuclear reactors convert the kinetic energy of their fission fragments into heat inside the nuclear fuel, requiring heat energy conversion, and adequate primary cooling is essential for nuclear safety. An alternative design (along with early-phase fuel development data) have been proposed that avoids today’s nuclear safety concerns entirely, since no major thermal energy conversion pathway is involved.
  - Conventional nuclear heat is converted to useful work / electricity, with a ‘thermal conversion (TC)’ efficiency of only about ~ 34%
  - ‘Direct Conversion (DC)’ design converts charged particles from the nuclear reaction to electricity directly, with potential efficiencies of almost twice the TC efficiencies. This requires a new nanometer-scale nuclear fuel design.
  - One possible generator is a ‘sub-critical’ particle-beam-pumped design for proportional, on-demand power delivery, and fast on / off
  - Nanoparticle fuel confinement is an unresolved issue, but we are attempting to address this limitation now
- Our design and early-phase development removes that intrinsic limitations to the safety and efficiency of current-day nuclear energy through an innovative, sub-critical, aerogel fuel containment design

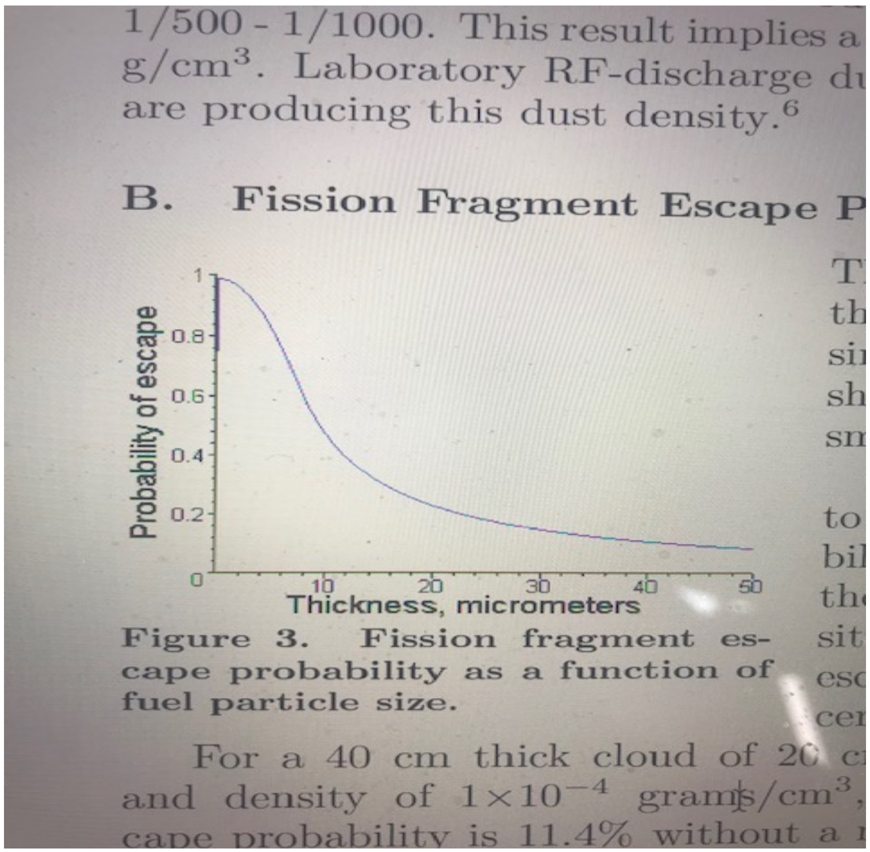
Safer, efficient approach to nuclear power

- An entirely different fuel cycle based upon fissile nanoparticles
  - Fission fragments escape the fuel particles producing very little heat, using aerogel containment of the nanoparticles
  - Fission fragments undergo DC to electrical power direct conversion, or provide propulsion in the nuclear rocket application
  - Primary fission neutron production is controlled by a particle beam in this sub-critical configuration
  - Nuclear run-away, melt-down, and control-rods are eliminated
  - Thorium nanoparticles maybe used to eliminate high-level radioactive materials from the stored / transported reactor before use: $^{232}Th \rightarrow {}^{233}U$
- We have expanded on the 'dusty – plasma' innovation by Clark and Sheldon (41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, July 10-13, 2005, Tucson, AZ). This was a critical reactor design, with a large, conventional BeO reflector / moderator. We are attempting to vastly improve on these limitations.

