MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY.

EDITED BY

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VOL. VIII.—1883.

WILLIAMS AND NORGATE,

14 HENRIETTA STREET, COVENT GARDEN, LONDON;

AND 20 SOUTH FREDERICK STREET, EDINBURGH.

1883.

tunity". He is of course right in objecting to *laissez faire* doctrine, but just as much of course wrong in identifying Mr. Spencer's view with this, as he does on p. 231. Nor, as Mr. Spencer long ago pointed out, is the title "Administrative Nihilism" rightly applicable to his doctrine of Specialised Administration.

Repetition of an obscure principle does not always produce clearness, any more than it evolves truth out of error. In Mr. Kilpatrick's "Pessimism and the Religious Consciousness," the repetition is excessive. That the individual must be grounded in the universal consciousness, and that no criticism of the world is sound except from the point of view of the constitutive principle—these enunciations form no small ingredient in his exposition. From the point of view of this universal consciousness the truth of pessimism ceases, for the world no longer answers to its description. The synthesis of humanity as in Comte's Positivism is deemed insufficient, but the writer does not consider the wider view manifest in the doctrine of Evolution, which can hardly be called pessimistic. One of the main points of the essay seems to involve a contradiction. On p. 269 we find that all the evil of which an individual is conscious is due to his own act, and it is so due because the individual "is lifted out of his individuality, and is united to the principle through which the world is for him". But on p. 272 evil is asserted to be "the assertion of the self in its individuality" against this principle, and on p. 275 it appears that his deliverance from evil is his union with that principle. To him for whom the cosmos creaks along on a pivot of pain, I cannot see that any help is given by these vaguenesses about a universal consciousness. The comparative solidity manifest in the opening essays of the book is here frittered away into a febrile and futile phraseology, and the essay serves as a final indication of how words may be taken for thoughts, and of the critical state of a theory exhibiting the attempt at philosophy without a mastery of the meaning of psychology.

RICHARD HODGSON.

Studies in Logic. By MEMBERS OF THE JOHNS HOPKINS UNIVERSITY. Boston: Little & Brown, 1883. Pp. 203.

Mr. C. S. Peirce's name is so well known to those who take an interest in the development of the Boolian or symbolic treatment of Logic that the knowledge that he was engaged in lecturing upon the subject to advanced classes at the Johns Hopkins University will have been an assurance that some interesting contributions to the subject might soon be looked for. And such assurance is justified in the volume under notice, which seems to me to contain a greater quantity of novel and suggestive matter than any other recent work on the same or allied subjects which has happened to come under my notice.

It is a collection of papers or essays by several different authors —pupils, it may be inferred, of Mr. Peirce—concluding with an essay on the nature and foundations of Inductive Logic contributed by Mr. Peirce himself.

The two contributions which deal specially with symbolic non-quantitative Logic are furnished respectively by Miss Ladd and Mr. O. H. Mitchell. The former of these adopts substantially the Boolian calculus as modified by Herr Schröder, but with considerable additions. The main departures from the original calculus are to be found in the adoption of the now familiar mode of expressing alternatives in an inexclusive notation, and in the introduction of a symbolic procedure for expressing the true particular proposition. Of the former of these two points little need be said here, as the procedure in question is distinctly the popular one at present, and indeed ever since it was first familiarised to us by Jevons. That it produces a great simplification in procedure is undeniable, but it has what seems to me the serious defect, from a speculative point of view, that it does not lend itself to the use of inverse formulæ, such for example as those analogous to the operation of division. Against the use of such formulæ, however, the writers before us show a rather unreasonable antipathy, and Miss Ladd is certainly wrong in declaring that the only justification for them is based on the analogy of mathematics.[1] That the use of the division-sign was first suggested by virtue of such analogy is doubtless true; but each can now stand equally on its own ground of justification, for not a shadow of reason can be alleged why the notion of an inverse operation should not exist in purely qualitative inference. I can hardly conceive any more valuable speculative exercise than that of clearly realising and determining the conditions of an inverse process in Logic—say that of ascertaining, after we have defined and symbolised the process of determination or restriction, what must be the starting point, and what the limits of indefiniteness in the step of such restriction in order to arrive at a given result: in other words, having defined xy, of putting our interpretation upon $x \div y$. I am confirmed in this opinion by the fact that a writer such as Jevons, in his last logical work, should have still regarded such expressions as "impossible" ones. To my thinking, the value of the mental exercise in question outweighs the merits of the very elegant devices (of which a good account is given by Miss Ladd) for denoting the contradictory of any complicated expression, which are available on the non-exclusive method.

