Quantum Oscillations and Quantum Hall Effect

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Classical/Semiclassical Transport

Classical Drude

$$j = nev$$

$$\sigma = \frac{ne^2\tau}{m} = ne\mu$$

Semiclassical (Metal)

$$< j >= \int dE\ f(E) g(E) e\ v(E)$$

Boltzmann Equation

$$= \frac{e^2\tau}{dim} \int dE\ g(E)\ v^2 \left(-\frac{\partial f_{FD}}{\partial E}\right)$$

$$\sigma = \frac{v_F^2 e^2 \tau}{dim} g(E_F)$$

$\left(n_{eff} = \frac{2E_F}{dim} g(E_F)\right)$ For metal

Density of States

$$g(E) = 2\sum_{n}\int \frac{d^{dim}k}{(2\pi)^{dim}}\delta(E - E_n\,(k))$$

$$g(E) = \frac{mk}{2\pi^2\hbar^2}$$

3D (Sphere Fermi Surface)

$$g(E) = \frac{m}{2\pi\hbar^2}$$

2D (Circle Fermi Surface)

Not all Fermi surfaces are spheres and circles! These are just textbook examples!

$$0 = e(\vec{\mathcal{E}} + \vec{v} \times \vec{B}) - \frac{m^* \vec{v}}{\tau}$$

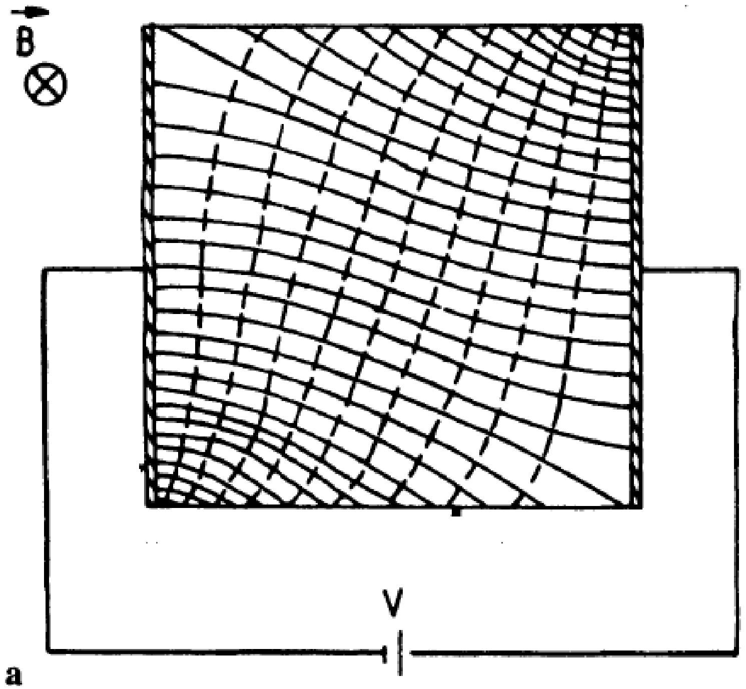
$$(ne\mu)\mathcal{E}_x = \mu B J_y + J_x,$$

$$(ne\mu)\mathcal{E}_y = -\mu B J_x + J_y,$$

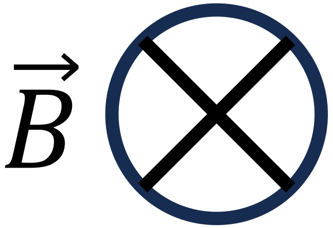
$$\vec{J} = \frac{ne\mu}{1 + (\mu B)^2} \begin{pmatrix} 1 & \mu B \\ -\mu B & 1 \end{pmatrix} \vec{\mathcal{E}}$$

If $j_y = 0$, $R_{\mathrm{Hall}} = \frac{V_y}{I_x} = \frac{\mathcal{E}_y W}{J_x W} = -\frac{B}{ne}.$

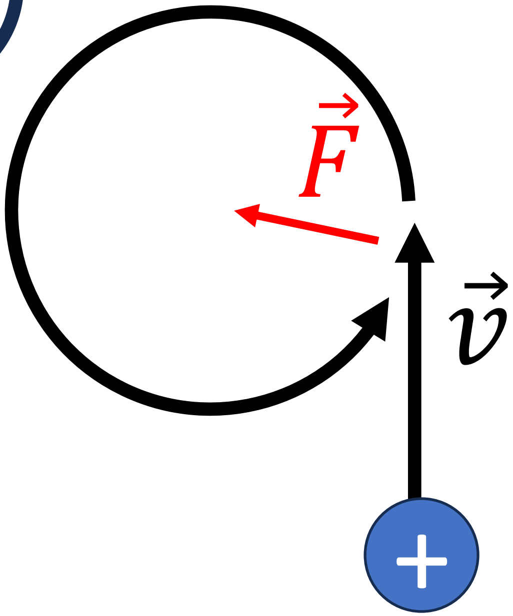
Electron Experiencing Lorentz Force



Seeger, Semiconductor Physics (2004)



$$\omega_c = \frac{eB}{m^*}$$



At high magnetic fields or high mobility samples, carriers can form a cyclotron orbit

What happens if you solve quantum mechanically?

What are you solving?

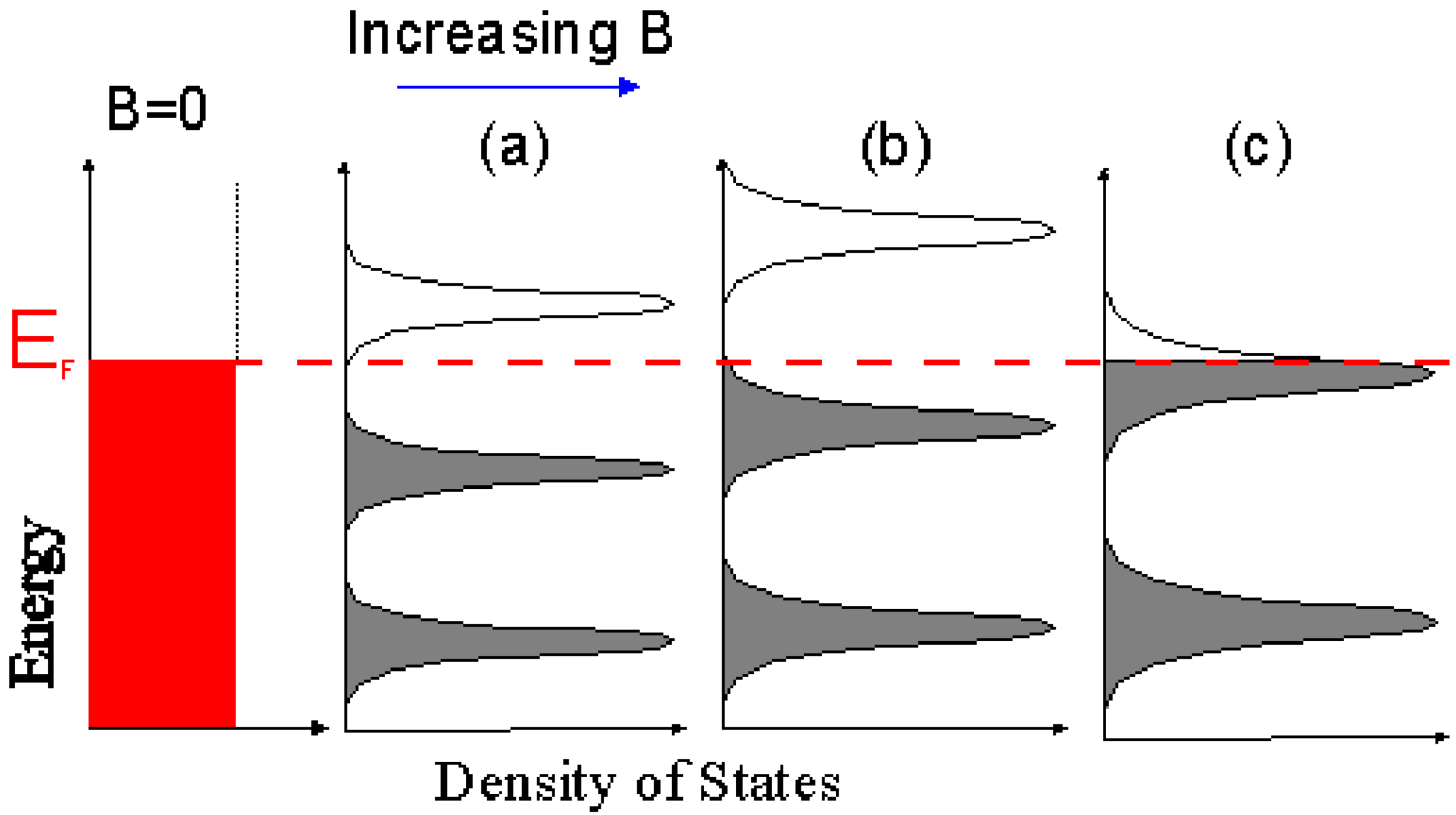
$$\hat{H} = \frac{1}{2m}(\hat{p}^2 - q\vec{A})^2$$

$$\psi(x, y) = f(x)e^{-iky}$$ And choose $$\vec{A}(x, y) = (Bx)\hat{y}$$ (Landau Gauge)

$$-\frac{\hbar^2}{2m}f''(x) + \frac{1}{2}m{\omega_c}^2(x + \frac{\hbar k}{eB})^2 f(x) = Ef(x)$$

$$E_n = \hbar\omega_c(n + \frac{1}{2})$$ Same as Harmonic Oscillator!

Density of States



Onsager Relation

$$g(E) = C\sum_n \delta(E - (n + \frac{1}{2})\hbar\omega_c),$$

Poisson Formula

$$g(E) = \frac{C}{\hbar\omega_c}[1 + 2\sum_{p=1}^{\infty}\cos(2\pi(\frac{E}{\hbar\omega_c} - 1/2)p)]$$

$$\frac{\Delta\sigma(B)}{\sigma(B=0)} = D_T D_D \cos[2\pi(\frac{F}{B} - \frac{1}{2})]$$

$$F = \frac{\hbar}{2\pi e}(\pi k_F^2) = \frac{\hbar}{2\pi e}A_{FS}$$

Oscillation with 1/B and Frequency is proportional to the Fermi surface size

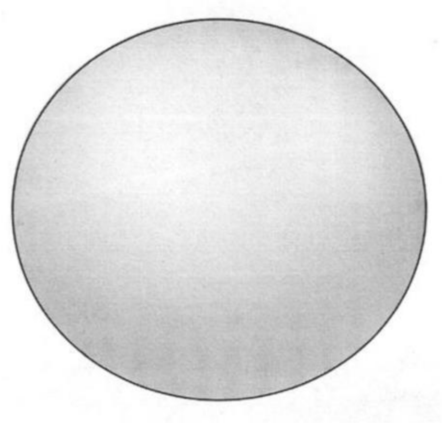
This is pretty cool but is it useful?

$$F = \frac{\hbar}{2\pi e}(\pi k_F^2) = \frac{\hbar}{2\pi e} A_{FS}$$

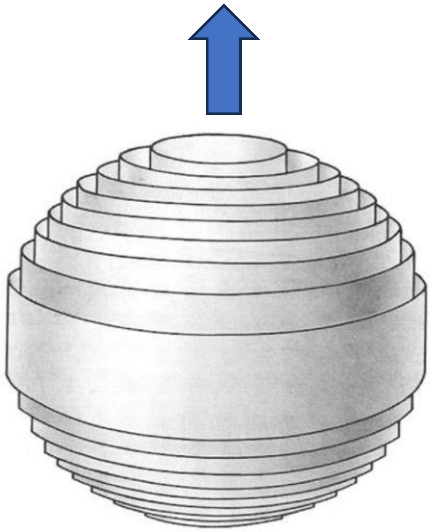
$$n = 2\left(\frac{1}{2\pi}\right)^3\left(\frac{4}{3}\pi k_F^3\right)$$ 3D

$$n = 2\left(\frac{1}{2\pi}\right)^2\left(\pi k_F^2\right)$$ 2D

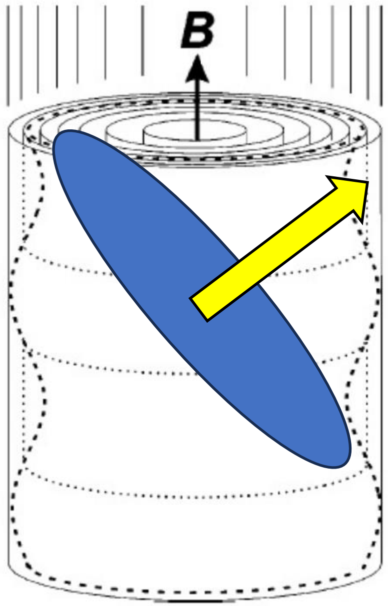
This is pretty cool but is it useful 2 ?



