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ART. XX.—*On the Absolute Wave-length of Light;* By LOUIS BELL, Fellow in Physics in Johns Hopkins University.

UP to the present time, Ångström's map of the solar spectrum and with it his determination of absolute wave length, has remained the final standard of reference in all spectroscopic matters. But since Ångström's work was published, optical science, particularly that part of it which deals with the manufacture and use of diffraction gratings, has made enormous progress. It is now possible with the concave grating to measure relative wave-lengths with an accuracy far greater than can be claimed for any one of the absolute determinations. The numbers given by Ångström are now known to be too small by as much as one part in seven or eight thousand, as has been shown by Thalén in his monograph "Sur le spectre du Fer," and since Ångström's work but one careful determination has been made. This is by Mr. C. S. Peirce and was undertaken some eight years since for the U. S. Coast and Geodetic Survey. No full report of this work has as yet been published, though it is evidently very careful and has already consumed sev al years. Certain results were communicated to Prof. Rowland of this University to serve as a standard of reference for his great map of the solar spectrum now nearly completed, and it was to serve as a check on these results and to furnish a value of the absolute wave length as nearly as possible commensurate in accuracy with the micrometrical observations, that the experiments detailed in the present paper were undertaken. Only the work with glass gratings has been as yet completed, but since the relative wave lengths, which are intrinsically of far greater importance, are now ready for publication and have been reduced by the value herein given, the result is published, leaving for further work with speculum metal gratings, its final confirmation or correction.

This portion of the determination is delayed awaiting better facilities for carrying it out, but the writer intends undertaking it at the earliest possible moment and hence leaves for a future paper the complete discussion of the problem.

The writer desires here to express his deep obligations to Prof. Rowland, under whose guidance the work has been carried on and to whom a very important correction is due, and to Profs. W. A. Rogers and C. S. Peirce for information given and courtesies extended.

Experimental.

The determination of absolute wave-length involves two quite distinct problems—first, the exact measurement of the angle of deviation of the ray investigated, and second, the measurement of the absolute length of the gratings used. Each portion of the work involves its own set of corrections, frequently quite complicated and difficult, but it is the latter part that is peculiarly liable to errors, which will be treated in detail further on. As to the former part, several important questions arise at the very outset. First is the choice between transmission and reflection gratings. The principal work heretofore has been done with the former, but metallic gratings possess certain advantages, notably from the ease with which their temperature can be accurately measured, and the fact that they can easily be made of a size much larger than glass gratings, and consequently a small inaccuracy in measuring them involves much less error in the result.

On the other hand the coefficient of expansion of speculum metal is more than twice as great as that of glass, and being a good conductor it is far more sensitive to small changes of temperature. And this property increases the liability to irregularities in the ruling, particularly in large gratings which require several days for completion. In ruling on glass change of temperature is less serious but this advantage is more than offset by the faults caused by the wearing away of the diamond point, which breaks down so rapidly that it is enormously difficult to produce a glass grating free from flaws and at all comparable in optical excellence with those upon speculum metal. The determination of absolute wave-length should rest on measurements made with both classes, and with sufficiently exact instruments and very careful experimentation, the better results

can probably be obtained from the metallic gratings. For the reasons previously stated, this paper is confined to the results from glass ones.

Now there are two quite distinct ways of using transmission gratings—first, perpendicular, or nearly so, to collimating or observing telescope; and second, in the position of minimum deviation. The method in the first case is familiar—the properties of the second are as follows:

The general relation between the incident and the diffracted ray is:

$$\sin i + \sin(\delta - i) = \frac{m\lambda}{a}.$$

When $i = o°$, this gives the ordinary formula for normal incidence. Putting it in the form—

$$\lambda = \frac{2(a)}{m} \sin\frac{\delta}{2} \cos\left(i - \frac{\delta}{2}\right),$$

the deviation represented by the angular term will evidently be a minimum when $i = \frac{\delta}{2}$ and the wave-length will then be given by the formula

$$\lambda = \frac{2(a)}{m} \sin\frac{\delta}{2}.$$

It is not easy to say which method of procedure is preferable, but on the whole the ordinary plan of normal incidence offers fewer experimental difficulties and therefore was adopted, particularly as the spectrometer used was specially well suited to that method. It is quite certain that either method will with proper care give the angular deviation with a degree of exactness far surpassing that attainable in the measurement of the gratings.