Concept of the fission fragment reactor / rocket

- G. Chapline, "Fission Fragment Rocket Concept," Nuclear Instruments and Methods in Physics Research Section A **271**, 1 (1988)
- R. L. Clark and R. B. Sheldon, "Dusty Plasma Based Fission Fragment Nuclear Reactor," in American Institute of Aeronautics and Astronautics, Inc., Tucson, AZ (2005)
- G. Zu, J. Shen, W. Wang, L. Zou, Y. Lian, Z. Zhang, B. Liu and F. Zhang, "Robust, Highly Thermally Stable, Core-Shell Nanostructured Metal Oxide Aerogels as High-Temperature Thermal Super insulators, Adsorbents, and Catalysts," *Chemistry of Materials* 26, 19 (2014)
- An entire new edition of Frontiers (London, UK) will be published on this soon, along with alternative nuclear pathways

Sheldon and Clark, 2005, simulations



1/500 - 1/1000. This result implies a
g/cm^3. Laboratory RF-discharge du
are producing this dust density.[6]

B. Fission Fragment Escape P

Figure 3. Fission fragment escape probability as a function of fuel particle size.

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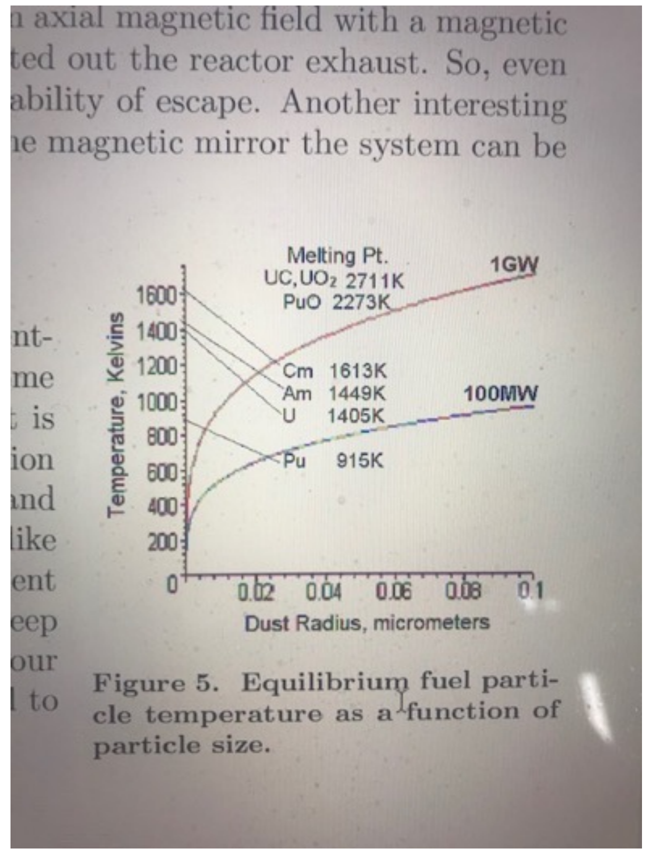
For a 40 cm thick cloud of 20 c
and density of 1×10^{-4} grams/cm^3,
cape probability is 11.4% without a



axial magnetic field with a magnetic
ted out the reactor exhaust. So, even
ability of escape. Another interesting
e magnetic mirror the system can be