[1] "It is only on account of a supposed resemblance between the logical and the mathematical processes that an attempt to introduce them has been made" (p. 19). Considering the extent to which they *are* introduced by Boole, and with what no one denies to be consistent usage and correct results, something more than an "attempt" might be admitted.

The most important and characteristic point insisted on in this volume is the necessity of introducing an additional form of copula (or predicate) for the due expression of particular propositions. As the reader probably knows, it is a characteristic of one main form of the Symbolic Logic that it aims at throwing propositions into the form of denying that a certain class-combination exists: thus 'All x is y' is regarded as denying that there is any x not-y. This plan does well enough so long as we deal with universals, but when we come to particulars we find that if we are to retain the full signification, in respect of indefiniteness, of the word 'some,' such propositions cannot be expressed without recourse to some new device. Both Boole and Jevons indeed claim to introduce particulars, but scarcely any one who will carefully analyse the full import of the expressions they employ can allow them to be successful. In Boole's $vx = vy$, it is distinctly admitted that v is not to be treated as an ordinary class-symbol that may be equated to zero, and this restriction invalidates the introduction of such an expression into his most important generalisations. Jevons's CA = CB escapes this difficulty by the denial that *any* of his single letter-symbols can be equated to zero. In this respect he stands alone amongst modern symbolists, and the consequences are fatal to the true generality of his expression of universal propositions. An indefinite number of perfectly consistent groups of propositions would have to be refused admittance by him on the ground of mutual inconsistency, because they would lead to the obliteration of some single letter or class-symbol.

It is one great merit of the writers under review to have fully recognised this fact and to have grappled with the attempt at the general introduction of such propositions in their true indefinite form. With consistency, therefore, the fundamental distinction is found to lie, not between affirmative and negative, but between universal and particular propositions. It is the function of the former to *deny*, whatever may be their character in their common signification—to deny, that is, not a predicate of a subject, but the existence of a certain combination made out of the subject and predicate taken together. It is the function of the latter to *affirm*, *viz.*, to save the existence of a similar compound. 'All x is y' and 'No x is y' respectively deny the existence of the compounds $x\bar{y}$ and xy; whilst 'Some x is not y' and 'Some x is y' respectively save these compounds. Accordingly whilst mere class-reference leads to a dichotomy only, in that a thing must belong to the class or not, the act of judgment or assertion leads to a trichotomy, for we may affirm or deny or be in doubt about the matter. I do not mean that this distinction is in any way peculiar to the conception of logic in question, but it seems to be much more strongly emphasised here; for when we break up the universe into all the possible sub-classes yielded by the class-terms, it would seem that to save or destroy any one

of these classes makes as complete a pair of alternatives as to include or exclude an object in reference to them. Why this difference, then, it may be asked, between class-reference and predication; and is the three-fold character of the latter a necessary and permanent one? To enter fully into this inquiry would be a digression in a brief review of this kind, but a few hints may be permitted. In respect of the distinction between class-reference and predication, the answer would seem to be that subjectively regarded we should equally have to admit a three-fold division in the case of the former; for we must either know that an object does belong to a given class, or that it does not, or we must be in doubt about it. But we find it simpler to take a completely objective standing-point here, by admitting only the two alternatives, one or other of which must exist physically, and by referring the element of doubt to the region of judgment, *viz.*, of predication. As regards the finality of any such logical convention, it must be frankly admitted that it is not final, but then the science of logic itself belongs to a progressive or imperfect state of knowledge. Were our knowledge absolutely complete we should know for certain of any object whether it did or did not belong to any assigned class; its x and *not-x* status would not only exist but be known. And in the same way we should know of every assignable sub-class whether it existed or not. Accordingly, not to affirm would be to deny and not to deny would be to affirm: that is, on the Boolian conception of the logical universe, the class-elements which were not destroyed would be known to be saved. But this is of course a merely ideal state of knowledge, and its realisation would do away with all the utility and significance of Logic.

