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# *THE CONTRIBUTIONS OF CHARLES S. PEIRCE*

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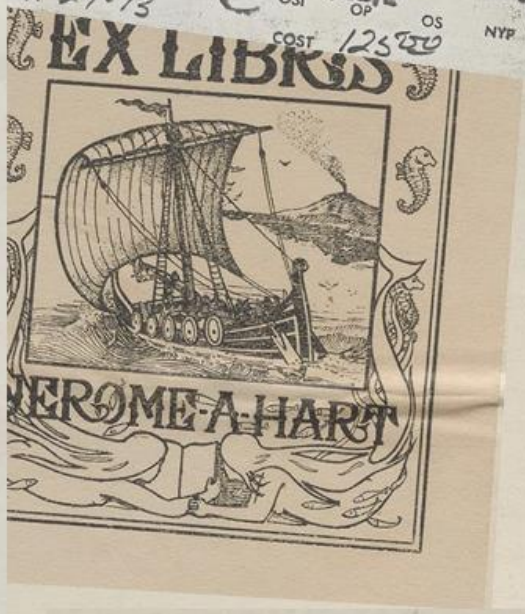

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Mr C S Peirce

Dear Sir:-

We are in receipt of your obliging order of Nov. 4 for Vol I of "Baldwins Dictionary of Philosophy and Psychology" As you are probably aware this work is sold only in sets and as Vol II has now been published, shall we send you both volumes at our terms? Immediately upon receipt of your reply the books will be for-

Milford Pa 1900 Oct 20

Prof Baldwin
Princeton, N.J.

Dear Sir:

I will do the work called for in yours of the 9th, which just reaches me.

It is indispensible that I should have all the preceding articles on Logic before me, — very desirable that I have the whole so far as in type.

The list of topics sent omits sundry well known terms of the traditional logic, as well as pretty much all that are at all out of the way, many terms of modern systems which have had great vogue (as Quantifier Predicate) and all other that have not had great vogue except out of a narrow line, and substantially the entire terminology of "exact" logic. In addition, many of the topics require sub-topics. I should recommend your allowing me to add what seems to me necessarily, under such general instructions as to fullness as you may give or which I may derive from the earlier proofs.

The French synonyms are without importance in Logic, while the Greek ones are highly so.

I shall not be able to get fully to work till you send me the proofs of all of Prof Adamson's mss. —

Yours very truly C S Peirce

## List of Terminology

**Economy** (logical principle of). A principle maintained by E. Mach that general concepts are merely an adaption for the economy of mental process. That they have that effect was noticed by Locke (C.S.P.)

**Empirical Logic**: Ger. *empirische Logik;* Fr. *logique empirique*; Ital. *logica empirica*. The treatment of logic on the basis or from the point of view of a sensationalist or other markedly empiricist theory of knowledge. (R.A. – C.S.P.)

**Equipollence or -cy** [Lat. *aequus*, equal, + pollere, to be able]: Ger. Aequipollenz; Fr. equipollence; Ital. equipollenza. The relation between two propositional forms which represents the same fact. It translated the Gr. ἰσοδυναμῶν. (C.S.P.)

**Genus** (in logic) [Lat. *genus*, birth]: Ger. *Genus*; Fr. *genre*; Ital. *genere*. A class which contains within its extension, or is divisible into, smaller classes, called relatively species.

The significance of the term has always shared the ambiguity which is discernible in classification. Genera have been distinguished partly by reason of the obvious differences in the larger types of natural forms, partly by reference to the relatively arbitrary process of arranging in accordance with selected marks. The first or empirical factor is pre-dominant in the popular sense of the term, and in much of the Aristotelian and Scholastic logic; the second has been insisted on in the strictly formal logic. The divergence of the two views makes itself manifest at the limits of classification, at the conception of a *summum genus* and an *infima species*, which ten on the one view to be regarded as having a place *in rerum natura*, while on the other they are but ideal boundaries to an arbitrary process. (R.A.)

One of the Aristotelian rules of DIVISION (q.v.) in logic is that the differences of different genera are different, that is to say, cross-divisions are not to be made. This rule is signally violated in the modern classification of chemistry, mathematics, and logic itself, but in biology, owing to the common origin of species, the classification is hierarchical, as Aristotle required. Cf. PREDICABLES. (C.S.P.)

**Given** [AS. *gifan*]: Ger. (das) *Gegebene*; Fr. (la) *donneé*; Ital. (11) dato. One of the hypotheses of a problem; used also in the Latin form *datum* (of which it is a translation). In Greek mathematics, the corresponding word was also extended to whatever is determined in certain specified ways by a given hypothesis. The plural *data* is loosely applied to any unquestioned knowledge upon which a judgement is based, and in particular to our percepts, in the phrase 'data of experience.'

The English adjective, *given*, has an exceedingly convenient use to indicate that that which its noun denotes must be understood as specified (in the verification of what is said) previously to the specification of something mentioned before. Thus, 'Some woman is adored by any given man, is said to avoid all possibility of understanding the statement as 'some woman is adored by every man.' (C.S.P.)