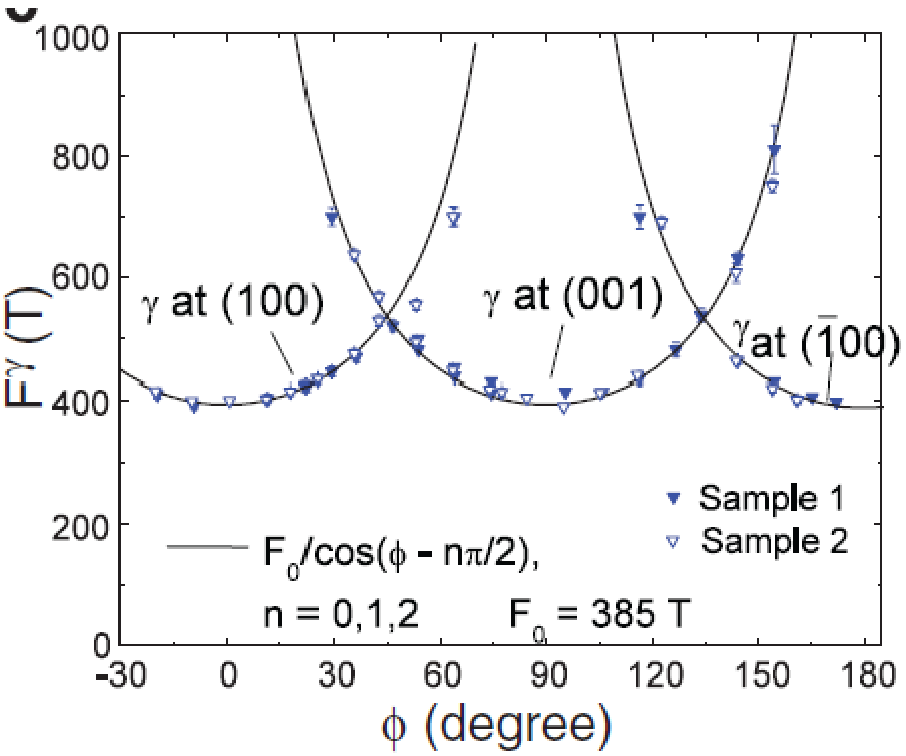
B = 0



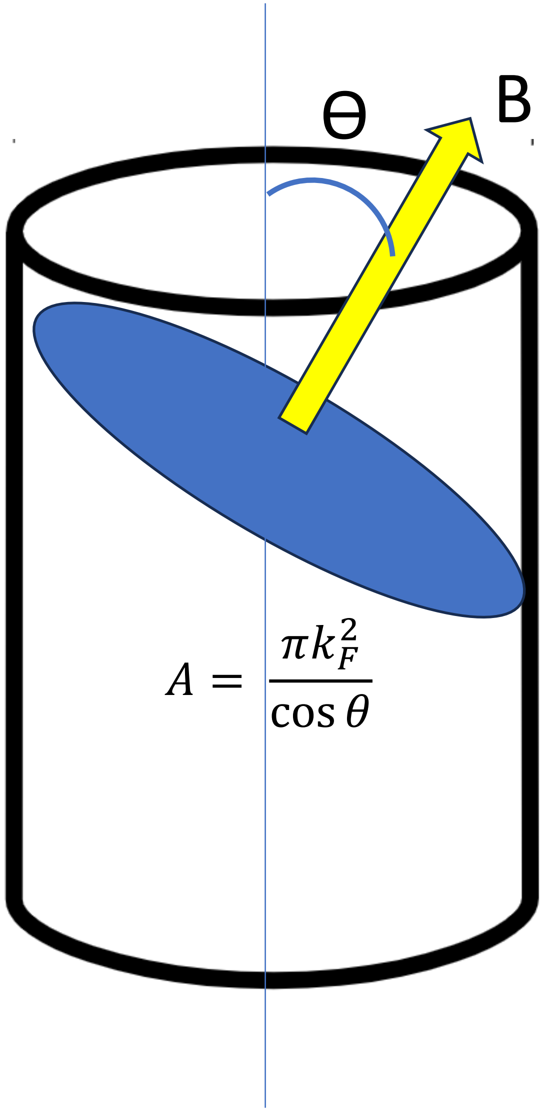
B ≠ 0



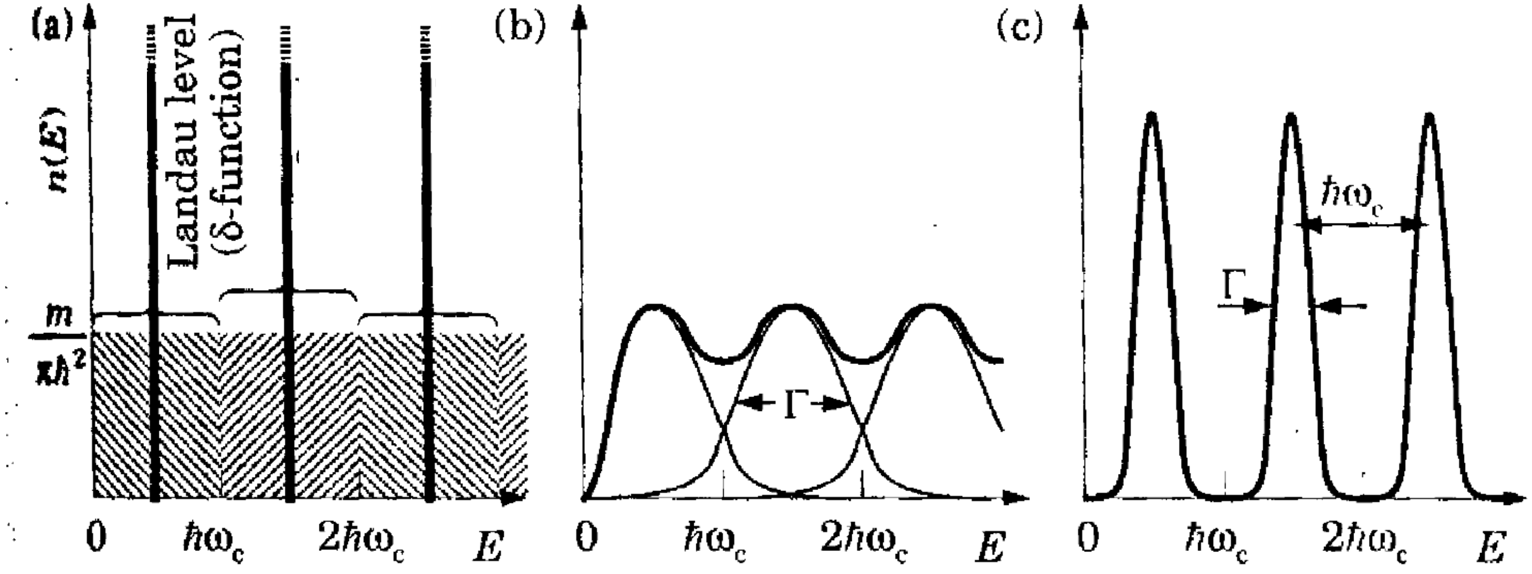
If Fermi Surface is Cylindrical or 2D



G. Li et al. Science (2014)



Reality: Disorder and Temperature Broadening



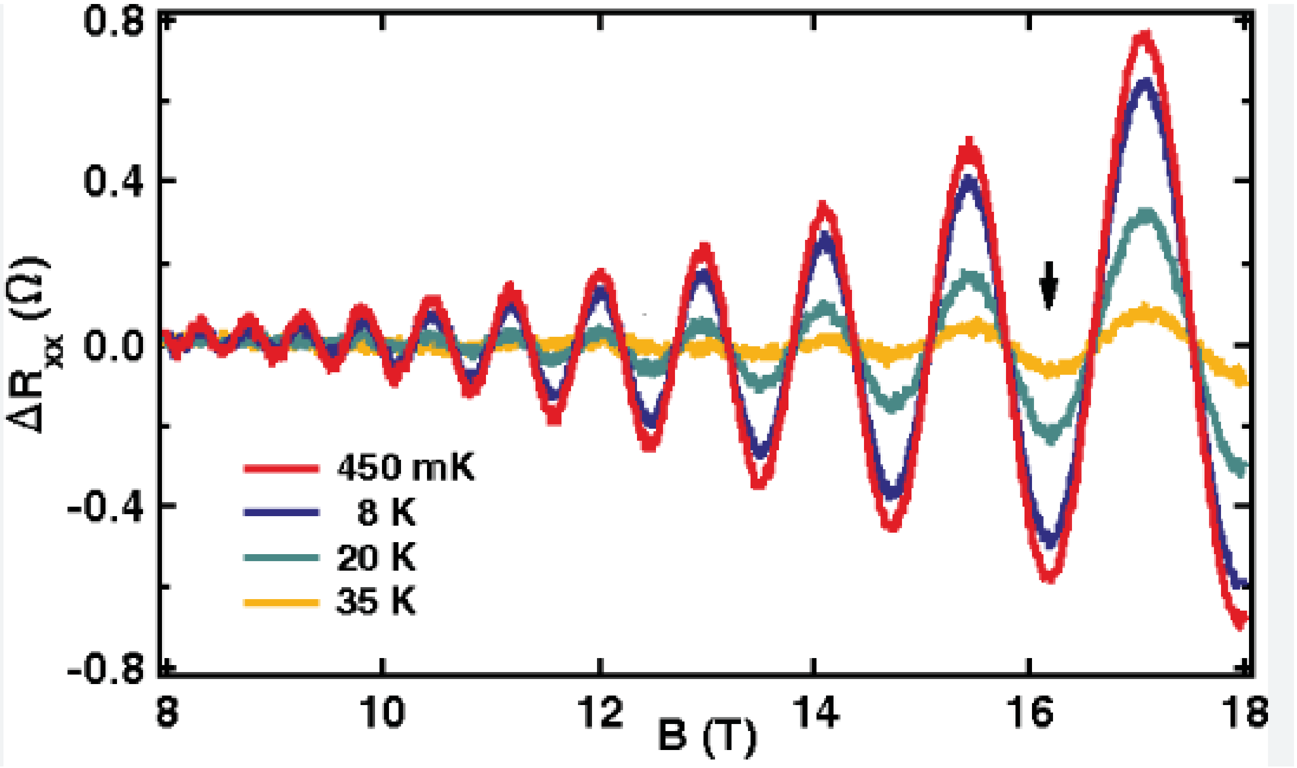
$$\frac{\Delta\sigma(B)}{\sigma(B=0)} = D_T D_D \cos[2\pi(\frac{F}{B} - \frac{1}{2})]$$