The copula-symbols employed by Miss Ladd are $\vee$ and $\bar{\vee}$: A $\vee$ B is to be read 'A is in part B,' or 'A is not-wholly excluded from B,' always with the special signification above assigned to the particular proposition. On the other hand, A $\bar{\vee}$ B is to be read 'A is not B,' or 'A is wholly excluded from B,' and gives no implication that either A or B exists. That is, these are the expressions for the particular affirmative and universal negative, whilst the universal affirmative 'All A is B,' is written A $\bar{\vee}$ $\bar{\text{B}}$, *viz.*, A excludes not-B. This mode of writing propositions is, I think, more consistent and intelligible when the constituent class-symbols are all gathered to the left side of the expression, and the whole is read as a declaration of existence or non-existence of the compound class-term thus produced. For instance, we may substitute 'AB *is*' for 'A is in part B,' when the particular proposition is interpreted under the conditions above-stated; and 'AB is not' for 'No A is B'. The symbolic expressions for these forms necessitate of course some symbol for the 'universe' and its absence, for which Miss Ladd employs, in common with some other writers, the mathematical forms ∞ and 0. But, as is pointed out, the introduction of a double copula symbol enables us to make use of one only of these universe symbols, and the result is simplified by taking ∞ for this purpose, and (by taking it for granted) avoiding the necessity of actually introducing it into our equations. Thus, 'There is x' and 'There is no x,' being written in full $x\vee\infty$ and $x\overline{\vee}\infty$, we may take the latter symbol for granted and simply write $x\vee$ and $x\overline{\vee}$. When for x here we substitute a complex expression, *e.g.*, $x\bar{y}$, we obtain propositions which break up into the familiar subject and predicate form. Thus, $x\bar{y}\vee$ means 'There is x which is not y,' or 'Some x is not y'; and $x\bar{y}\overline{\vee}$ means 'There is no x which is not y,' namely, 'All x is y'.[1]

It should be remarked that the different symbols thus aggregated on the left side are necessarily commutative and transferable at will to the other side. Thus, for $x\bar{y}z\overline{\vee}$ which means 'There is no $x\bar{y}z$,' we may substitute such equivalent forms as $xz\overline{\vee}\bar{y}$, *viz.*, 'All xz is y,' or $x\bar{y}\overline{\vee}z$, *viz.*, 'No x which is not y is z,' and so on to any extent which the available number of permutations may permit. A large part of the essay is devoted to the exhibition and comparison of convenient equivalent or inferrible expressions founded on this leading idea.

The remarks already made about the expression of particular propositions naturally suggest the inquiry whether any perfectly general treatment of them is available, that is, corresponding in generality and brevity to those which Boole has given and which have been simplified in their practical employment by a succession of writers. I am inclined to think that it is not; at least I do not remember to have seen anything at all rivalling the completeness with which groups of universal propositions can be grappled with. The passage in which one would most expect to find the desired formulæ in the volume before us is at page 45, under the head of "Resolution of Problems". As the passage seems to me obscure, I quote it in full:—

"From a combination of universal propositions, the conclusion, irrespective of any term or set of terms to be eliminated, x, consists of the universal exclusion of the product of the co-efficient of x—by that of the negative of x, added to the excluded combinations which are free from x as given. If the premisses include an alternation of particular propositions, the conclusion consists of the partial inclusion of the total co-efficient of x in the particular propositions by the negative of that of x in the universal propositions, added to the included combinations which are free from x as given."