The Spectrometer.

This was a large and solid instrument by Meyerstein with a circle on silver 32^{cm} in diameter divided to tenths of a degree. This is read by two micrometer microscopes 180° apart. The pitch of the micrometer screws is such that one turn equals about 2′, and as the head is divided into sixty parts each of these represents 2″. The micrometer can, however, be set with certainty to less than half this amount. The collimating and observing telescopes are of 4^{cm} clear aperture and 35^{cm} focal length and the lenses are well corrected. The collimator is fixed to the massive arms which carry the reading microscopes while the observing telescope is attached to a collar on the axis of the main circle and moves freely upon it or can be firmly clamped so as to move with the circle. The grating is carried on an adjustable platform with a circle 12·5^{cm} in diameter, divided to 30′ by verniers to 1′ and moving either upon or with the large circle.

This arrangement of parts does not admit of fixing the grating rigidly normal to the collimator, so in all the experiments it was placed normal to the observing telescope, a position which was particularly advantageous in the matter of adjustment. The instrument was set up in a southern room in the physcial laboratory and throughout the experiments the collimator pointed about south-southeast. With the eyepiece used the observing telescope had a power of very nearly sixteen diameters.

Gratings.

Very few glass gratings have ever been ruled on Prof. Rowland's engine, since for most purposes they are much inferior to the metallic ones, and are very much more difficult to rule, as they run great risk of being spoiled by the breaking down of the diamond point. A very few, however, were ruled in 1884 with special reference to wave-length determination, and of these the two best were available for these experiments. They are both ruled upon plane sextant mirrors, and are of very nearly the same size—thirty millimeters long, with lines of about nineteen millimeters. Each hundredth line is longer, and each fiftieth line shorter than the rest, so that the gratings are very easy to examine in detail. The ruling of both is smooth and firm, without breaks or accidental irregularities and almost without flaws. They were ruled at different temperatures and on different parts of the screw, and while one was ruled with the ordinary arrangement of the engine, the other was ruled to a very different space by means of a tangent screw. This great diversity of conditions in the two gratings is far from favoring a close agreement in the results, but tends to eliminate constant errors due to the dividing engine, and hence to increase the value of the average result. It must be remembered that two gratings ruled on the same part of the screw are in most respects little better than one. The grating designated I in this paper contains 12,100 spaces, at the rate of very nearly 400 to the millimeter, and was ruled (by tangent screw) at a temperature of 6°·7 C. in January, 1884. It gives excellent definition with almost exactly the same focus for the spectra on either side, and is quite free from ghosts or other similar defects.

The grating designated II has 8,600 spaces, at the rate of about 7,200 to the inch, and was ruled in November, 1884, at 11°·6 C. Its definition and focussing are very nearly as good as in I and like it, it shows no trace of ghosts or false lines; they

are both exquisite specimens of the work which Prof. Rowland's engine is capable of doing, though as the event showed, I is decidedly the better grating, in the matter of regularity of ruling.

Angular Measurements.

