How to predict when thermal and quantum effects become important

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The scientific method has been remarkably successful in advancing modern technology at an outstanding rate. For example, it took only 66 years from the date of the first heavier-than-air powered aircraft flight (1903) to the first human moon walk (1969). This was achieved by the simple application of the scientific method, applied over and over again. This method involves proposing a hypothesis based on the best current understanding of the system, and then trying to design a clear experiment with an outcome that is predictable based upon the hypothesis. If the experimental outcomes are not as expected based on this hypothesis (which is usually the case), then the hypothesis is refined and this process is repeated. Science is advanced when the hypothesis is defeated and hence modified, not when the hypothesis is 'proven'. A good scientist is always trying to disprove what he/she thinks, since this is the process that leads to discovery and the advancement of knowledge.

It is very important to know how to estimate when known effects, such as thermal and quantum effects, become important to the design of a system. This is done by comparing different physical processes to each other in order to determine the cross-over point when one dominates over the other. This is often called scaling, since we are determining the dominant process as our adjustable parameters in the system (such as temperature, length, dimensions, conductivity, etc.) are changed. Here (below) is an example of a scaling analysis of a tiny capacitor that is designed to be charged by a single electron. We will predict where thermal effects become important at temperature T, which will set the maximum size of the capacitor C. Then we will see that the quantized charge states of this capacitor become 'sharp' only when this tiny capacitor is adequately isolated from its surrounding environment by a sufficiently large electrical resistance. These devices are useful in their own right, but the main thing I want to stress is the method of predicting when the scale is correct to observe these effects. You will often need to do a similar analysis when you design new apparatus to observe other effects at the nanoscale in the future.

Consider the simple conceptual drawing in the figure. A small, perfectly conducting sphere of radius r is located above a perfectly conducting ground plane, shown in brown. A layer of dielectric with permittivity ε, shown in yellow, is deposited on the conducting ground plane to provide support of the sphere at a distance d above the ground plane, as shown. This dielectric isolates the sphere partially from the ground plane by providing an electrical resistance R between the sphere and the ground plane.

We want to charge this capacitor with n electrons, where n is typically equals one, and we do not what thermal fluctuations to discharge this capacitor, or to swamp it with more thermally fluctuating electrons. How small must C be? The charging energy E_n, where n is the number of electrons on the capacitor, is given by:

$E_n = \frac{1}{2}CV^2$ By definition, $C = Q/V$, so $E_n = Q^2/(2C)$. We want to charge this capacitor with n electrons, where n is usually one, so Q = ne, where e = 1.6 x 10^{-19} Coulombs (C). Hence, $E_n = n^2e^2/(2C)$. First, we want to be sure that this charge on the capacitor is not washed out by thermal fluctuations at the ambient temperature T. This requires that $E_n \gg kT$ where Boltzman's Constant k = 1.38x10^{-23} J/K. So, from a scaling perspective, this capacitor starts to become robust against thermal fluctuations when n^2 e^2 $/(2C) = kT$. So the capacitance C of this device must be much smaller than n^2 e^2 / $(2kT)$ to avoid thermal wash-out. For n =1 and T = 300K this means that C must be much smaller than 3x10^{-18}F (3 aF), so to be safe C must be less than about 0.3 aF. This would be an challenging capacitor device to grow in a controlled fashion, as we will see later this semester. Single electron capacitors have been developed and used in interesting devices in the past, but they have operated at very low temperatures (often T < 1 K) in order to make this capacitance larger, and hence this device much easier to build. But that is less of an issue now that we have better nano-fabrication techniques.

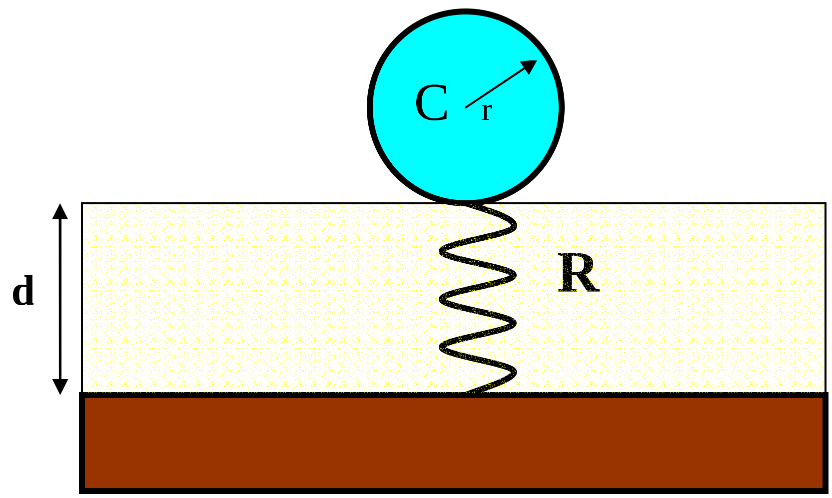

Figure: The model of a single electron capacitor, as described in the text.