**Imaging (in logic):** Ger. *Abbildung*; Fr. (in mathematics) *représentation*; Ital. *rappresentazione*. A term proposed to translate Abbildung in its logical use. In order to apprehend this meaning, it is indispensable to be acquainted with the history of the meanings of Abbildung. This word was used in 1845 by Gauss for what is called in English a map-projection, which is an incorrect term, since many such modes of representation are not geometrical rectilinear projections at all; and of

those which Gauss had in view, but a single one is so. In mathematics Abbildung is translated *representation*; but this word is preempted in logic. Since Bild is always translated *image*, *imaging* will answer very well for Abbildung. If a map of the entire globe were made on a sufficiently large scale, and out of doors, the map itself would be shown upon the map; and upon that image would be seen the map of the map; and so on, indefinitely. If the map were to cover the entire globe, it would be an image of nothing but itself, where each point would be imaged by some other point, itself imaged by a third, &c. But a map of the heavens does not show the map itself at all. A Mercator's projection shows the entire globe (except the poles) over and over again in endlessly recurring strips. Many maps, if they were completed, would show two or more different places on the earth at each point of the map (or at any rate on a part of it), like one map drawn upon another. Such is obviously the case with any rectilinear projection of the entire sphere, excepting only the stereographic. These two peculiarities may coexist in the same map.

Any mathematical function of one variable may be regarded as an image of its variable according to some mode of imaging. For the real and imaginary quantities correspond, one to one and continuously, to the assignable points on a sphere. Although mathematics is by far the swiftest of the sciences in its generalizations, it was not until 1879 that Dedekind (in the 3rd edition of his recension of Lejeune-Dirichlet's *Zahlentheorie*, § 163, p. 470; but the writer has not examined the second edition) extended the conception to discrete systems in these words: 'It very often happens in other sciences, as well asin mathematics, that there is a replacement of every element *w* of a system Ω of elements or things by a corresponding element *w*' [of a system Ω']. Such an act should be called a substation….But a still more convenient expression is found by regarding Ω' as the image of Ω, and w' of w, according to a certain mode of imaging.' And he adds, in a footnote: 'This power of the mind of comparing a thing w with a thing w', or of relating w tot w', or of considering w' to correspond to w, is one without which no though would be possible.' [We do not translate the main clause.] This is an early and significant acknowledgement that the so-called ' logic of relatives-then deemed beneath the notice of logicians-is an integral part of logic. This remark remained unnoticed until, in 1895, Schröder devoted the crowning chapter of his great work (*Exakte Logik*, iii. 553-649) to its development. Schröder says that, in the broadest sense, any relative whatever may be considered as an imaging- 'nämlich al seine eventuell bald "undeutige," bald "eindeutige," bald "mehredeutige" Zuordnung.' He presumably means that the logical universe is thus imaged in itself. However, in a narrower sense, he says, a mode of imaging is restricted to a relative which fulfils one or other of the two conditions of being never *undeutig*, or being never *mehrdeutig*. That is, the relation must belong to one or other of two classes, the one embracing such that every object has an image, and the other such that no object has more than one image. Schröder's definitions (however interesting his developments) break all analogy with the important property of the imaging of continua noticed above. If this is to be regarded as essential, an imaging must be regarded as essential, an imaging must be defined as generic relation between an object-class and an image-class, which generic relation consists of specific relations, in each of which one individual, and no more, of the image-class stand to each individual of the object—class, and in each of which every individual of the image-class stand to one individual and to no more, of the object class. This is substantially a return to Dedekind's definition, which makes an *imaging* a synonym for a substitution. (C.S.P.,H.B.F.)

(C.S.P.)

**Implicit (in logic).** Said of an element or character of a representation, whether verbal or mental, which is not contained in the representation itself, but which appears in the strictly logical (not merely in the psychological) analysis of that representation.

Thus, when we ordinarily think of something, say the Antarctic continent, as real, we do not stop to reflect that every intelligible question about it admits of a true answer; but when we logically analyse the meaning of reality, this result appears in the analysis. Consequently, only concepts, not percepts, can contain any implicit elements, since they alone are capable of logical analysis. An implicit contradiction, or contradiction *in adjecto*, is one which appears as soon as the terms are defined, irrespective of the properties of their objects. Thus there is, strictly speaking, no implicit contradiction in the notion of a quadrilateral triangle, although it is impossible. But, owing to exaggeration, this would currently be said to involve not merely an implicit, but an explicit contradiction, or contradiction in terms.

Any proposition which neither requires the exclusion from nor the inclusion in the universe of any state of facts or kind of object except such as a given second proposition so excludes or requires to be included, is implied in that second proposition in the logical sense of implication, no matter how different it may be in its point of view, or otherwise. It is a part of the meaning of the copula 'is' employed in logical forms of proposition, that it expresses a transitive relation, so that whatever inference from the proposition would be justified by the dictum de omni is implied in the meaning of the proposition. Nor could any rule be admitted as universally valid in formal logic, unless it were a part of the definition of one of the symbols used in formal logic. Accordingly, whatever can be logically deduced from any proposition is implied in it; and conversely. Whether what is implied will, or will not, be suggested by the contemplation of the proposition is a question of psychology. All that concerns logic is, whether all the facts excluded and required by the one proposition are among those so excluded or required by the other. (C.S.P.)

**Inconsistency** [Lat. in + con + sister, to stand]: Ger. *Unvereinbarkeit*; Fr. *inconsistence*; Ital. *incpmpatibilità*. The relation between twi assertions which cannot be true at once, though it may not be a direct contradiction: as between a statement of items and a statement of their total. Cf. CONSISTENCY.

