Famous Development by the Great Inventor Edwin Armstrong!

Any nonlinear characteristic in a circuit element can serve as a 'mixer', through the squared term in the Taylor's Series:

$$f(E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left. \frac{\partial^n f}{\partial E^n} \right|_{E=E_c} \cdot (E-E_c)^n \quad \text{Set } E_c = 0$$

(Maclauren series)

So, the first "mixer" term is E^2

Consider a strong field $E_1 = E_{1_0} \cos \omega_1 t$ mixing with a very weak field $E_2 = E_{2_0} \cos(\omega_2 t + \phi)$, but, for now, let $\phi = 0$. $E = E_1 + E_2$ and $(E_1+E_2)^2$ is:

$\cos \omega t = \dfrac{e^{i\omega t} + e^{-i\omega t}}{2}$, so $E_1 = E_{1_0}\left(\dfrac{e^{i\omega_1 t} + e^{-i\omega_1 t}}{2}\right)$

$E_2 = E_{2_0}\left(\dfrac{e^{i\omega_2 t} + e^{-i\omega_2 t}}{2}\right)$ and so:

$$(E_1+E_2)^2 = \left(E_{1_0}\frac{e^{i\omega_1 t} + e^{-i\omega_1 t}}{2} + E_{2_0}\frac{e^{i\omega_2 t} + e^{-i\omega_2 t}}{2}\right)^2$$

$$= \frac{1}{4}\Big[E_{1_0}^2(2 + 2\cos(2\omega_1 t)) + E_{2_0}^2(2 + 2\cos(2\omega_2 t)) + E_{1_0}E_{2_0}\left(e^{i(\omega_1+\omega_2)t} + e^{i(\omega_1-\omega_2)t} + c.c.\right)\Big]$$

$$= \frac{1}{2}\Big[E_{1_0}^2 + E_{2_0}^2 + E_{1_0}^2\cos(2\omega_1 t) + E_{2_0}^2\cos(2\omega_2 t) + E_{1_0}E_{2_0}\cdot\left[\cos(\omega_1-\omega_2)t + \cos(\omega_1+\omega_2)t\right]\Big]$$

If ω_1 is only slightly greater than ω_2, then $\Delta\omega = \omega_1 - \omega_2$ is small. Typically, ω may be an RF, and $\Delta\omega$ may fall in the audible range. We will discuss AM and FM radio, given what we have just learned. Notice that the $E_{1_0}E_{2_0}\cos\Delta\omega t$ term's amplitude is the product of a very strong "Local Oscillator (LO)" signal E_1 and the very weak E_2 signal to be received + amplified.

The product of these two terms have $E_{1_0} E_{2_0}$ amplitude, which is $\gg E_{2_0}^2$. Hence, heterodyne amplification of the weak signal E_2. Also, E_{2_0} may change slowly at audio frequencies, but E_{1_0} does not change. Put $\omega_1 = \omega_2$ and put $E_{1_0} E_{2_0}$ across a capacitor to "hear" only the time variation in E_{2_0}. This is "AM". How about FM?: Well, keep both E_{1_0} and E_{2_0} constant, and vary $\Delta\omega$ at audio frequencies. The result is "FM".

For radioastronomy, $L_J = \frac{h}{2e}(I^2 - I_c^2)^{-1/2}$, and convert I to E for the arguement above. Now, expand about $I = I_c$ (or, better yet, $I = 0.9 I_c$) to get a stronger nonlinearity.

Also, <u>Homodyne and Phase-lock amplification</u>

Here, the local oscillator E_1 is transmitted ~~out~~ into the circuit (environment) to produce some variation in the phase ϕ, $\omega_1 = \omega_2$, so $\Delta\omega = 0$ and only the phase between E_1 and the received-back E_2 is detected. But only the signal that is ~~in phase~~ at exactly the same frequency $\omega_1 = \omega_2$ is received. The important term is $E_{1_0} E_{2_0} \cos\phi$. The noise exclusion may be <u>huge</u>. since the frequency $\omega_1 = \omega_2$ is very sharp. $\left(\int dt \left(E_1(t)\, E_2(t)\right) = \delta(\omega_1 - \omega_2)\right)$

Homodyne radar:

The phase shift ϕ originates due to the round-trip distance of the reflected radar wave. If a π-radian phase shift occurs on reflection, then $\phi - \pi = \frac{\text{distance}}{c} \cdot \omega$

<u>Much</u> more to discuss here!