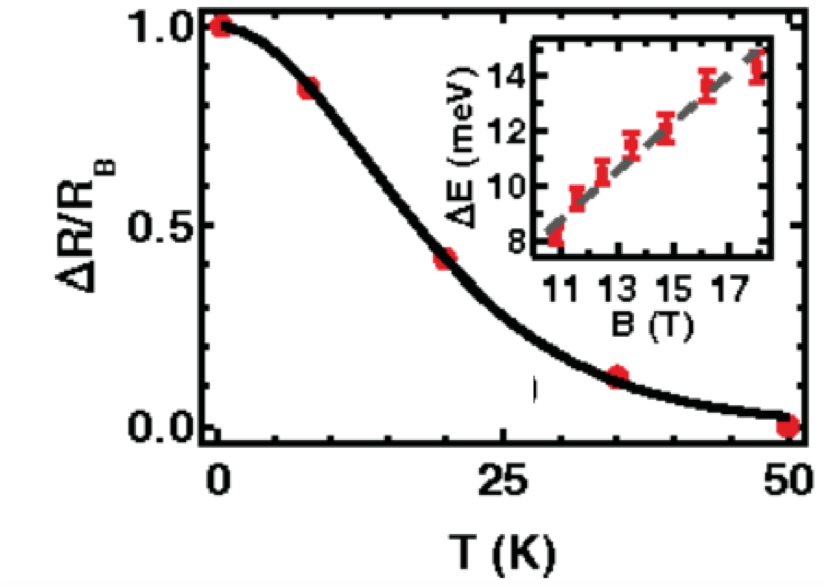
$$D_T = \frac{\frac{2\pi^2 k_B T}{\hbar\omega_c}}{\sinh[\frac{2\pi^2 k_B T}{\hbar\omega_c}]},$$

Temperature Damping Factor
(Amplitude becomes bigger with Temperature)

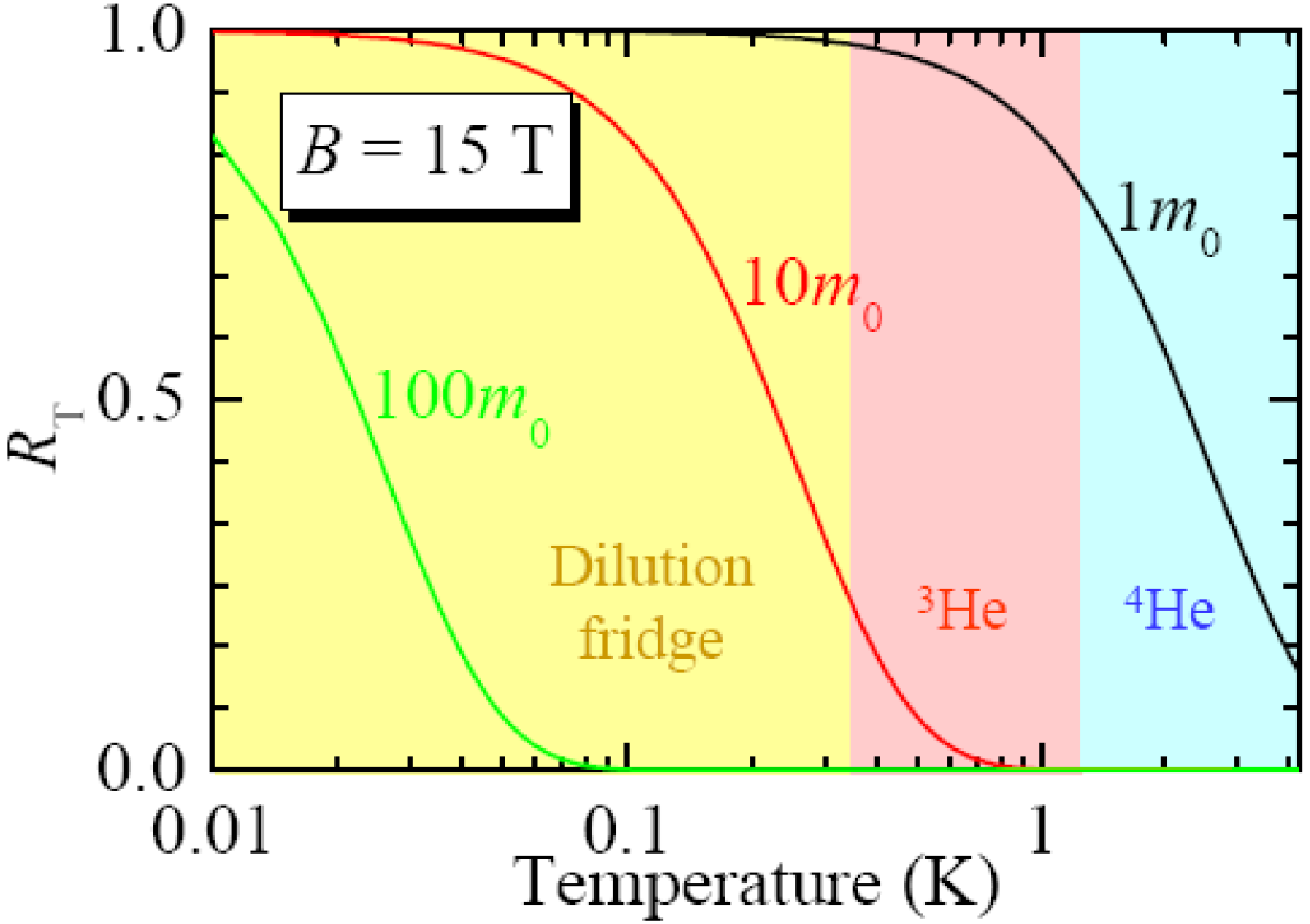
$$D_D = \exp(-\frac{\pi}{\omega_c \tau_Q}) = \exp(-\frac{\pi}{\mu_Q B})$$

Dingle Damping Factor
(Amplitude becomes bigger with B-field)



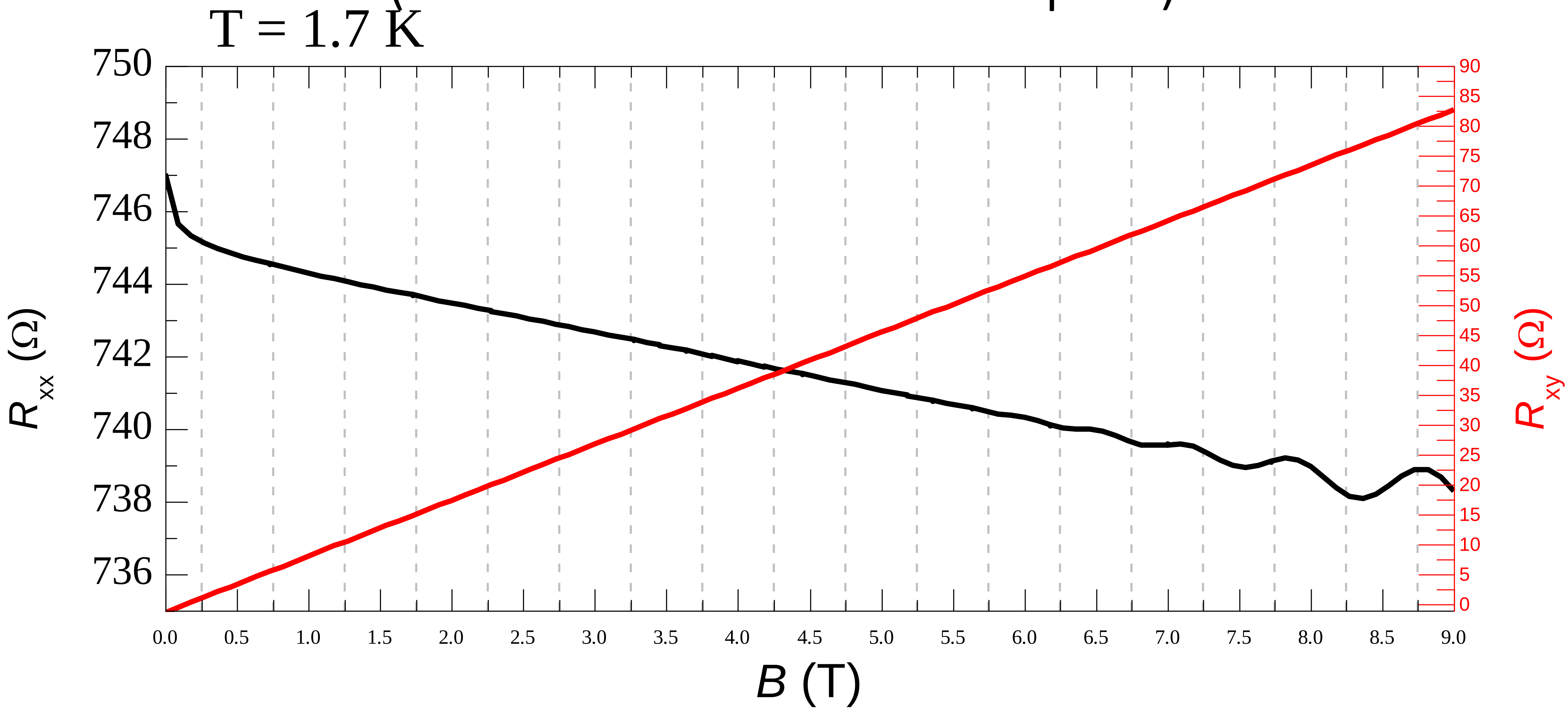


Bi_2Se_3 H. Cao PRL 108, 216803 (2012)



B = 15 T
1.0
0.5
0.0
R_T
100m_0
10m_0
1m_0
Dilution fridge
³He
⁴He
0.01
0.1
1
Temperature (K)

Mysterious Thin Film Material (3D)
(GaN with Bismuth doped)

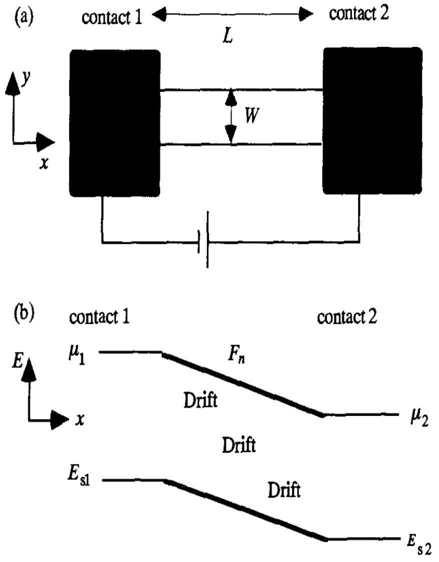


Some Interesting Questions

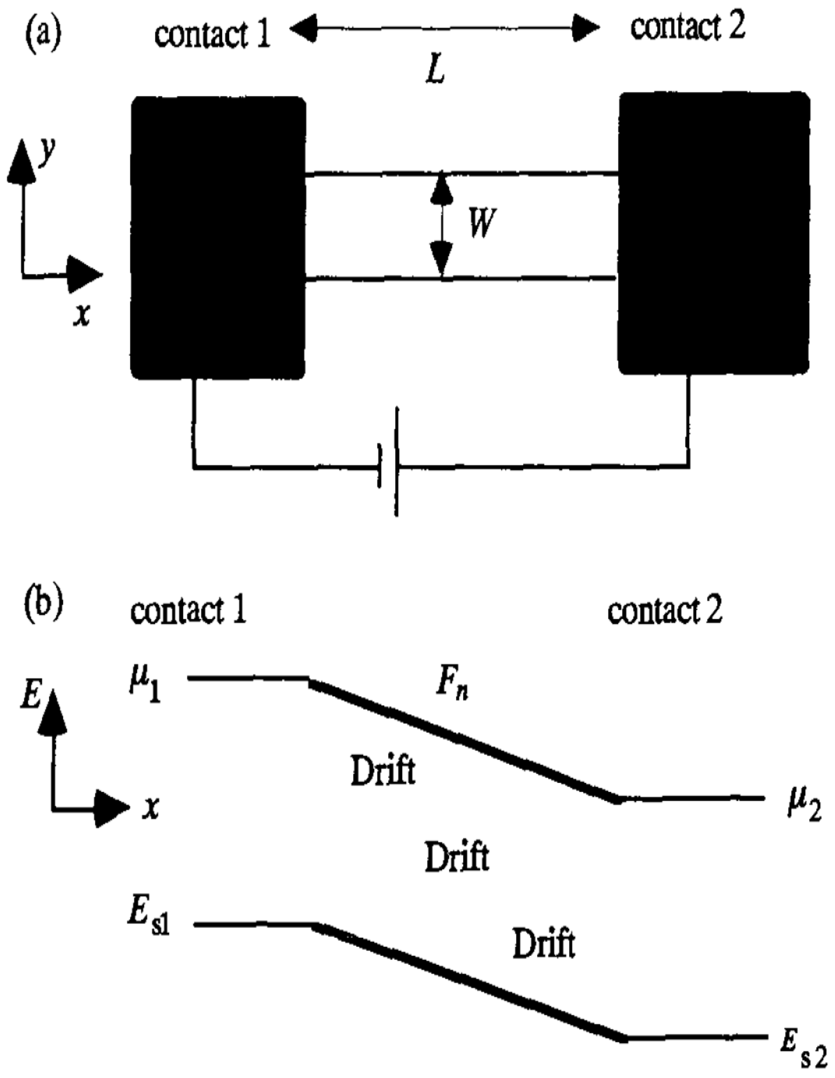
- Roughly Speaking, when does the Quantum Oscillations Start? Can one estimate what quality of the sample is needed to see quantum oscillations at 9T?
- By eyeballing the oscillations, can one estimate the Frequency?
- From the frequency, what is the carrier density? (Assume the Fermi surface is a sphere)
- Does this agree with the Hall effect data? (Thickness = 200 nm)