A logical discrepancy on the other hand, is a difference between two statements either diificult or impossible to reconcile with the credibility of both. It is said to be negative if one assertion omits an inseparable part of the fact stated in another; as when one witness testifies that *A* pointed a pistol at *B*, and another that *A* shot at *B*. It is positive if one asserts what the other denies. But even then it may often conciliable (verträglich); that is, may not prove that either statement is in other respects untrustworthy. See Bachmann, *Logik*, §§ 214 ff.

'Inconsistent' is applied to an assertion, or hypothesis, which either in itself, or in copulation with another proposition with which it is said to be inconsistent, might be known to be false by a man devoid of all information except the meanings of the words used and their syntax.

Inconsistent differs from contradictory (see CONTRADICTION) in being restricted usually to propositions, expressed or implied, and also in not implying that the falsity arises from a relation of negation. 'That is John' and 'It is Paul' are inconsistent, but hardly contradictory. Moreover, contradictory is also used in a peculiar sense in formal logic. Cf. OPPOSITION. (C.S.P)

(C.S.P.)

**Independence** [Lat. *in* + *de* + *pendere*, to hnag]: Ger. *Unabhängigkeit*; Fr. *indépendance*; Ital. *indipendenza*. (1) Two subjects are independent in so far as the possession of any character by the one does not require not prevent the possession of any character by the other, unless these characters are directly or indirectly relative to the other individual.

(2) Two events are independent if either is equally probable whether the other takes place or not. (C.S.P.)

**Index** (in exact logic). A sign, or representation, which refers to its object not so much because of any similarity or analogy with it, nor because it is associated with general characters which that object happens to possess, as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other hand.

No matter of fact can be stated without the use of some sign serving as an index. If *A* says to *B*, 'There is a fire,' *B* will ask, 'where?' Thereupon *A* is forced to resort to an index, even if he only means somewhere in the real universe, past and future. Otherwise, he has only said that there is such an idea as fire, which would give no information, since unless it were known already, the word 'fire' would be unintelligible. If *A* points his finger to the fire, his finger is dynamically connected with the fire, as much as if a self-acting fire-alarm had directly turned it in that direction; while it also forces the eyes of *B* to turn that way, his attention to be riveted upon it, and his understanding to recognize that his question is answered. If *A*'s reply is, 'within a thousand yards of here,' the word 'here' is an index; for it has precisely the same force as if he had pointed energetically to the ground between him and *B*. Moreover, the word 'yard,' though it stands for an object of a general class, is indirectly indexical, since the yard-sticks themselves are signs of the parliamentary standard, and that, not because they have similar qualities, for all the pertinent properties of a small bar are, as far as we can perceive, the same as those of a large one, but because each of them has been, actually or virtually, carried to the prototype and subjected to certain dynamical operations, while the associational compulsion calls up in our minds, when we see one of them, various experiences, and brings us to regard them as related to something fixed in length, though we may not have reflected that that standard is a material bar. The above considerations might lead the reader to suppose that indices have exclusive reference to objects of experience, and that there would be no use for them in pure mathematics, dealing, as it does, with ideal creations, without regard to whether they are anywhere realized or not. But the imaginary constructions of the mathematician, and even dreams, so far approximate to reality as to have a certain degree of fixity, in consequence of which they can be recognized and identified as individuals. In short, there is a degenerate form of observation which is directed to the creations of our own minds — using the word observation in its full sense as implying some degree of fixity and quasi-reality in the object to which it endeavours to conform. Accordingly, we find that indices are absolutely indispensable in mathematics; and until this truth was comprehended, all efforts to reduce to rule the logic of triadic and higher relations failed; while as soon as it was once grasped the problem was solved. The ordinary letters of algebra that present no peculiarities are indices. So also are the letters *A*, *B*, *C*, &c., attached to a geometrical figure. Lawyers and others who have to state a complicated affair with precision have recourse to letters to distinguish individuals. Letters so used are merely improved relative pronouns. Thus, while demonstrative and personal pronouns are, as ordinarily used, 'genuine indices,' relative pronouns are 'degenerate indices;' for though they may, accidentally and indirectly, refer to

(C.S.P.)

existing things, they directly refer, and need only refer, to the images in the mind which previous words have created.

Indices may be distinguished from other signs, or representations, by three characteristic marks: first, that they have no significant resemblance to their objects; second, that they refer to individuals, single units, single collections of units, or single continua; third, that they direct the attention to their objects by blind compulsion. But it would be difficult, if not impossible, to instance an absolutely pure index, or to find any sign absolutely devoid of the indexical quality. Psychologically, the action of indices depends upon association by contiguity, and not upon association by resemblance or upon intellectual operations. See Peirce, in *Proc. Amer. Acad. Arts and Sci.*, vii. 294 (May 14, 1867). (C.S.P.)

**Individual** (in logic) [as a technical term of logic, *individuum* first appears in Boethius, in a translation from Victorinus, no doubt of ἄτομον, a word used by Plato (sophistes, 229 D) for an indivisible species, and by Aristotle, often in the same sense, but occasionally for an individual. Of course the physical and mathematical senses of the word were earlier. Aristotle's usual term for individuals is τὰ καθ' ἕκαστα, Lat. *Singularia*, Eng. *singulars*.] used in logic in tow closely connected senses. (1) According to the more formal of these an individual is an object (or term) not only actually determinate in respect to having or wanting each general character and not both having and wanting any, but is necessitated by its mode of being to be so determinate. See PARTICULAR (in logic).