At the beginning of the work a serious question of adjustment arose. There are two ways of using a grating perpendicular to one of the telescopes. In the first place it may be placed and kept accurately in that position, and secondly it may be placed nearly in the position for normal incidence, and the error measured and corrected for. Ångström used the latter method, which involved a measurement on the direct image of the slit as well as on the lines observed. Using Ångström's notation—let α and α' be the readings on the spectra, and M that on the slit. Let, also,

$$\frac{\alpha+\alpha'}{2} - M = \Delta \quad \text{and} \quad \frac{\alpha-\alpha'}{2} = \varphi.$$

Then if γ is the angle made by the incident ray with the normal to the grating, and N the order of spectrum,

$$\frac{N\lambda}{\varepsilon} = \cos(\gamma+\Delta)\sin\varphi;$$

also $$\sin\gamma = \sin(\gamma+\Delta)\cos\varphi;$$

and $$\tan\gamma = \frac{\cos\varphi}{1-\cos\varphi}\Delta.$$ But from the second of the above equations,

$$\sin(\gamma+\Delta) = \frac{\sin\gamma}{\cos\varphi}.$$

Now it was found that with the collimating eyepiece belonging to the spectrometer, γ would never exceed and seldom reach 10″, while the angles of deviation observed were about 45°. Substituting these values in the last equation, it at once appeared that the cosine of $(\gamma+\Delta)$ was a quantity differing from unity by considerably less than one part in a million, and hence entirely negligible. Further, it was found that the grating once set could be trusted to remain perpendicular through a series of measurements, and though at the end of each series the grating was adjusted to a new part of the circle, and a close watch was kept for its slipping out of adjustment, it was never found necessary to reject a series from that cause.

The grating was centered and adjusted with reference to the circles and their axes by the ordinary methods. Throughout the experiments the light was concentrated on the slit by an achromatic lens of about half a meter focus, which was placed behind a sheet of deep yellow glass, which served to cut off the overlapping blue rays which might otherwise have proved troublesome. A heliostat enabled the sun's image to be kept centrally upon the slit.

The method of observation was as follows: When instrument and grating were in exact adjustment, readings were taken on D, in the spectra on either side of the slit, and the angle measured from three to six times in rapid succession, the last reading being of course on the same side as the first.

Then the grating was rotated about ten degrees, readjusted, and the process repeated.

The angles observed in one series were combined to eliminate errors of setting, while the use of all portions of the circle served to correct errors of subdivision, since the number of independent series of observations was quite large.

To eliminate any errors which might be due to imperfections of figure in the gratings they were used in all the four possible positions. No such error, however, became apparent either from critical examination of the gratings themselves, or from the results obtained in the different positions.

Observations with grating I were begun late in October, 1885, and occupied the clear days for a month. Forty-eight series of measurements were made, and the agreement between them was very satisfactory. After correcting for temperature, thirty-six of the number fell within a range of three seconds and the rest were clustered closely about them. Observations on the various days were as follows:

Date.		Number of Series.	Angle.
Oct.	19	1	45° 1′ 47″·2
	20	1	45° 1′ 48″·4
	22	2	45° 1′ 48″·2
	23	1	45° 1′ 49″·8
	26	4	45° 1′ 49″·3
	27	3	45° 1′ 48″·2
	31	1	45° 1′ 50″·1
Nov.	3	1	45° 1′ 48″·6
	4	3	45° 1′ 47″·4
	5	2	45° 1′ 47″·9
	10	4	45° 1′ 47″·8
	11	6	45° 1′ 49″·7
	16	8	45° 1′ 48″·2
	17	5	45° 1′ 47″·5
	20	6	45° 1′ 47″·5

All the above were in the third spectrum to which measurements were in the main confined as in it the definition was particularly good, and it being the highest order which could be conveniently observed, an error in the angle would produce the

minimum effect. The spectra on both sides of the slit were about equal in brilliancy and definition.

The observations were weighted as nearly as possible according to the favorable or unfavorable conditions under which they were made, and when finally combined gave as the value of the angle of deviation for grating I,

$$\varphi = 45° \; 1' \; 48''{\cdot}24 \pm 0''{\cdot}11.$$

The above probable error is equivalent to a little less than one part in a million and can introduce no sensible error into the resulting wave length.

Other work intervened and the measurements with grating II were not taken up until early in the succeeding March. Precisely the same method of observation was employed and the results were nearly as consistent and satisfactory.