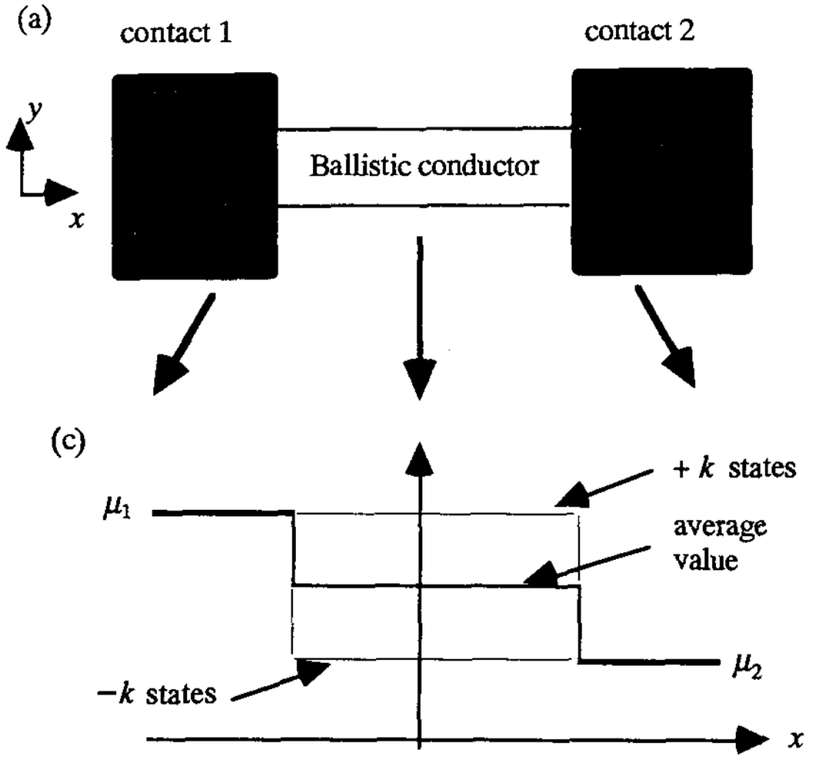
Ballistic Transport

- What happens if the electrons do not scatter in the channel? Is there no voltage drop?



- How should we understand current and voltage (Ohm's law)?


(a)
contact 1
contact 2
L
y
x
W
(b)
contact 1
contact 2
E
x
μ_1
F_n
Drift
μ_2
Drift
E_{s1}
Drift
E_{s2}


(a)
contact 1
contact 2
y
x
Ballistic conductor
(c)
μ_1
+ k states
average
value
μ_2
−k states
x

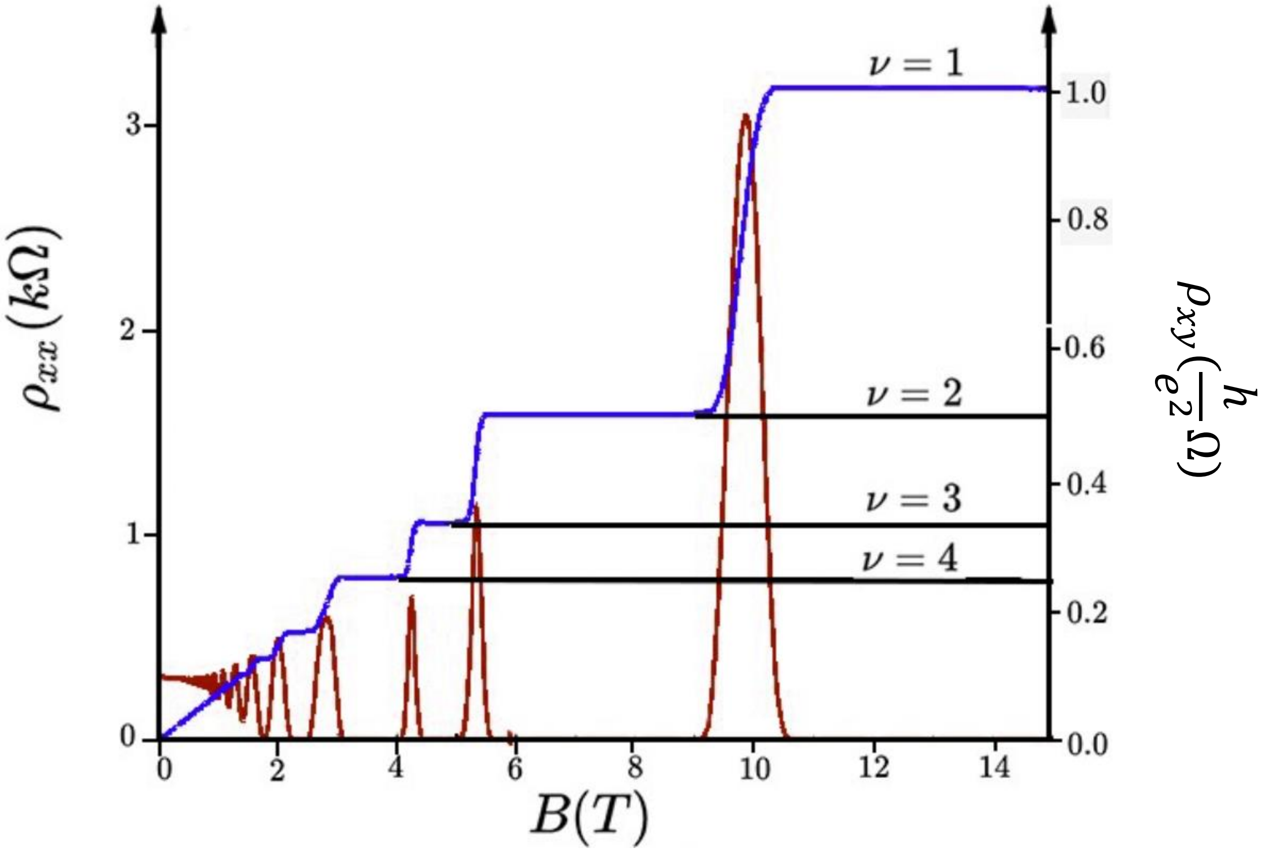
$$I = \frac{e^2}{h} MV$$

Two-dimensional Electron Gas (2DEG)

- What is a 2DEG?
  - Electrons are confined so that it has no *z*- degree of freedom.

  - How do you realize 2DEG systems?
    - By confinement: Designing a quantum well

2DEG under Magnetic Field?