This definition does not prevent two distinct individuals from being precisely similar, since they may be distinguished by their necceities (or determinations not of a generalizable nature); so that Leibnitz' principle of indiscernibles is not involved in this definition. Although the principles of contradiction and excluded middle may be regarded as together constituting the definition of the relation expressed by 'not,' yet they also imply that whatever exists consists of individuals. This, however, does not seem to be an identical proposition or necessity of thought; for Kant's Law of Specification (*Krit. d. reinen vernunft*, 1st ed., 656i 2nd ed., 684; but it is requisite to read the whole section to understand his meaning), which has been widely accepted, treats logical quantity as a continuum in Kant's sense, i.e. that every part of which is composed of parts. Though this law is only regulative, it is supposed to be demanded by reason, and its wide acceptance as so demanded is a strong argument in favour of the conceivability of a world without individuals in the sense of the definition now considered. Besides, since it is not in the nature of concepts adequately to define individuals, it would seem that a world from which they were eliminated would only be the more intelligible. A new discussion of the matter, on a level with modern mathematical thought and with exact logic, is a desideratum. A highly important contribution is contained in Schröder's *Logik*, iii, *vorles*. 10. What Scotus says (*Quaest. In Met.*, VII. Q, xiii and xv) is worth consideration.

(2) Another definition which avoids the above difficulties is that an individual is something which reacts. That is to say, it does react against some things, and is of such a nature that it might react, or have reacted, against my will. This is the stoical definition of a reality; but since the Stoics were individualistic nominalists, this rather favours the satisfactoriness of the definition than otherwise. It may be objected that it is unintelligible; but in the sense in which this is true, it is a merit, since an individual is unintelligible in that sense. It is a brute fact that the moon exists, and all explanations suppose the existence of that same matter. That existence is unintelligible in the sense in which the definition is so. That is to say, a reaction may be experienced, but it cannot be conceived in its character of a reaction; for that element evaporates

(C.S.P.)

from every general idea. According to this definition, that which alone immediately presents itself as an individual is a reaction against the will. But everything whose identity exists in a continuity of reactions will be a single logical individual. Thus any portion of space, so far as it can be regarded as reacting, is for logic a single individual; its special extension is no objection. With this definition there is no difficulty about the truth that whatever exists is individual, since existence (not reality) and individuality are essentially the same thing; and whatever fulfills the present definition equally fulfills the former definition by virtue of the principles of contradiction and excluded middle, regarded as more definitions of the relation expressed by 'not.' As for the principle of indiscernibles, if two individual things are exactly alike in all other respects, they must, according to this definition, differ in their spatial relations, since space is nothing but the intuitional presentation of the conditions of reaction, or of some of them. But there will be no logical hindrance to two things being exactly alike in all other respects; and if they are never so, that is a physical law, not a necessity of logic. This second definition, therefore, seems to be the preferable one. Cf. PARTICULAR (in logic). (C.S.P.)

**Inference** [Lat. *in* + *ferre*, to bear]: Ger. *Schliessen*, *Schlussi*; Fr. *inférence*; Ital. *illazione* (*conclusione*). (1) In logic: (*a*) the act of consciously determining the content of a cognition by a previous cognition or cognitions, in a way which seems generally calculated to advance knowledge.

In this sense the word differs from REASONING (q.v.) only in referring strictly to a single step of the process, or to what seems a single step. Unless the act is consciously performed, no logical control can be exercised; and this is sufficient reason for separating such acts from any operations otherwise analogous which may take place in the formation of percepts. To be conscious of determining a cognition by another, and not merely of making the one follow after the other, involves some more or less obscure judgment that the pair of representations, the determining and determined, belong to a class of analogous pairs, so that a general maxim is virtually obeyed in the act. There is, besides, a purpose of learning more of the truth./ The representations concerned in inference are, it appears, always judgments (or propositions). Probably, if a pair of percepts were, in the very act of determining the one to accord with the other, looked upon as special cases of a class of pairs of percepts so related to one another that if one were true the other ought to be accepted, they would, *ipso facto*, become judgments.

(*b*) A pair (or larger set) of judgments, of which one (or all of them together but one) determines the remaining one, as in (*a*) above, the whole set being regarded as constituting together a cognition more complete than a judgment.

In this sense, inference is synonymous with argument. The latter word, it is true, only implies that the set of propositions might be thought, perhaps written down and no longer even accepted by the author, while the former word implies that the movement of thought takes place. Moreover, an inference creates belief in the mind that makes it, while an argument may be a system of propositions put together with a view of creating belief in another mind, or perhaps merely to exhibit the logical relation between different beliefs. But these distinctions often vanish or lose all importance. When the determining judgement is a copulative proposition, its members may either be called the PREMISE (q.v.). But when different beliefs are brought together in thought for the first time to form a copulative judgment, the premises must be taken as plural.

Several other logical meanings are in general use as more or less permissible inaccuracies of language. Thus, the determined judgment, or conclusion, may sometimes be conveniently

called an ‘inference.’ The popular use of the word for a dubious illation, as in such a sentence as ‘This is proof positive, while that is only an inference,’ is quite inadmissible. (C.S.P.)