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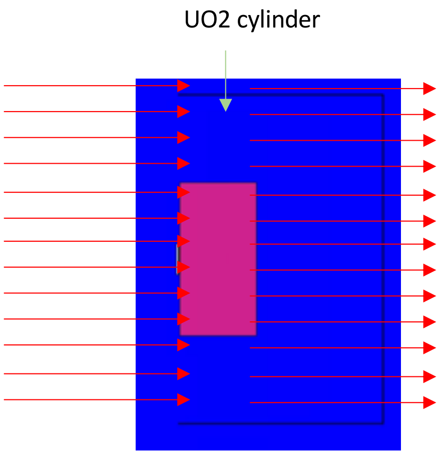
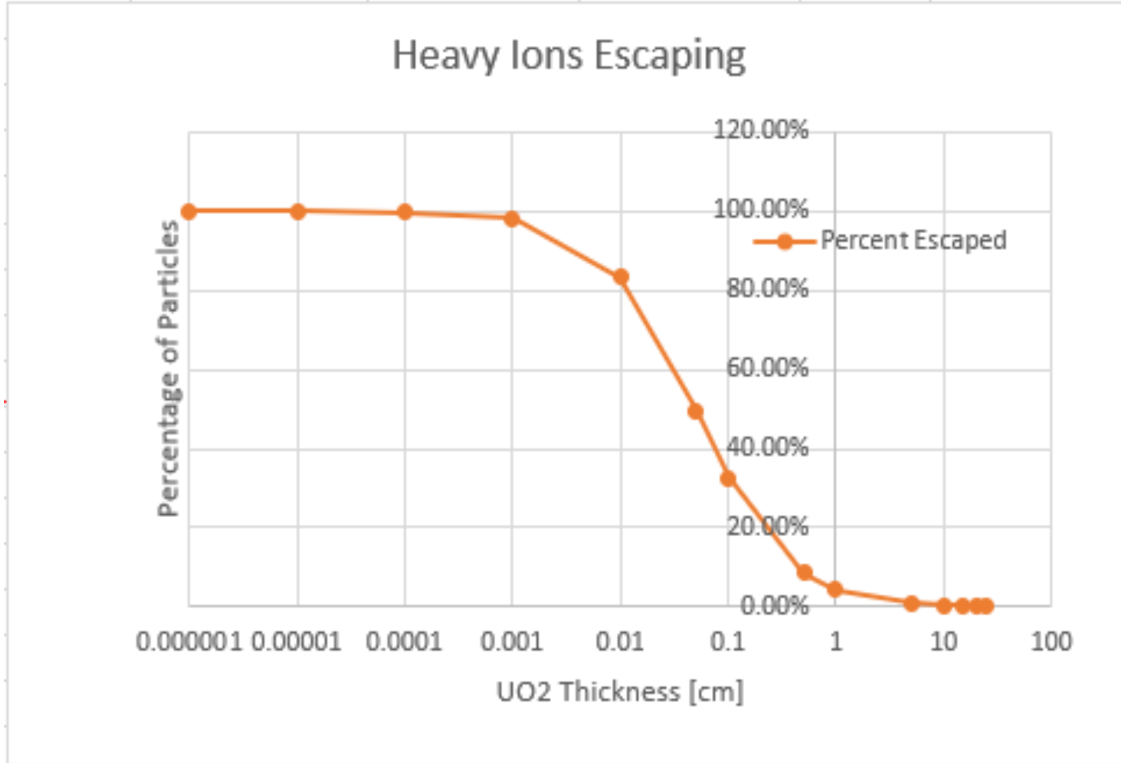
Figure 5. Equilibrium fuel particle temperature as a function of particle size.

Simulations using MCNP6.2 Code @ TTU



Source neutrons directed +x
Mag. Field also directed in +x



Summary

- New innovations in nuclear physics and engineering are occurring today at an unprecedented rate
- Nano-Nuclear Science provides for new fuel cycles, direct (non-thermal) energy conversion, and intrinsically safer nuclear reactors
- The discovery of lattice-confined, collisional secondary nuclear reactions open up an entirely new approach to better nuclear power cycle design
- A lithium deuteride fission / fusion cycle will benefit from the current at-scale lithium mining and processing industry
- Our energy / climate sustainability crisis today may have been induced by a prior innovation / inventorship crisis in nuclear science
- But we will change this now!

Responding to question regarding cross sections and the unit of 'barn (b)'

- For computational information regarding how the nuclear cross section may be used to determine the rate of a nuclear reaction, see (for example): https://en.wikipedia.org/wiki/Nuclear_cross_section
- The unit of a 'barn (b)' is the approximate cross-sectional area of a uranium nucleus (~ 10^{-12} cm)^2, and one barn = 10^{-24} cm^2. See: https://en.wikipedia.org/wiki/Barn_(unit)
- It is interesting to note that the Bohr radius $r_B \sim 5\text{x}10^{-11}$ m = $h^2/(4\pi^2 me^2)$; the Compton wavelength divided by $2\pi = \lambda_C/2\pi = h/(2\pi mc) \sim 4\text{x}10^{-13}$ m (which is the distance below which quantum electrodynamic effects become important); and the classical radius of the electron = $r_e = e^2/mc^2 \sim 3\text{x}10^{-15}$ m. Notice that $r_e = \alpha^2 r_B$, and $\lambda_C/2\pi = \alpha r_B$, where $\alpha = 2\pi e^2/(hc) \sim 1/137$. Finally, the shortest length scale discussed in physics is the plank length, r_P, where quantum gravity is predicted to dominate: $r_P = (hG/2\pi c^3)^{1/2} \sim 1.6\text{x}10^{-35}$ m.