The first part of this statement is simple enough. It is the well-known rule, For $f(x)$ write $f(1)f(0)$, put into its simpler concrete form; for when $f(x)$ is represented as a logical expression involving x and not-x, it stands $Ax + B\bar{x} + C$, and the above formula becomes (regard had to the suggestions of simplification which Boole recommends) $AB + C = 0$.[1] But the latter sentence does not seem to me at all clear. Take a simple instance embracing one universal proposition, or resultant of several such, and one alternation of particulars. The fullest expression for the former, regard being had to any class-term x and its negation, is as above $Ax + B\bar{x} + C = 0$; or as Miss Ladd writes it $Ax + B\bar{x} + C\overline{\vee}$. The correspondingly full form for the latter would then be $Ex + F\bar{x} + G$ = something, or $Ex + F\bar{x} + G\vee$; *viz.*, it intimates that something comprised in this aggregate of class-combinations is to be saved. Now "the total co-efficient of x in the particular propositions" is here E, and "the negative of that of x in the universal proposition" is $\bar{A}$, and "the combinations which are free from x as given" are C and G, and by "partial inclusion" is meant combination into a particular proposition. According to this, the solution of the problem would be $EA + C + G\vee$, if the "addition of the included combinations free from x" is to be carried out as when we are dealing with universals alone. But this clearly will not do. The rule may possibly be intended for the much simpler case of two such premisses as $Ax + B\bar{x}\overline{\vee}$, and $Ex + G\vee$. A value (not the full value) of x, determined from the former, is that it is $\bar{A}$: substitute this in the latter, and we should have $E\bar{A} + G\vee$, which answers to the rule given.

It may be worth while to digress from the present treatment for a few minutes in order to point out how we might attempt to represent diagrammatically the results of combining any number of universal propositions and any number of alternatives of particulars. I have suggested elsewhere a plan for thus representing any combination of universals by drawing a system of circles or other closed and mutually intersecting figures, and shading out all the compartments which are shown to be destroyed when the propositions are interpreted with the usual symbolic signification. If we introduce particular propositio[ns] also, we must of course employ some additional form of diagrammatical notation, just as the writers under notice find it neces-

[1] The originality of treatment here consists mainly in the notation and certain consequences thereof. The general conception of forming a scheme of propositions by equating all the possible combinations of class-terms to 0 and *something*, has been suggested before. For instance, I have suggested such a scheme myself, on a plan modified with less departure from Boole's plan.

[1] This formula is sometimes (as here) called Schröder's *modification* of Boole's formula of elimination. It hardly appears to me to deserve a distinct name. No doubt it seems at first sight, as Miss Ladd says, to differ from Boole's by the omission of the redundant terms AC and BC given by the multiplication of the factors $(A + C)(B + C)$ or $f(1)f(0)$. But Boole himself recommends (p. 130) the omission of such terms in the practical application of his rules. The simple fact is that the formula $f(1)f(0)$ is the most abstract or general form, and $AB + C$ the form suggested for adoption when our expression is one degree more concrete in its statement.

sary to employ two forms of copula. We might, for example, just draw a bar across the compartments declared to be saved; remembering of course that, whereas destruction is distributive, *i.e.*, *every* included sub-section is destroyed, the salvation is only alternative or partial, *i.e.*, we can only be sure that some of the included sub-sections are saved. Thus, 'No x is y,' leading to the destruction of xy, will destroy both xyz and $xy\bar{z}$, if z has to be taken account of. But 'Some x is y,' saving a part of xy, does not in the least indicate whether such part is xyz or $xy\bar{z}$. Thus, if we had the general alternative premiss that $Ax + B\bar{x} + C$ exists, this means that either there is x which is A, or not-x which is B, or that there is C. Draw a line of some recognisable kind through, or in any way put a mark on the list of elements included above, and the import of the proposition is complete when we are informed that some or other of their contents is to be reserved. We may then proceed to fill in all the information furnished by the universal propositions, by erasing the compartments with which they deal, and the full information of the combined propositions is represented to the eye. Of course, if several groups of such alternatives of particulars are given in the premisses we must employ a distinctive line or mark for each such group. If it were worth while thus to illustrate complicated groups of propositions of the kind in question, it could, I fancy, be done with very tolerable success.

Miss Ladd's paper is illustrated by a good selection of examples. Some of these are of considerable difficulty and complexity, and show very forcibly the progress that has been made within the last few years, as logicians have begun to acquire ease and dexterity in the manipulation of their rules, and in the invention of intermediate formulæ and practical simplifications. They seem, so far as I have tried them, to be concisely and correctly worked out, and several of the devices adopted represent real simplifications in procedure.