**Insolubilia** [Lat. *in* + *solvere*, to loose; trans. of Aristotle’s aπopia; used mainly in plural]. A class of sophisms in which a question is put of such a nature that, whether it be answered affirmatively or negatively, an argument unimpeachable in form will prove the answer to be false.

The type is this. Given the following proposition.

This assertion is not true:

is that assertion, which proclaims its own falsity, and nothing else, true or false? Suppose it true. Then,

Whatever is asserted in it is true,
But that it is not true is asserted in it;
∴ By Barbara, That it is not true is true;
∴ It is not true.

Besides, if it is true, that it is true is true. Hence,
That it is not true is not true,
But that it is not true is asserted in the proposition;
∴ By barapti, something asserted in the proposition is not true;
∴ The proposition is not true.

On the other hand, suppose it is not true.
In that case,

That it is not true is true,
But all that the proposition asserts is that it is not true.

∴ By Barbara, All that the proposition asserts is true.

∴ The proposition is true.

Besides, in this case,
Something that the proposition asserts is not true,
But all that the proposition asserts is that it is not true;
∴ By Bokardo, That it is not true is not altogether true;

∴ That it is true is true;
∴ It is true.

Thus, whether it be true or not, it is both true and not. Now, it must be either true or not, hence it is both true and not, which is absurd.

Only two essentially distinct methods of solution have been proposed. One, which is supported by Ockham (*Summa totius logices*, 3rd div. of 3rd part, cap. 38 and 45), admits the validity of the argumentation and its consequence, which is that there can be no such proposition, and attempts to show by other arguments that no proposition can assert anything of itself. Many logical writers follow Ockham in the first part of his solution, but fail to see the need of the second part. The other method of solution, supported by Paulus Venetus (Sophismata Aurea, sophisma 50), diametrically denies the principle of the former solution, and undertakes to show that every proposition virtually asserts its own truth. This method, therefore, denies the premise of the antithesis that ‘all that the proposition asserts is that it is not true,’ since, like any other proposition, it also asserts its own truth, and therefore contradictory and false, not in what it expressly asserts, but in what it implicitly asserts. Some writers (as Fries) hold that because every proposition asserts its own truth, therefore nothing is a proposition which asserts its own

(C.S.P.)

falsity. See Aristotle, *Sophisticae Elenchi*, cap. 25. Other proposed solutions of little importance are given by Paulus Venetus, loc. cit. (C.S.P.)

**Intention** (in logic) [Lat. *Intentio*, with the same meaning in Aquinas (*Summa* Theol., I. 9. 53, is the principal passage); in classical writers as an act of attention (and so Aquinas, ibid., I. ii. 9. 38, art. 2, and elsewhere); *from* in + *tendere*, to stretch. Aquinas seems sometimes to use the term for a mode of being (ibid., I. 9. 29, art. I, 9. 76, art. 3, and esp. art. 4)]. A concept, as the result of attention.
*First intentions* are those concepts which are derived by comparing percepts, such as ordinary concepts of classes, relations, &c. *Second intentions* are those which are formed by observing and comparing first intentions. Thus the concept "class" is formed by observing and comparing class-concepts and other objects. The special class-concepts, *ens*, or what is, in the sense of including figments as well as realities can only have originated in that way. Of relative second intentions, four are prominent-identity, otherness, co-existence, and incompossibility. Aquinas defined logic as the science of second intentions applied to first. (C.S.P.)

**Involution** [Lat. *in* + *volvere*, to roll]: Ger. *Involution*; Fr. *involution*, Ital. *involuzione*. A term of SYMBOLIC LOGIC (q.v.) borrowed from algebra, where it means the raising of a base to a power. In logic it has two different senses. (1) Relative involution: let *lwm* denote any lover of a well-wisher of a man. That is, any individual *A* is denoted by *lwm*, provided there are in existence individuals *B* and *C* (who may be identical with each other or with *A*) , such that *A* loves *B*, while *B* wishes well to *C*, and *C* is a man. Further, let $l^w m$ denote any individual *C*, who is a man, and who is such that taking any individual *B* whatever, if *B* is a well-wisher of *C*, then *A* is a lover of *B*. The operation of combining *l* and *w* in this statement is termed 'progressive involution.' Again, let $l^w m$ denote any individual *A*, if, and only if, there is in existence and individual *B*, who is loved by *A*, and who is such that taking any individual *C* whatever, if *C* is wished well by *B*, then *C* is a man. The operation of combining *w* and *m* in this statement is termed 'regressive involution.' These designations were adopted because of the analogy of the general formulae to those of involution in the algebra of quantity.
These kinds of involution are not, at present, in use in symbolical logic; but they are, nevertheless, useful, especially in developing the conception of continuity. These two kinds of involution together constitute relative involution.
(2) Non-relative involution: consisting in the repeated introduction of the same premise into a reasoning; as, for example, the half-dozen simple premises upon which the Theory of Numbers is based are introduced over and over again in the reasoning by which its myriad theorems are deduced. In exact logic the regular process of deduction begins by non-relatively multiplying together all the premises to make one conjunctive premise, from which whatever can be deduced by using those premises as often as they are introduced as factors, can be deduced by processes of 'immediate inference' from that single conjunctive premise. But the general character of the conclusion is found to depend greatly upon the number of times the same factor is multiplied in. From this circumstance the importance and the name of non-relative involution arise (C.S.P.)