Let's take a moment to consider how to build a 0.3 aF capacitor: This is clearly not a parallel plate capacitor, but from a scaling perspective we can estimate the capacitance roughly (to within a factor of three or so) using this simple approximation. The capacitance of the blue sphere relative to the brown ground plane is approximately $C \approx \varepsilon A/d = \varepsilon \pi r^2/d$ where ε is the permittivity of the yellow dielectric layer. The permittivity of germanium near room temperature is ε = 1.4x10^{-10} F/m, and d is typically one micron (10^{-6} m), but it can be made much smaller.

Hence, to achieve a 0.3 aF capacitor, r must be about 2×10^{-8} m (about 20 nm). It is difficult, but certainly feasible, to produce uniform, small spheres (often called 'quantum dots') at this level. Many devices use a number of these quantum dots arranged near each other and often separated by thin lithographic metal lines, and the uniformity of these quantum dots is critical to achieve proper device functionality. A typical metalization process in use today to produce microprocessors uses a metal linewidth of about 10 nm (https://www.tel.com/museum/magazine/material/150227_report04_01/), and it is difficult to control dimensions below about 3 nm in commercial mass production. Moreover, it is very difficult to eliminate accidental stray capacitances at the level of 0.3 aF in a circuit. But, if T = 3 K, then this capacitance can be as large as 30 aF to meet the same criterion discussed above (do you see why?), and such capacitances are relatively easy to produce in a controlled manner.

Being robust against thermal fluctuations is a necessary but not sufficient condition to develop single-electronic devices. It is important that the energy states of this capacitor be 'sharp', meaning that the charges placed on the capacitor must stay there for a sufficient length of time τ so that the energy of these charged states is much less uncertain than the energy of this single-electron charged capacitor. This sets a lower bound on how small the resistance R may be, since a small value of the resistance between the quantum dot and the ground plane will cause the electrons to flow off in an unacceptably short time. The uncertainty of the energy of this singe-electron charged state is ΔE, and $E_1 = e^2/(2C)$ as we saw above. So we want $\Delta E/E_1 \ll 1$. We consider the scaling situation when $\Delta E = E_1$. The time $\tau = RC$ for this simple circuit drawn in the figure, and by the Heisenberg Uncertainty Relation: $\Delta E\ \tau\ \simeq \hbar$ where $\hbar = 1.06 \times 10^{-34}$ Js is Planck's Constant h divided by 2π. Planck's constant is a very important constant, since it sets the scale for all quantum processes in nature. We will see Planck's constant throughout this course on many occasions. The condition $\Delta E = E_1$ then gives, substituting in $\tau = RC$, that R >> h/e^2. For a properly operating device, we would like this resistance to be about ten times larger in order to assure $E_1 \gg \Delta E$.

We have just tripped upon another very important combination of the fundamental constants, namely $h/e^2 = 26\ k\Omega$. This value is called the 'Von Klitzing' in honor of the physicist who discovered the quantum Hall effect. Typically we need about three Von Klitzings, or about a value of R greater than or equal to 75 kΩ, in order to see sharp quantum effects in our single-electron capacitor. Notice that this level of required electrical isolation from the surrounding world to see sharp quantum states is independent of the size of the capacitance!

So now we know a lot about how to design a single-electron quantum capacitor. But why would we like to make one? Well, in such a device the quantum states can be tuned to whatever level we desire. The structure and energies of atomic and molecular quantum states are given to us by nature, but now we have a way to custom design the energy levels of these states. Optical transitions occur between two electronic states in atoms and molecules, and now an optical transition may occur between the n = 2 and n = 1 state of our quantum capacitor as well. So now we can build custom quantum optical devices. We can also string these quantum capacitors along separated by tiny electrical lithographically made wires. The ‘gate potentials’ on these wires can be phased such that single electrons hop from quantum dot to quantum dot is an exceptionally well controlled way. This sort of device, called an ‘electron turnstile’ (https://physics.aps.org/articles/v9/s44) may be used to control electrical currents with an absolutely unprecedented accuracy, but that day has yet to come. Practical devices are burdened by stray charge and stray capacitance effects, but as technology advances this short-coming should be eliminated soon.

Exercises for the reader: Hop onto Google or go to the library and read-up on single-electron transistors, other single-electron devices, and on electron turnstiles. Now you know fundamentally how they work! The hard part is in controlling your fabrication process well enough to build them, and in developing accurate circuits ot characterize them. You will learn how ot approach these fabrication and measurement challenges this semester.

Enjoy!

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