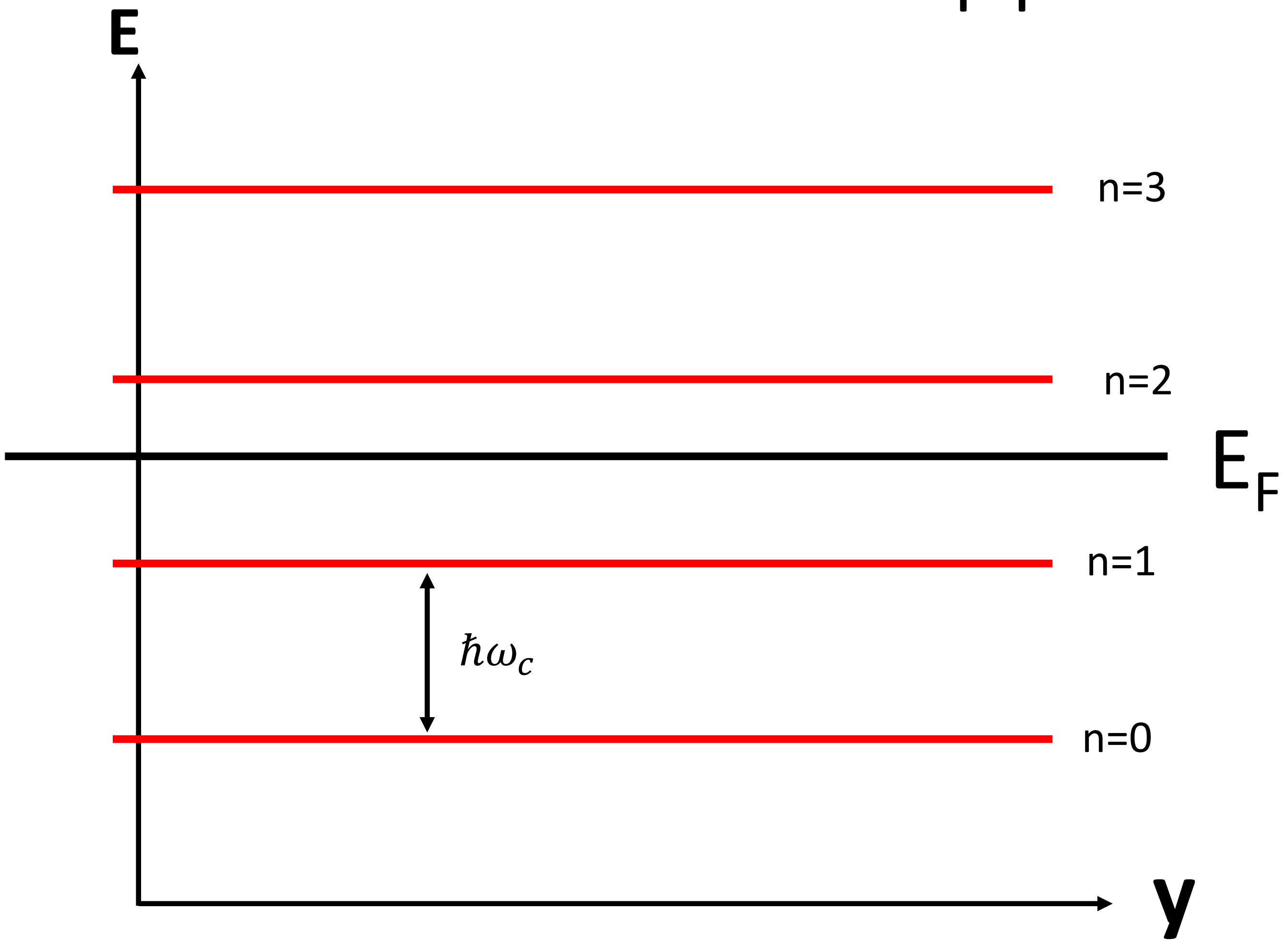


Hall effect (Rxy) is not a straight line. There are plateaus

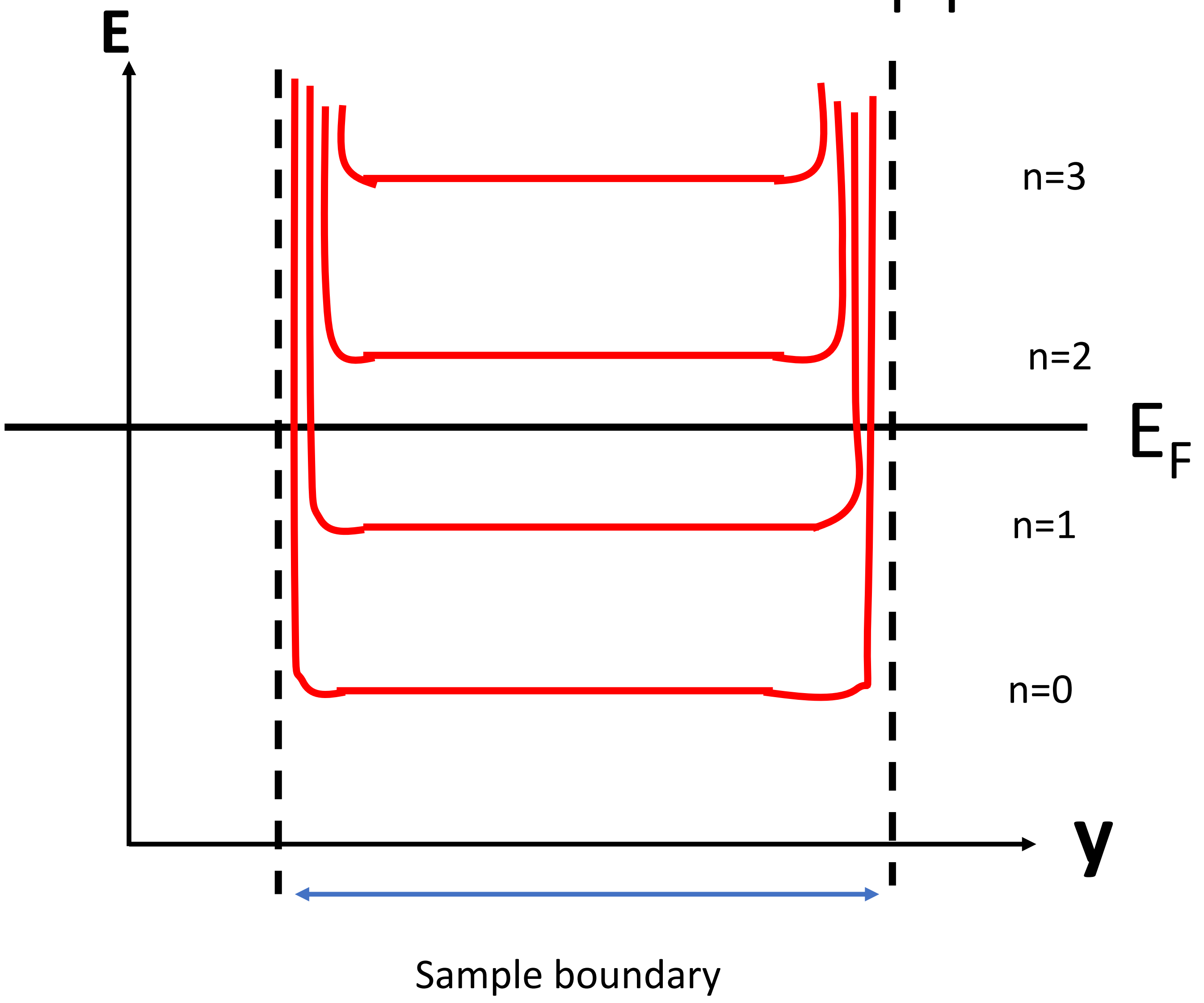
h/e^2 = 25.812807 kOhm

Rxx don't look like cosines: drops to zero

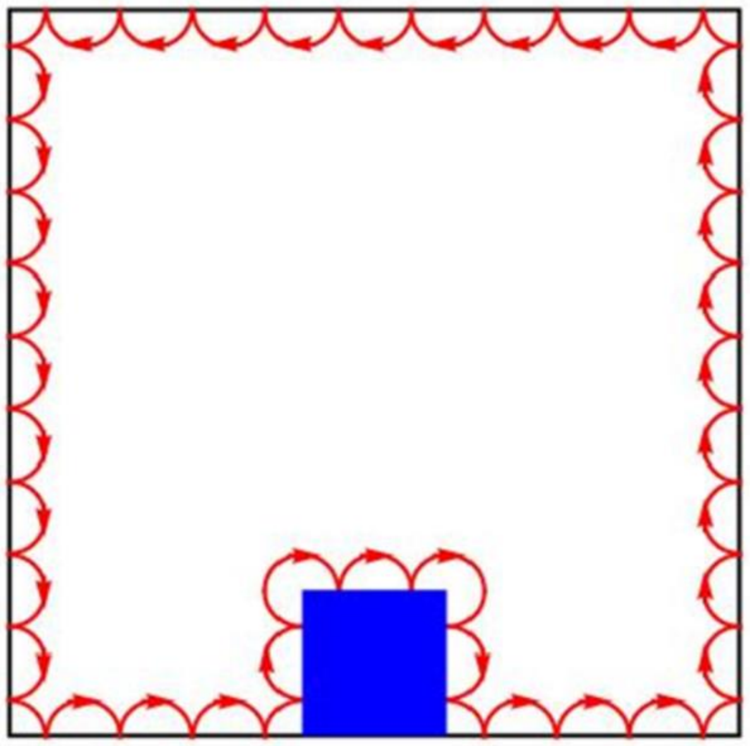
What happens in 2D?



What happens in 2D?



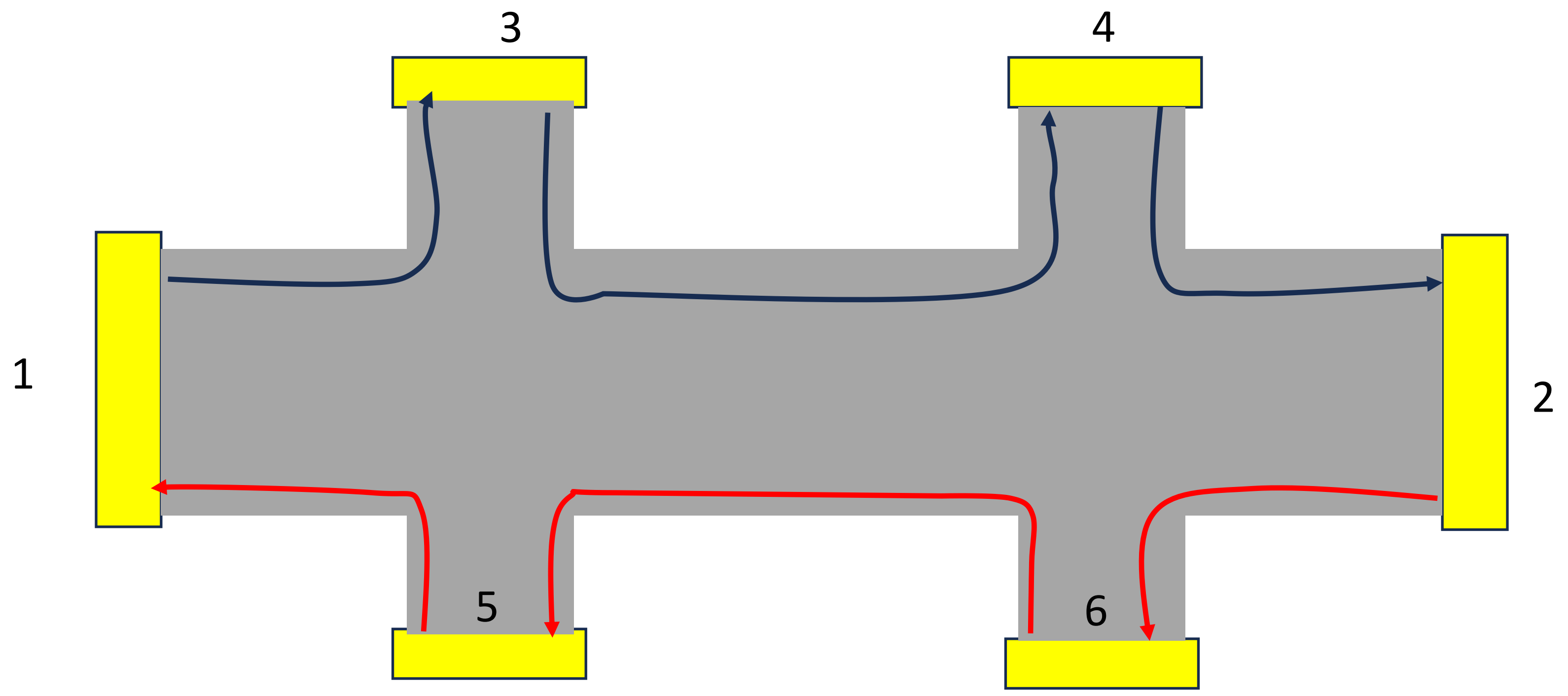
What happens in 2D? Classical Picture



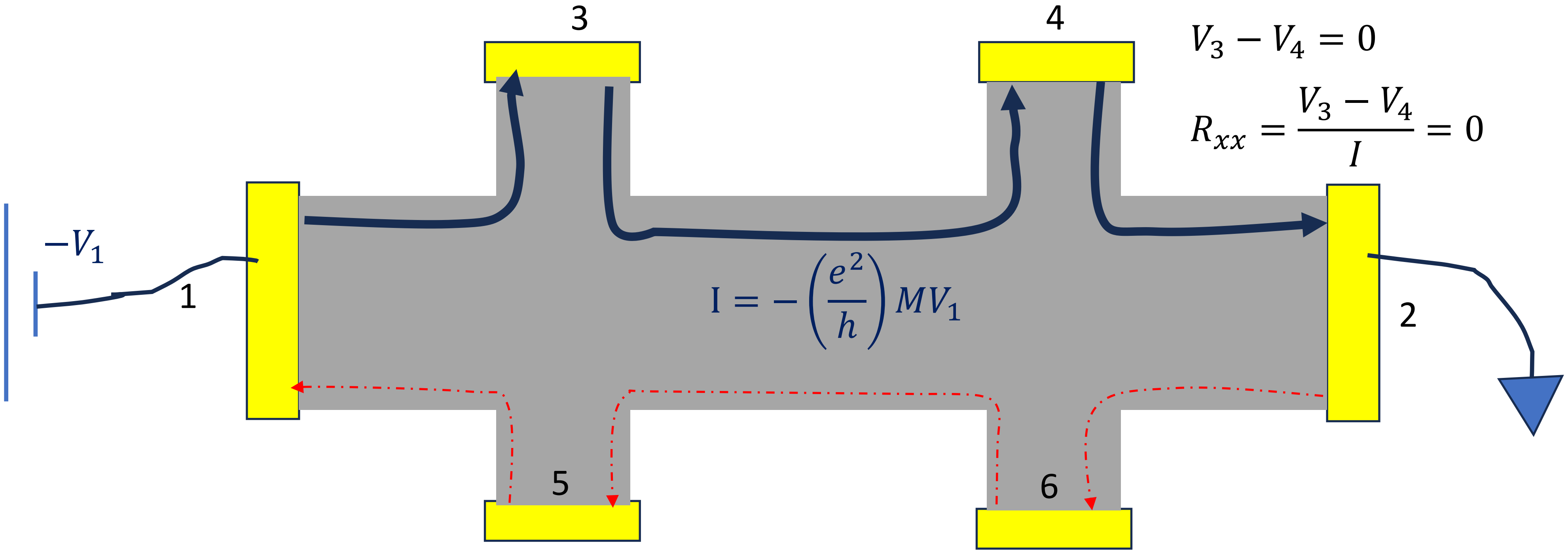
$$I = \frac{e^2}{h} VM$$

(Need to use transport in the ballistic regime)

What if we apply bias Voltage with leads?