The following minor points seem to me to call for revision or reconsideration. On p. 42, the example is certainly wrong, or else $\bar{x}$ has been written by mistake for x. On p. 27, it is maintained that the notation employed will suit the expression of negative propositions whether interpreted in extension or intension; "the proposition *no stones are plants* means that the objects denoted and the qualities connoted by the term *stone* are inconsistent with the objects denoted and the qualities connoted by the term *plant*". This seems to me a very misleading statement. The objects denoted are of course numerically entirely distinct, but in what appropriate sense they can be called "inconsistent," I cannot perceive, for each may necessarily imply the existence of the other, *e.g.*, 'No husbands are wives'. On the other hand, the qualities connoted are by no means necessarily distinct as wholes, the utmost we can say being that in one at least of the

two groups there must be some attribute which is not found in the other.

The second of the two principal papers on Symbolic Logic is by Mr. O. H. Mitchell, and seems to me one of the most valuable and original in the volume. Its fundamental method turns upon an ingenious modification of the Boolian plan of expressing propositions. Starting from the fundamental expression for the full development of a combination of two terms, $xy + x\bar{y} + \bar{x}y + \bar{x}\bar{y} = 1$, the import of a universal proposition is most usually read off as destroying one or more of these elements. Thus, 'All x is y,' or $x\bar{y} = 0$, expunges $x\bar{y}$, and so on with the others. But it is an exactly equivalent alternative form to say that the aggregate of the remaining elements constitute the logical universe, for $x\bar{y} = 0$ is obviously the same result as $xy + \bar{x}y + \bar{x}\bar{y} = 1$. So much is of course familiar to every student of the subject, but what I confess was new to me was the ease with which groups of propositions could be combined on this plan, and the fact that on this mode of procedure the process of multiplication corresponds to that of addition on the ordinary process; another of the interesting parallelisms which have been pointed out as existing between + and × in the logical algebra. Thus, if we take the two propositions 'All x is y' and 'All y is x,' they give $x\bar{y} = 0$, $\bar{x}y = 0$, and the combination of the two is given by *adding* the results. But when we equate the remainders to unity, or the universe, we must *multiply*, for $(xy + \bar{x}y + \bar{x}\bar{y} = 1) \times (xy + x\bar{y} + \bar{x}\bar{y} = 1)$ gives $xy + \bar{x}\bar{y} = 1$, which is of course the same as if we had added $x\bar{y} = 0$ to $\bar{x}y = 0$. The process, obvious in this simple case, admits of an easy symbolic generalisation. And this leads to a further development by which we may not only represent particular propositions but also combine them with universals. The notation adopted for this purpose is to write F_1 for $F = 1$, where F is any logical polynomial or aggregate of class-terms, so that that aggregate is declared to constitute the whole universe; and F_u for the assertion that F simply exists, that is, that some one or more of its constituent elements is represented as existent. The former therefore belongs to universal propositions and the latter to particular. A large number of derivative formulæ are then given, such as the following: $F_1G_1 = (FG)_1$, $F_u + G_u = (F + G)_u$, to quote two only of the simplest.

This method of procedure sometimes gives a decidedly more compact and convenient expression than the more familiar one, sometimes the reverse, according to the nature of the propositions dealt with; for as more elements have to be equated to zero the residue that have to be equated to unity, or the universe, become fewer. For instance, the expression on this scheme of 'No x is y' is decidedly more cumbrous, since we have to put it $(\bar{a} + \bar{b})_1$ *viz.*, to make it assert that not-a and not-b make up the whole.

The problem of elimination on this scheme becomes simple. It rests upon the same fundamental basis, of course, *viz.*, that logical elimination is simply the dropping of irrelevant information, only here it assumes the form of expanding or widening the extent of any class term in the aggregate. For instance if xy and z together fill the universe, it is clear *à fortiori* that y and z together will do so, *viz.*, we may drop the constituent element x. The element z, which stands alone, cannot be thus dropped; or rather it must be regarded as (what it is) a constituent of 1, *viz.*, as $1z$, so that the elimination of z above would be given as $xy + 1 = 1$, which is of course a truism. This is expressed by saying "If F be a polynomial of the class-terms $a, b, c, \ldots$ x, y, z, then x, y, z, may be eliminated from F by erasure, provided no aggregant term is thereby destroyed" (p. 80).

The best way of illustrating the peculiarities of this scheme of procedure and notation will be to take one of Mr. Mitchell's simpler examples as he does it, and to give the solution as it might be worked out on Boole's plan.