**Kind** [AS. *cynd*, nature, from *cynde*, natural; same root as Gr. γένος, Lat. *genus*]: Ger. Art (the word 'kind' is also used to translate. Ger. *Gattung*, for which see HEGEL'S TERMINOLOGY); Fr. *genre*; Ital. *genere specie*. Before 'class' acquired its logical signification in Queen Anne's

(C.S.P.)

reign, kind was sometimes used for any collection of objects having a common and peculiar general character, simple or complex.

Thus, in Blundevile's *Arte of Logicke*, we read: '*Genus* is a general kind which may be spoken of many things differing in special kind.' At other times, and more accurately, it was restricted to the species, or narrowest recognized class, or that which was supposed to be derived from one stock. Thus Wilson's *Rule of Reason* (1551) has: '*Genus* is a generall woorde vnder the whichedillerse kindes or sortes of things are comprehended.'

But before persons who picked their words had become ready to use 'class' as a mere logical extension, they had begun to avoid 'kind,' except when the emphasis of attention was placed upon the logical depth rather than the breadth. Watts's *Logick* (1724) illustrates this. This last is the ordinary popular sense of the word to-day; so that 'of this kind,' 'of this nature,' of this character' are interchangeable phrases. J.S. Mill however, in his *System of Logic*, BK. I chap. vii. § 4, erected the word into a technical term of logic, at the same time introducing the term 'real kind.' His meaning, so far as it was determinate, was that classes are of two orders, the first comprising those which, over and above the characters which are involved in their definitions and which serve to delimit their extension, have, at most, but a limited number of others, and those following as 'consequences, under laws of nature,' of the defining characters; and the second, the real kinds, comprising those each of which has innumerable common properties independent of one another. As instances of real kinds, he mentions the class of animals and the class of sulphur, as an instance of a kind not real, the class of white things. It is important for the understanding of Mill's though here, as throughout his work, to note that when he talks of 'properties,' he has in mind, mainly, characters interesting to us. Otherwise, it would not be true that all white things have few properties in common. By a 'law of nature' he means any absolute uniformity; so that it is hardly enough to assert that if all white things had any property *P*, this would be a 'consequence, under a law of nature,' of their whiteness; for it would be itself an absolute and ultimate uniformity. Mill says that if the common properties of a class thus follow from a small number of primary characters 'which, as the phrase is, *account for* all the rest,' it is not a real kind. He does not remark that the man of science is bent upon ultimately thus accounting for each and every property that he studies. The following definition might be proposed: Any class which, in addition to its defining character, has another that is of permanent interest and is common and peculiar to its members, is destined to be conserved in that ultimate conception of the universe at which we aim, and is accordingly to be called 'real.' (C.S.P.)

**Knowledge (in logic)**. This word is used in logic in two senses: (1) as a synonym for COGNITION (q.v.), and (2), and more usefully, to signify a perfect cognition, that is a cognition fulfilling three conditions: first, that it holds for true a proposition that really is true; second, that it is perfectly self-satisfied and free from the uneasiness of doubt; third, that some character of this satisfaction is such that it would be logically impossible that this character should ever belong to satisfaction in a proposition not true.

Knowledge is divided, firstly, according to whatever classification of the sciences is adopted. Thus, Kantians distinguish formal and material knowledge. See SCIENCE. Secondly, knowledge is divided according to the different ways in which it is attained as into immediate and mediate knowledge. See IMMEDIACY AND MEDIACY (logical). Immediate knowledge is a cognition, or objective modification of consciousness, which is borne in upon a man with such resistless force as to constitute a guarantee that it (or a representation of it) will remain permanent in the development of human cognition. Such knowledge is, if its existence be

granted, either borne in through an avenue of sense, external or internal, as a percept of an individual, or springs up within the mind as a first principle of reason or as a mystical revelation. Mediate knowledge is that for which there is some guarantee behind itself, although, no matter how fat criticism be carried, simple evidency, or direct insistency, something has to be relied upon. The external guarantee rests ultimately either upon authority, i.e. testimony, or upon observation. In either case mediate knowledge is attained by REASONING, which see for further divisions. It is only necessary to mention here that the Aristotelians distinguished knowledge ὅτι, or of the facts themselves, and knowledge διότι or of the rational connection of facts, the knowledge of the how and why (cf. the preceding topic). They did not distinguish between the how and they why, because they held that knowledge διότι is solely produced by SYLLOGISM (q.v.) in its greatest perfection, as demonstration. The term empirical knowledge is applied to knowledge, mediate or immediate, which rests upon percepts; while the terms philosophical and rational knowledge are applied to knowledge, mediate or immediate, which rests chiefly or wholly upon conclusions or revelations of reason. Thirdly, knowledge is divided, according to the character of the immediate object, into apprehensive and judicative knowledge, the former being of a percept, image, or Vorstellung, the latter of the existence or non-existence of a fact. Fourthly, knowledge is divided, according to the manner in which it is in the mind, into actual, virtual, and habitual knowledge. See Scotus, *Opus Oxoniense*, lib. I. dist. iii. quest. 2, paragraph beginning 'Loquendo igitur.' Fifthly, knowledge is divided, according to its end, into speculative and piratical. (C.S.P., C.L.F.)