What if we apply bias Voltage with leads?



$$V_3 - V_4 = 0$$

$$R_{xx} = \frac{V_3 - V_4}{I} = 0$$

$$V_3 - V_5 = -V_1$$

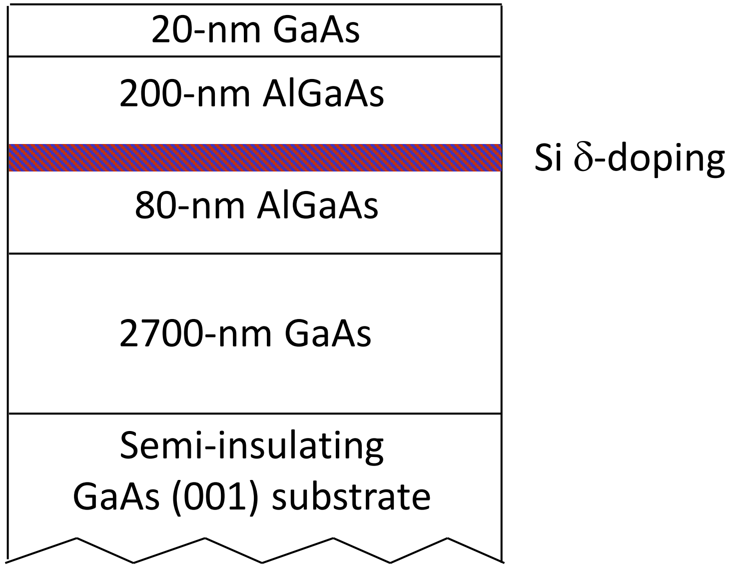
$$R_{xy} = \frac{V_3 - V_5}{I} = \frac{h}{e^2 M}$$

How do you make 2DEGs?

- Quantum Well Structure

Delta-doped GaAs/AlGaAs Hetrostructures with a 2DEG

GaAs/AlGaAs Heterostruture

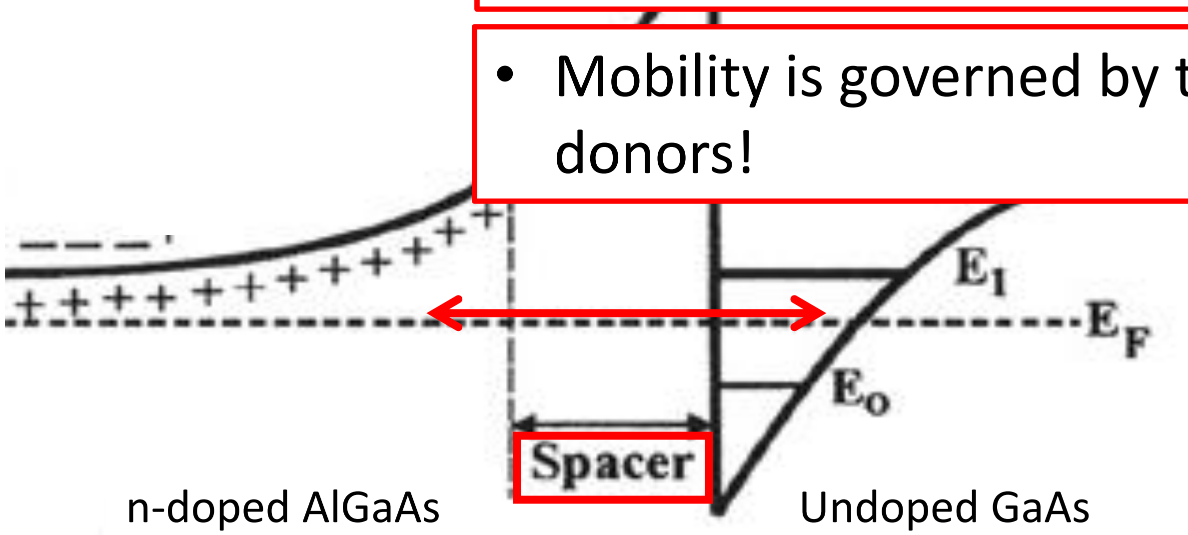


- Thin film grown by Loren Pfeiffer at Princeton University
- We used three different samples that have different doping density

Doping density = 1.5×10^{12} (cm^{-2})

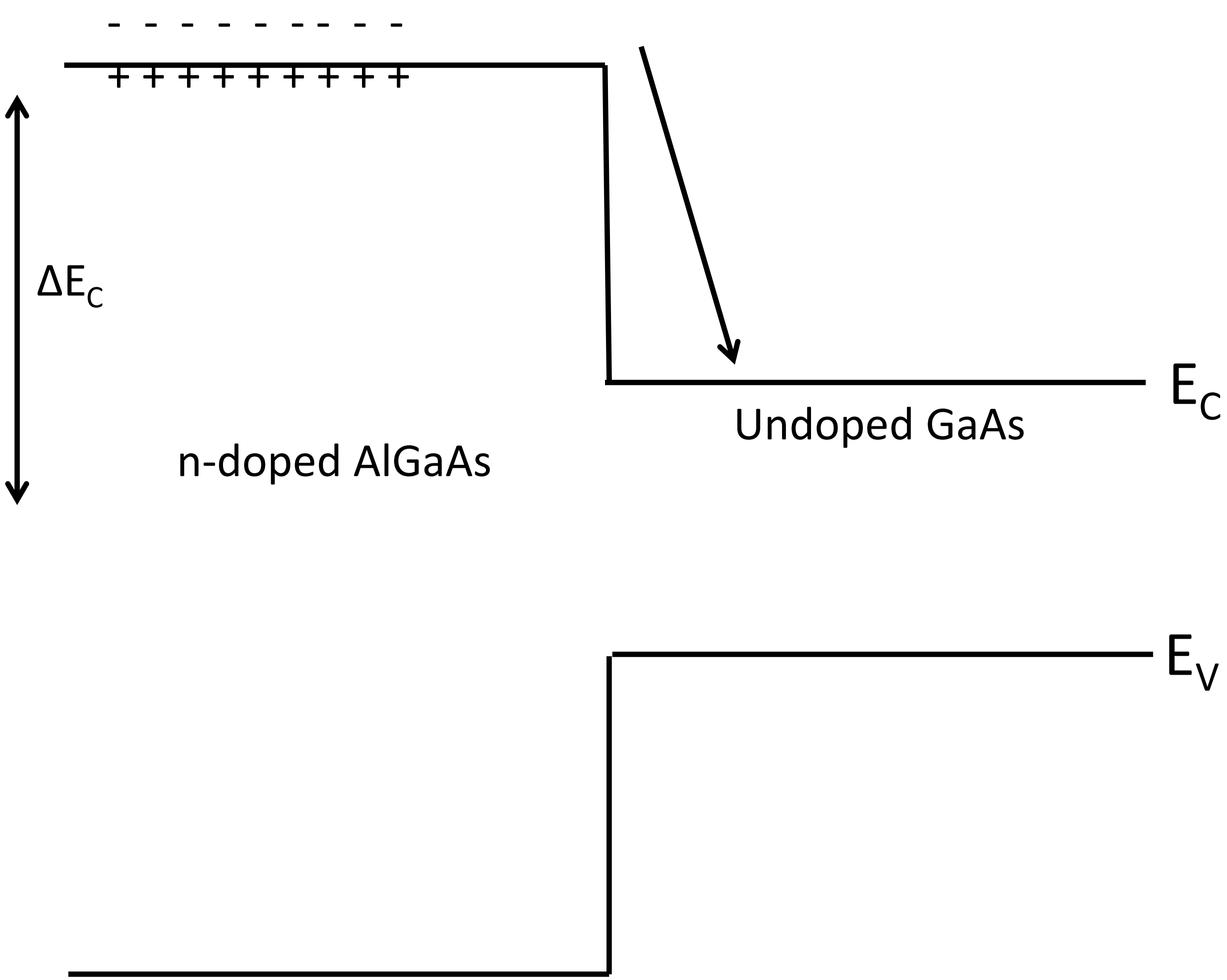
Remote Modulation Doping

- Carriers are separated from impurities.
- Forms a 2DEG.
- Mobility is governed by the potential of donors!



Remote Modulation Doping

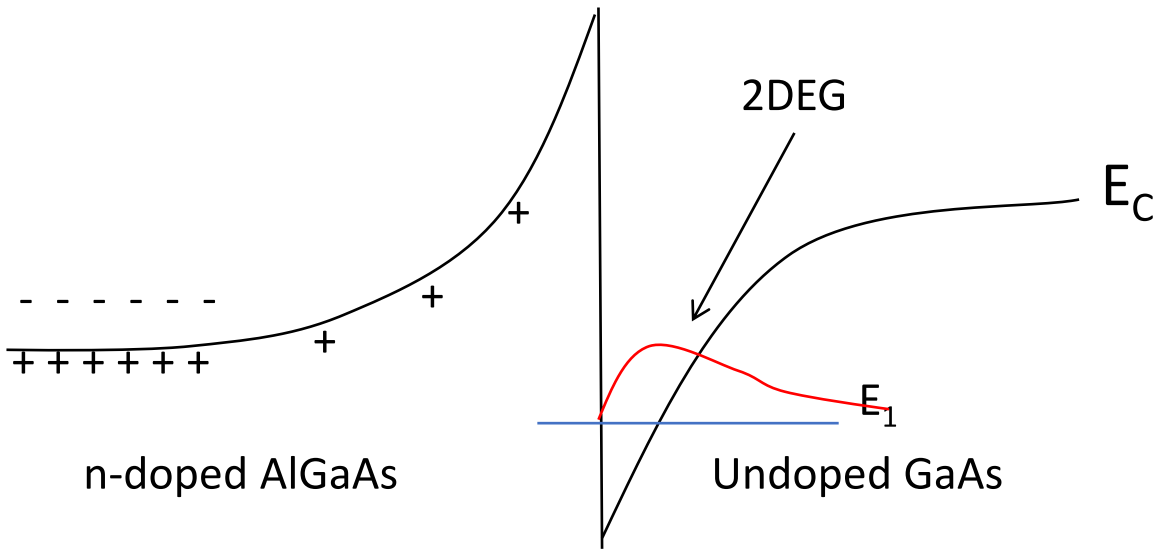
Lattice-matched Heterostructure



Type 1 alignment

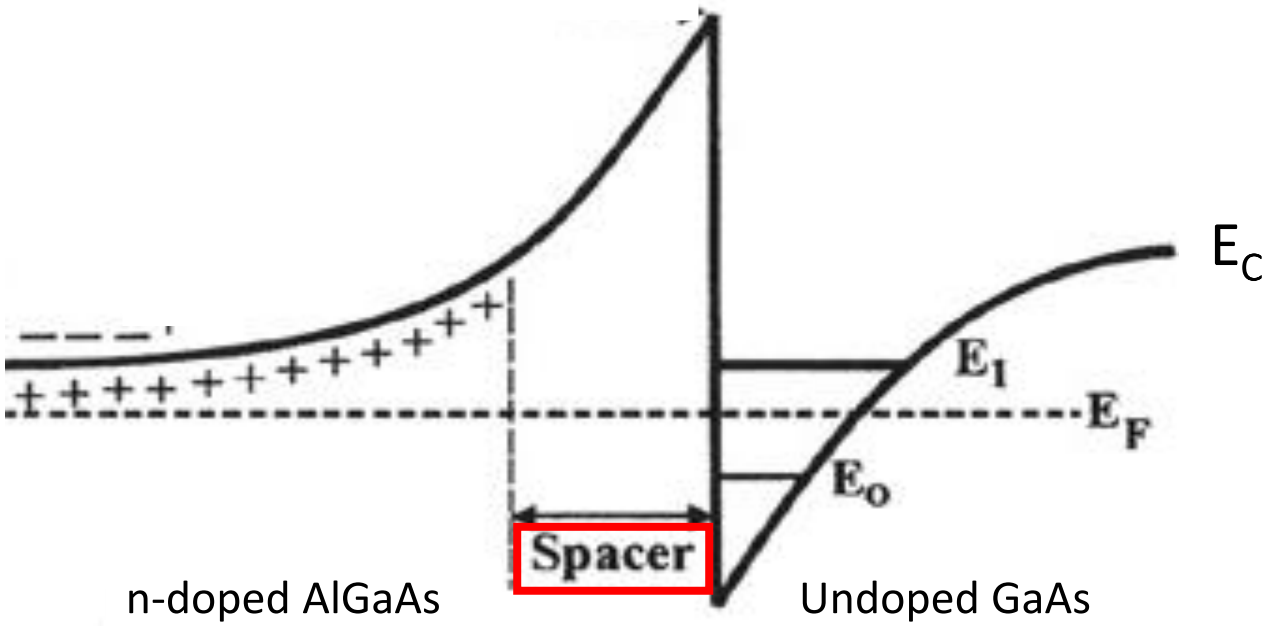
Remote Modulation Doping

2DEG Formation



Remote Modulation Doping

Adding Spacer Undoped-AlGaAs



How to actually calcultate?
Self Consistent Method

$$\rho(x) = (N_D - n_c)e$$

$$\nabla E = -\frac{\rho}{\varepsilon}$$

Creates voltage that bends the conduction band

$$E_c(x)$$

$$n_c(x)$$

Schrodinger Equation to calculate the Quantum Well

https://www3.nd.edu/~gsnider/

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- **Nanodevices Group Website This is the group website that actually gets maintained!**

Welcome to Greg Snider's WWW HomePage! You will find here information about my research and teaching interestes, as well as about my educational background. Just click the corresponding item below.
My apologies for "Missing Links", but things are still under construction.