"What may be inferred independent of x and y from the two premisses 'Either some a that is x is not y, or all d is both x and y,' and 'Either some y is both b and x, or all x is either not y or c and not b'? (p. 85).

"The premisses are

$$(ax\bar{y})_u + (\bar{d} + xy)_1,$$
$$(bxy)_u + (\bar{x} + \bar{y} + \bar{b}c)_1.$$

"By multiplication we get

$$(ax\bar{y})_u(bxy)_u + (bxy)_u + (ax\bar{y})_u + (dx + \bar{d}\bar{y} + \bar{b}c\bar{d} + \bar{b}cxy)_1.$$

"Whence, dropping x, y, and reducing, we get

$$(b + a)_u + (\bar{d} + \bar{b}c)_1$$

which may be interpreted in words, 'There is some b or a, or else all d is c and not b'."

The Boolian process is not at its best in dealing with particulars and hypotheticals, but, as it happens, this example yields little difficulty. Merely premising that 'Either a is β or γ is δ' means 'If a is not β, γ is δ,' and that this may be expressed $\gamma\bar{\delta}(1 - a\beta)$, the premisses stand,

$$d(1 - xy)(1 - ax\bar{y}) = 0$$
$$xy(1 - c\bar{b})(1 - bxy) = 0,$$

or, more simply,

$$d(x\bar{y}\bar{a} + \bar{x}) = 0$$
$$xy\bar{b}\bar{c} \qquad = 0$$

The elimination of x gives at once $y\bar{b}\bar{c} + d\bar{y}\bar{a} = 0$, and that of y gives $d\bar{a}\bar{b}\bar{c} = 0$, *viz.*, 'All d is either a or b or c'.

Mr. Mitchell's result is in reality the same as this, but it is expressed with needless prolixity. To say that 'There is some b or a, or else all d is c and not b,' is to say that 'If there is neither b nor a then d is c and not b,' and under these circumstances it it clearly needless to say that d is not b.

In addition to the essays which we have thus noticed, there are several others all of them deserving of study. Dr. Marquand contributes two: one of these describes a mechanical device for representing certain syllogistic results; and the other contains an account of the Epicurean system of empirical logic as this has been recovered from certain fragments of MSS. at Herculaneum, the substance being taken from the monographs of Gomperz and Bahnsch. Mr. Gilman contributes a paper upon the intricacies of the Logic of Relatives, that is, which deals with propositions not involving mere predication in the way of class-relation, but relation generally. The most interesting paper philosophically is the concluding one by Mr. Peirce himself. It deals with the nature and foundations of statistical reasoning and the mutual connexion between Probability and Induction, but it is too long and intricate to admit of a brief summary. It supplies an element which is somewhat missing in the other papers; for whilst the younger authors write more as mathematicians who have turned to the consideration of logical formulæ, Mr. Peirce is well acquainted with the history of the subject, and realises more clearly what are the special characteristics to be looked for in a symbolical or algebraical treatment of Logic.

J. VENN.

The Elements of Logic. By T. K. ABBOTT, B.D., Fellow and Tutor of Trinity College, Dublin. Dublin: Hodges; London: Longmans, 1883. Pp. 102.

IT is impossible to read this little treatise without arriving at two conclusions: first, that the author is a master of his subject, namely, Aristotelian Logic; and, secondly, that his work has been to a great extent injured by following too closely the outlines of Dr. Murray's treatise on the same subject, which forms the present text-book of the University of Dublin. Of that work Sir W. Hamilton wrote, just half a century ago, that while Cambridge was dependent on a treatise which dispensed "a muddy scantling of metaphysic, psychology and dialectic," "Dr. Murray's *Compendium Logicæ*, the Trinity College text-book, may show that matters are if possible at a lower pass in Dublin" (*Discussions*, p. 123). Had the great logician seen Walker's Commentary, which largely aggravated the faults of the original work, he would doubtless have omitted the "if possible". It is only now, however, that the University authorities are beginning to become alive to the fact, which has long been somewhat rudely expressed by the students, that the Provost and Mathematician whose work has been studied for nearly a century was (as regards this subject at least) not up to the mark; and, seeing that Mr. Abbott, like Mr. Walker, can write the magic words "Fellow and Tutor" after his name, it may be confidently hoped that the reign of Murray is at an end, while perhaps after the lapse of