**Laws of Thought**: Ger. *Denkgesetze*; Fr. *lois de la pensée*; Ital. *leggi del pensiero*. The three formulas of identity, contradiction, and excluded middle have been widely so known, though the doctrine that they are three co-ordinate and sufficient laws of all tough or of all reasoning has been held by a comparatively small party which hardly survives; and it is not too much to say that the doctrine is untenable. But the designation is so familiar and convenient that those formulas may very well be referred to as 'the so-called three laws of thought.' The formulas have usually been stated by those who upheld the doctrine as follows: —

I. *The Principle of Identity*: *A* is *A*

II. *The Principle of Contradiction*: *A* is not not-*A*

III. *The Principle of Excluded Middle or Excluded Third*: everything is either *A* or not-*A*.

It is noticeable that two of these propositions are categorical and the third disjunctive, a circumstance demanding explanation for those who hold the distinction of categorical, conditional, and disjunctive propositions to be fundamental.

The meaning of the formula of identity presents only one small difficulty. If the copula 'is' be taken in the sense of 'is, if it exists,' then the meaning of the formula is that no universal affirmative proposition having the same term as subject and predicate is false. If, however, the copula be understood to imply existence, the meaning is that no universal affirmative proposition is false in which the same term is subject and predicate, provided that term denotes any existing object. Or, the meaning may be that the same thing is true when the subject and predicate are the same proper name of an individual. In any case, it may properly be required that the precise meaning attached to the copula should be explained; and this explanation must in substance involve one or other of the above three statements; so that in any case the principle of identity is merely a part of the definition of the copula.

In like manner, if the word 'not' is to be used in logical forms, its force should be explained with the utmost precision. Such an explanation will consist in showing that the relation

is expresses belongs at once to certain classes of relations, probably not more than two, in view of the simplicity of the idea. Each of these two statements may be embodied in a formula similar, in a general way, to the formulas of contradiction and excluded middle. It has, therefore, seemed to Mill and to the 'exact' logicians that these two formulas ought together to constitute a definition of the force of 'not.'

Other writers have regarded all three laws as 'practical maxims.' But practically nobody needs a maxim to remind him that a contradiction, for example, is an absurdity. It might be a useful injunction to tell him to beware of latent contradictions; but as soon as he clearly sees that a proposition is self-contradictory, he will have abandoned it before any maxim can be adduced. Seeing, then, that such formulas are required to define the relation expressed by not, but are not required as maxims, it is in the former aspect that their true meanings are to be sought.

If it is admitted that they constitute a definition, they must conform to the rules of definition. Considered as part of a definition, one of the commonest statements of the principle of contradiction, *'A* non est non-*A*,' offends against the rule that the *definitum* must not be introduced into the definition. This is easily avoided by using the form '*A* est non non-*A*,' *'A* is not not-*A*,' or every term may be subsumed under the double negation of itself. If this from is adopted for the principle of contradiction, the principle of excluded middle ought to be 'What is not not-*A* is *A*.' If, however, we prefer to state the principle of excluded middle as 'Everything is either *A* or not-*A*,' then we should state the principle of contradiction as 'What is, at once, *A* and not-*A* is nothing.' There is no vicious circle here, since the term 'nothing,' or 'non ens,' may be formally defined without employing the particle 'not' or any equivalent. Thus, we may express the principle of contradiction as follows:

Whatever there may be which is both *A* and not-*A* is *X*, no matter what term *X* may be.

In either formula, *A* may be understood to be restricted to being an individual, or it may be allowed to be any term, individual or general. In the former case, in order to avoid conflict with the fundamental law that no true definition asserts existence, a special clause should be added such as 'if not-*A* there be.' In the latter case, it should be stated that by 'not-*A'* is not meant 'not some *A*,' but 'not any *A*,' or 'other than whatever *A* there may be.'

Bearing these points in mind, the formula *'A* is not-not-*A*,' or '*A* is other than whatever is other than whatever is *A*,' is seen to be a way of saying that the relation expressed by 'not' is one of those which is its own converse, and is analogous to the following:

Every rose is similar to whatever is similar to whatever is a rose; which again is similar to the following:

Every man is loved by whatever loves whatever is a man.

But if we turn to the corresponding formula of excluded middle, "Not-not-*A* is *A*,' or 'Whatever is not anything that is not any *A* is *A*,' we find that its meaning cannot be so simply expressed. Supposing that the relation *r* is such that it is true that

Whatever is *r* to whatever is *r* to whatever is *A* is *A*,

it can readily be proved that, whether the multitude of individuals in the universe be finite or infinite, each individual is either *non-r* to itself and to nothing else, or is one of a pair of individuals that are non-r to each other and to nothing else; and conversely, if the universe is so constituted, the above formula necessarily holds. But it is evident that if the universe is so constituted the relation *r* is converse to itself; so that the formula corresponding to that of contradiction also holds. But this constitution of the universe does not determine *r* to be the relation expressed by 'not.' Hence, the pair of formulas,

*A* is not not-*A*,

(C.S.P.)

Not not-*A* is *A*,

are inadequate to defining 'not,' and the former of them is mere surplusage. in fact, in a universe of monogamously married people, taking any class, the *A*'s,

Every *A* is a non-spouse to whatever is non-spouse to every *A*,

and

Whatever is non-spouse to whatever is a non-spouse to every *A* is an *A*.