- **Education and Background**
- **QCA Data**
- **Research and Publications**
- **Teaching and Courses**

- **Notre Dame Nanofabrication Facility**

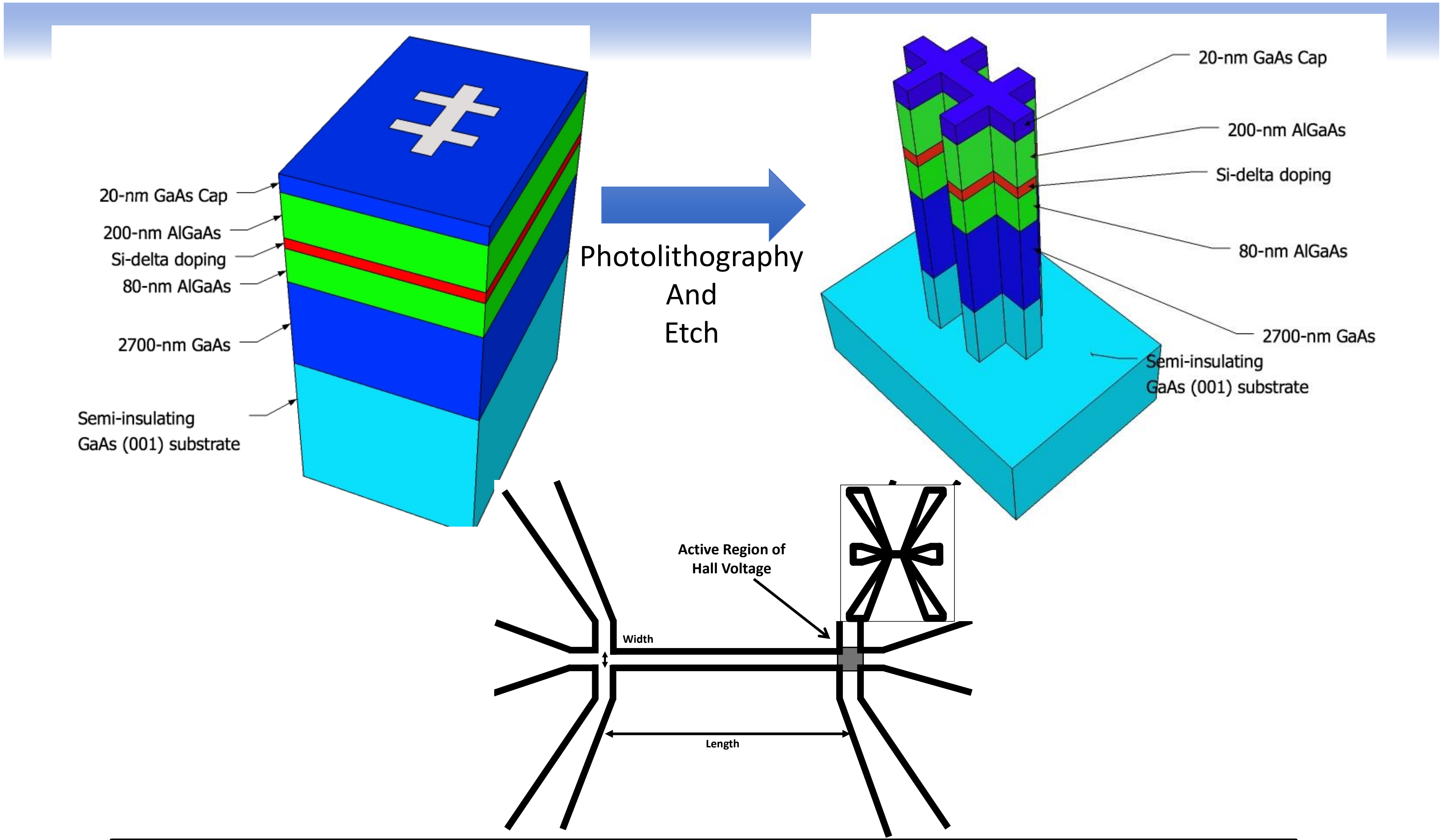
1D Poisson is a program for calculating energy band diagrams for semiconductor structures. It is a FreeWare program that I've written which solves the one-dimensional Poisson and Schrodinger equations self-consistently. The program is quite user friendly, and runs on a Macintosh, Linux or PC. These are the current versions of the program that have a number of nice new features. Click on one of the links below to download an archive containing a version of the program. The Windows and Linux versions have all the same features and use the same files as the Mac version. All currently run as terminal applications.

- Download Mac OSX (Universal) 1D Poisson (zip file) (Mac version beta 8j1)
- Download PC 1D Poisson (zip file) (PC version beta 8j1)
- Download PC 1D Poisson (zip file) (PC version beta 8k command line)
- Please contact me if you would like a Linux version.

This page is maintained by Gregory Snider.

Numerical Solver is available for free!