No such objection exists to the other pair of formulas:

Whatever is both *A* and not-*A* is nothing.

Everything is either *A* or not-*A*.

Their meaning is perfectly clear. Dividing all ordered pairs of individuals into those of the form *A*:*B* and those of the form *A*:*A*,

The principle of contradiction excludes from the relation 'not' all of the from *A*:*A*,

The principle of excluded middle makes the relation of 'not' to include all pairs of the form *A*:*B*.

From this point of view, we see at once that there are three other similar pairs of formulas defining the relations of identity, coexistence, and incompossibility, as follows:

Whatever is *A* is identical with *A*, i.e.

Identity includes all pairs *A*:*A*

Whatever is identical with *A* is *A*; i.e.

Identity excludes all pairs *A*:*B*

Whatever is *A* is coexistent with *A*;

i.e. Coexistence includes all pairs *A*:*A*.

Everything is either *A* or coexistent with *A*; i.e. Coexistence includes all pairs *A*:*B*.

Whatever is both *A* and incompossible with *A* is nothing; i.e. Incompossibility excludes all pairs *A*:*A*.

Whatever there may be incompossible with *A* is *A*; i.e. Incompossibility excludes all pairs *A*:*B*.

Much has been written concerning the relations of the thre principles to forms of syllogism. They have even been called Die Principien des Schliessens, and have often been so regarded. Some points in reference to the meanings they have borne in such discussions require mention. Many writers have failed to distinguish sufficiently between reasoning and the logical forms of inference. The distinction may be brought out by comparing the moods Camestres and Cesare (see MOOD, in logic). Formally, these are essentially different. The form of Camestres is as follows:

Every *P* is an *M*

Every *S* is other than every *M*;

∴ Every *S* is other than every *P*.

This form does not depend upon either clause of the definition of 'not' or 'other than.' For if any other relative term, such as 'lover of,' be substituted for 'other than,' the inference will be equally valid. The form of Cesare is as follows:

Every *P* is other than every *M*

Every *S* is an *M*;

∴ Every *S* is other than every *P*.

This depends upon the equiparance of 'other than.' For if we substitute an ordinary relative, scuh as loves, for 'other than' in the premise, the conclusion will be

(C.S.P.)

Every *S* is loved by every *P*.

(See De Morgan's fourth memoir on the syllogism, Cambridge Philos. Trans., x. (1860) 354.) The two forms are thus widely distinct in logic; and yet when a man actually performs an inference, it would be impossible to determine that he 'reasons in' one of these moods rather than in the other. Either statement is incorrect. He does not, in strict accuracy, reason in any form of syllogism. For his reasoning moves in first intentions, while the forms of logic are constructions of second intentions. They are diagrammatic representations of the intellectual relation between the facts from which he reasons and the fact which he infers, this diagram necessarily making use of a particular system of symbols — a perfectly regular and very limited kind of language. It may be a part of a logician's duty to show how ordinary ways of speaking and of thinking are to be translated into that symbolism of formal logic; but it is no part of syllogistic itself. Logical principles of inference are merely rules for the illative transformation of the symbols of the particular system employed. If the system is essentially changed, they will be quite different. As the Boolians represent Cesare and Camestres, they appear, after literally translating the algebraic signs of those logicians into words, as follows:

*A* that is *B* is nothing,
*C* that is not *B* is nothing;
∴ *A* that is *C* is nothing.

The two moods are here absolutely indistinguishable.

From the times of Scotus down to Kant more and more was made of a principle agreeing in enunciation, often exactly, in other places approximately, with our principle of contradiction, and in the later of those ages usually called by that name, although earlier more often *principium primum*, *primum cognitum*, *principium identitatis*, *dignitas dignitatum*, &c. It would best be called the *Principle of Consistency*. Attention was called to it in the fourth book of Aristotle's *Metaphysics*. The meaning of this, which was altogether different, at least in post-scholastic times, from our principle of contradiction, is stated in the so-called *Monadologie* of Leibnitz (§ 31) to be that principle by virtue of which we judge that to be false which involves a contradiction, and the denial of the contradiction to be true. The latter clause involves an appeal to the principle of excluded middle as much as the former clause does to the formal principle of contradiction. And so the 'principle of contradiction' was formerly frequently stated. But, in fact, neither is appealed to; for Leibnitz does not say that the contradiction is to be made explicit, but only that it is to be recognized as an inconsistency. Interpreted too strictly, the passage would seem to mean that all demonstrative reasoning is by the reductio ad absurdum; but this cannot be intended. All that is meant is that we draw that conclusion the denial of which would involve an absurdity — in short, that which consistency requires. This is a description, however imperfect, of the procedure of demonstrative REASONING (q.v.), and deos not relate to logical forms. It deals with first, not second, intentions. (C.S.P)

**Leading of Proof:** no concise foreign equivalents. The operation bringing up to attention, among propositions admitted to be true, certain relations between them which logically compel the acceptance of a conclusion. (C.S.P.)

**Leading Principle:** Ger. *leitendes Prinzip;* Fr. p*rincipe directeur*; Ital. *principio fondamentale.* It is of the essence of reasoning that the reasoner should proceed, and should be conscious of proceeding, according to a general habit, or method, which he holds would either (according to the kind of reasoning) always lead to the truth, provided the premises were true; or, consistently

(C.S.P.)