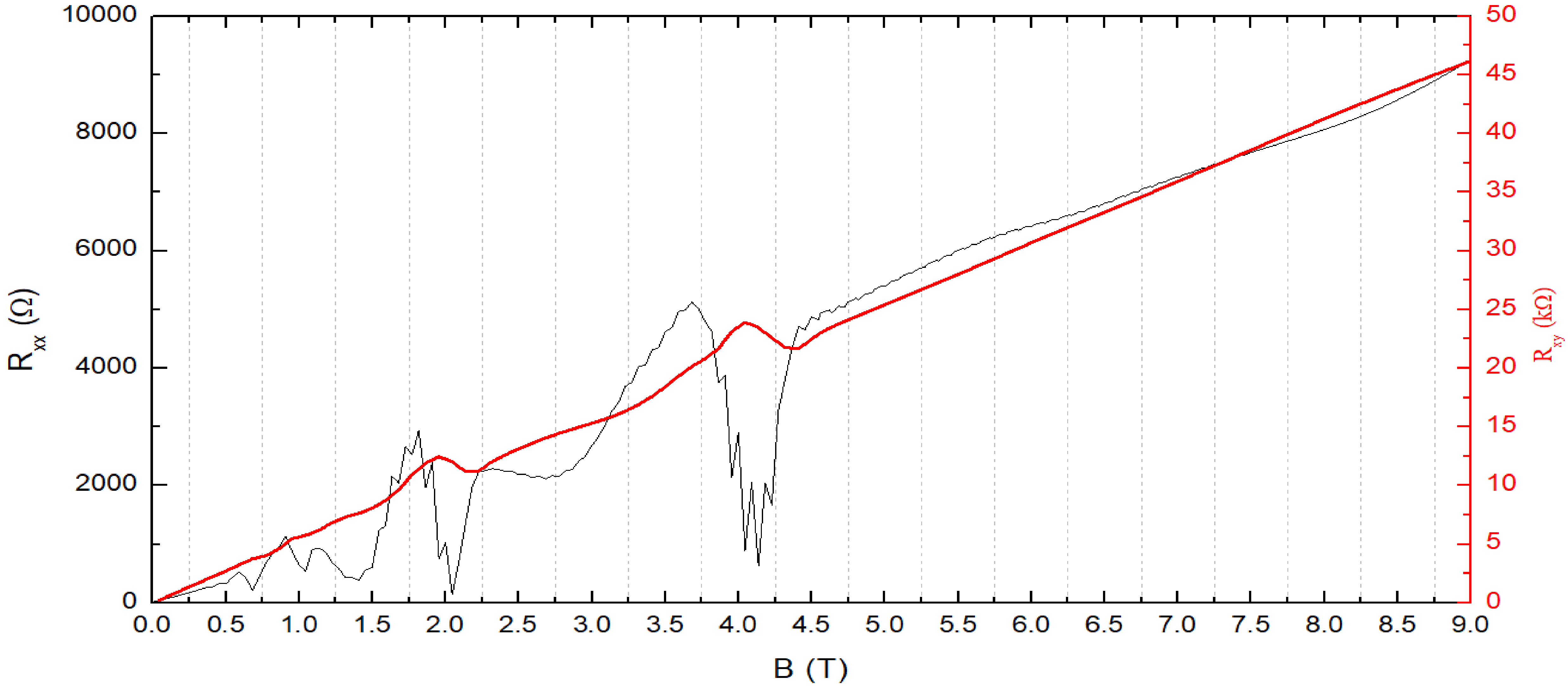
Hall Bar Fabrication



Etch away the unwanted region so that only the Hall bar pattern forms a 2DEG

GaAs/AlGaAs data

T = 1.8 K

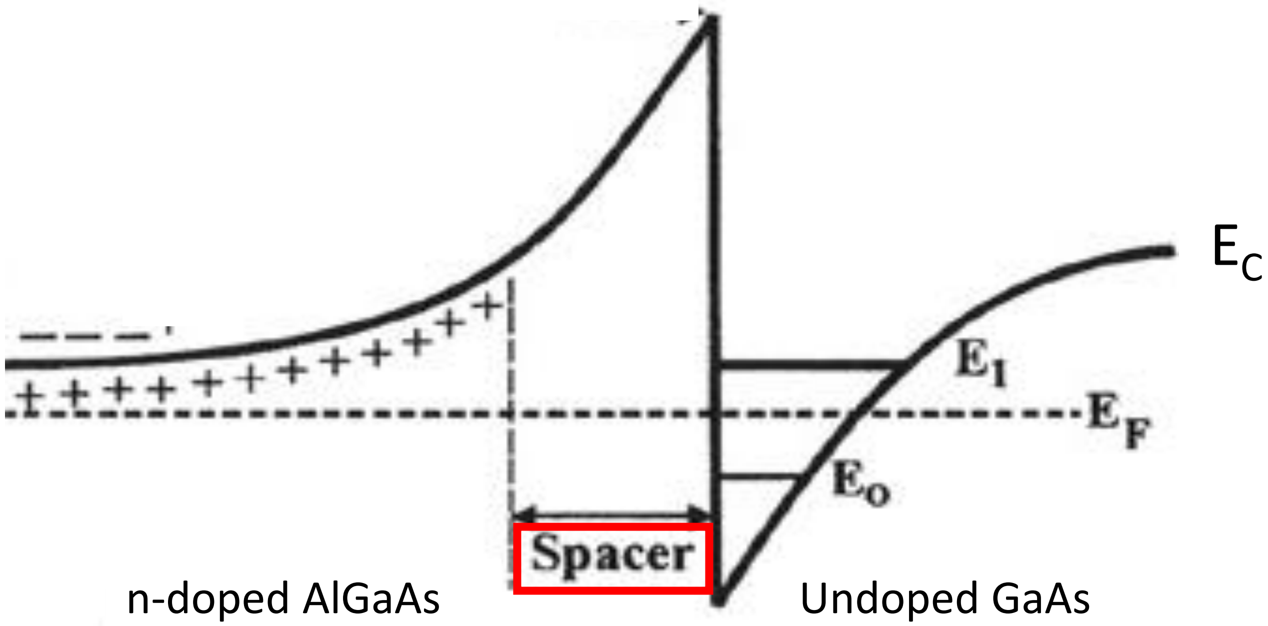


Interesting Questions

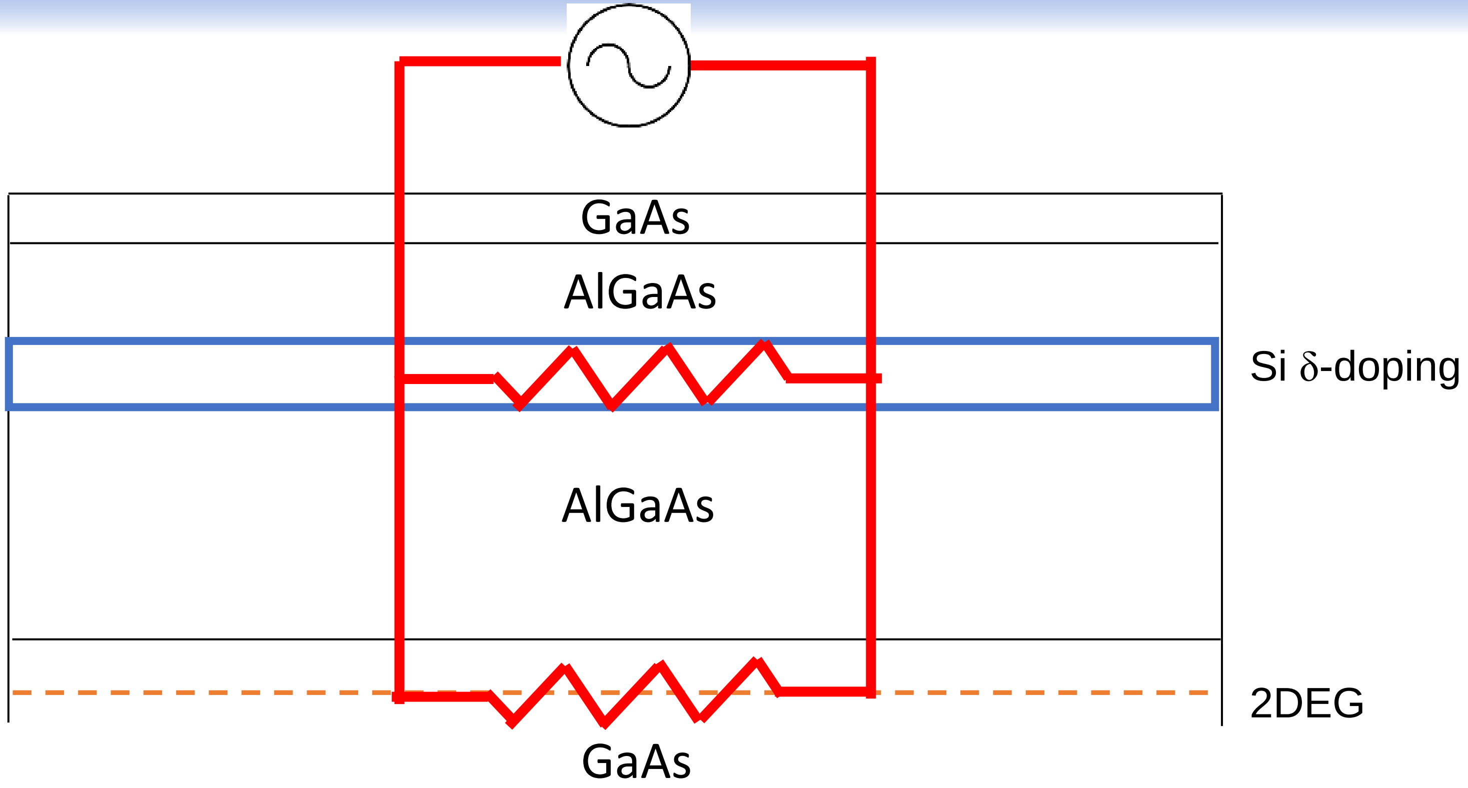
- Where is N = 1?
- What is the carrier density of this 2DEG?
- What doesn't look right in this data?

Remote Modulation Doping

Adding Spacer Undoped-AlGaAs



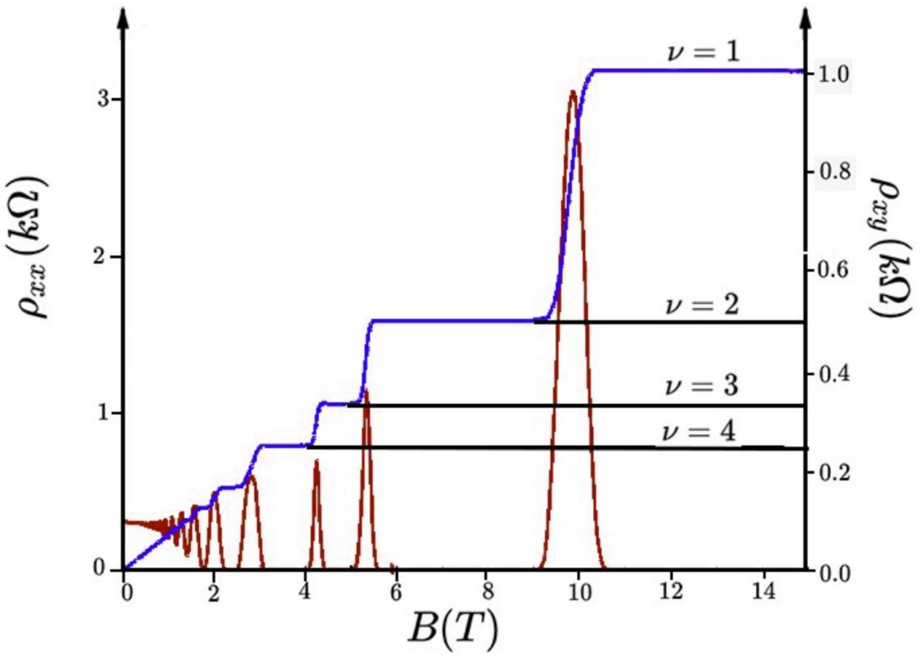
Parallel Conduction

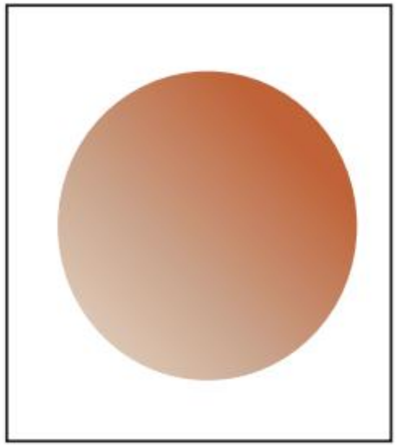


- Additional conduction channel is introduced to the system if the doping density is too large

Quantum Hall Effect: Ancestor of Topological Insulators

What is topological? Topology of the wave function Hilbert Space





$\nu = 2$ $\nu = 0$

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\nu = \frac{1}{2\pi}\int dk\Omega$$

Berry Curvature

$$\Omega = \nabla \times (i \langle u(k)|\nabla_k|u(k)\rangle)$$

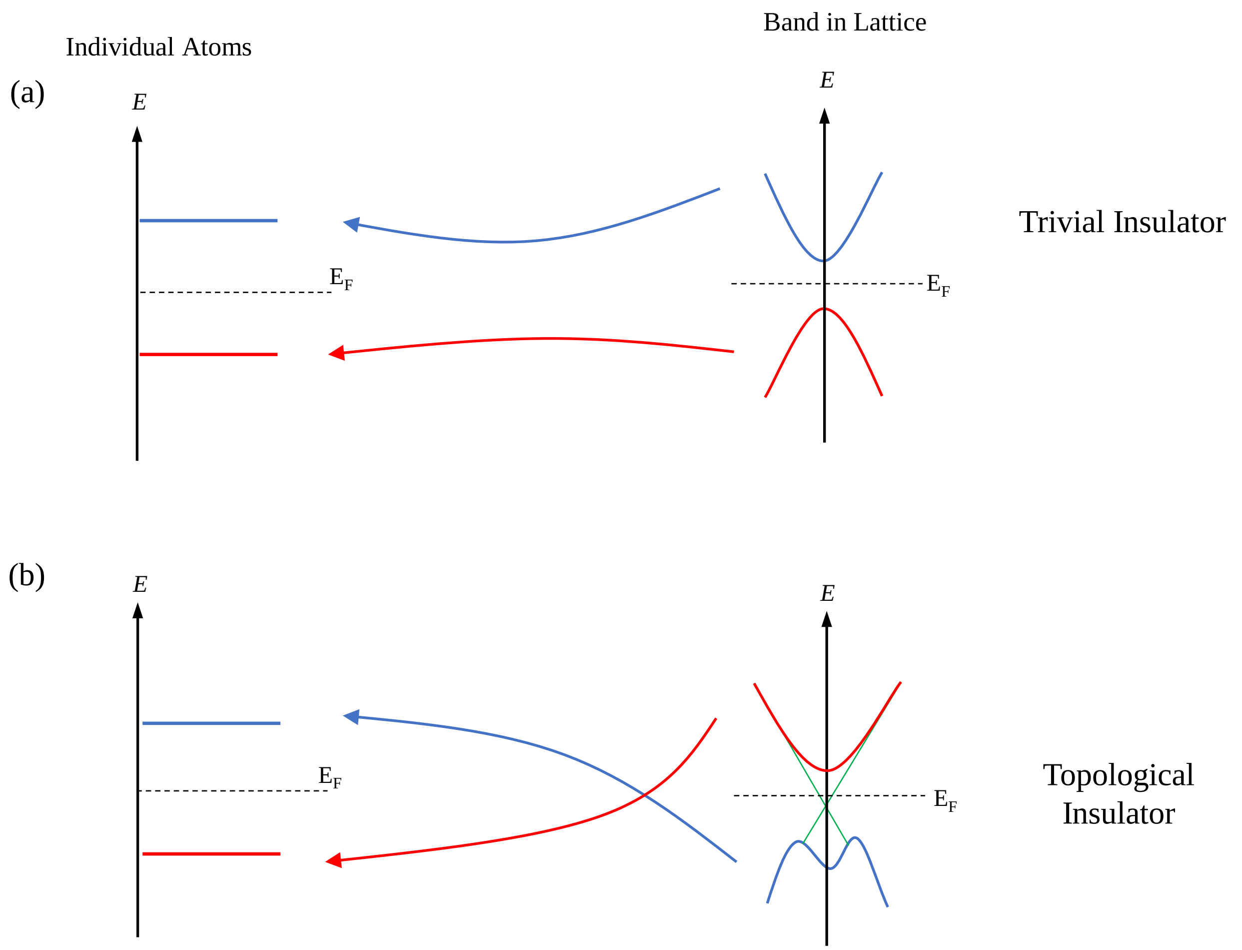
Berry Connection

Gauss-Bonnet Theorem

$$\nu = \frac{1}{2\pi}\int dS\ K$$

Curvature

3D Topological Insulators



- Realized without magnetic field (protected by Time Reversal Symmetry)
- Driven by strong spin-orbit coupling.
- Band Inversion

Theory

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Topological Insulators in Three Dimensions

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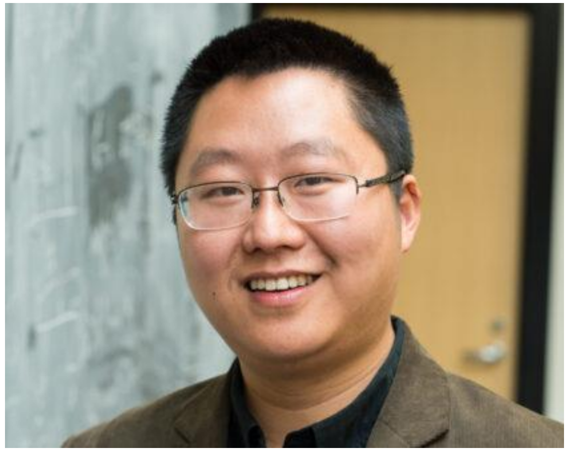

Liang Fu (MIT)