The observations by days were as follows:

Date, 1886.		Number of Series.	Angle.
March	6	2	42° 5′ 1″·2
	10	1	42° 4′ 58″·6
	14	7	42° 5′ 1″·4
	15	1	42° 5′ 4″·0
	16	6	42° 4′ 57″·8
	17	6	42° 4′ 58″·5
	18	7	42° 4′ 59″·1
	23	6	42° 4′ 58″·3

When collected thus by days the observations do not appear to agree nearly as well as those made with grating I, particularly since a solitary wild reading, that of March 15, is retained. The distribution of the various readings, however, is such that after weighing and combining the final result is by no means deficient in accuracy. It is,

$$\varphi = 42° \; 4' \; 59''{\cdot}28 \pm 0''{\cdot}2$$

The above probable error amounts to about one part in six hundred thousand. The observations with grating II were uniformly in the fourth order of spectrum.

Throughout the measurements with both gratings the temperature was kept within a few degrees. 20° C. had been selected as the standard temperature and the variation was rarely more than two or three degrees on either side of that figure. The question of temperature determination is a serious one in case of glass gratings, for it is very hard to tell what heating effect the incident beam has on the grating, and equally hard to measure that effect. It is hardly safe without extraordinary precautions to assume that the grating has the same temperature as the air near, and it is such a bad conductor that it would not easily assume the temperature of the apparatus. In these experiments a sort of compromise was effected. A small thermometer was attached to the thin metallic slip that held the edge of the grating, and shielded by cotton from air currents which of course would affect it much more than they would the grating. The thermometer was a small Fahrenheit graduated to quarter degrees and quite sensitive. It was carefully compared, throughout the range of temperatures employed, with thermometer Baudin 7312, which served as a standard in all the measurements regular and linear, and during part of the time was placed directly over the grating to give a check on the attached thermometer. This expedient was finally abandoned as unlikely to help the matter much.

The corrections for temperature were deduced from the assumed coefficient of expansion of glass, which was taken as 0·0000085. This was reduced to angular correction for the approximate value of φ and applied directly to the observed angles. Since the temperatures at which observations were made varied little from 20° C. and were quite equally distributed on both sides of that figure any error in the assumed coefficient would hardly affect the average result, but would appear, if at all, as a slight increase in the probable error.

760^{mm} (reduced) was taken as standard pressure and the values for the days of observation were taken from the U. S. Signal Service observations for the hours of 11 A. M. and 3 P. M. on those days. The average for the measurements made with grating I was 761^{mm}, and for those with grating II 760^{mm}, so that corrections for pressure were uncalled for.

The effect of the velocity of the apparatus through space is a subject concerning which there has been much discussion. Ångström deduced a correction, but van der Willigen in quite a lengthy discussion of the whole matter came to the conclusion that there was no error due to the above cause. Since that period the question has been raised from time to time, but no decisive investigations on the subject have yet been published. At present however it seems to be tolerably well settled that no correction is needed, as the error, if there be any, is of an order of magnitude entirely negligible, and in the present paper none has been applied.

The angular measurements, after all corrections were applied, may thus be regarded as determined with a high degree of accuracy—most probably to less than one part in half a million.

Measurement of the Gratings.

The exact determination of the grating space is by far the most difficult portion of a research on absolute wave length, and has been uniformly the most fruitful source of errors.

Besides the experimental difficulties of the task, it is far from an easy matter to secure proper standards of length. The standards used in various former investigations have proved to be in error, sometimes by a very considerable amount, and indeed very few of the older standards are above suspicion. As Peirce has very justly remarked in connection with this subject, "All exact measures of length made now must wait for their final correction until the establishment of the new metric prototype." Short standards of length are in some respects peculiarly liable to error, since they must be compared with the subdivisions (often not sufficiently well determined) of secondary standards, and small sources of uncertainty such as poor defining lines, slight changes in the apparatus and the like, of course are much more serious as the length is less.

Fortunately there were available for the measurement of the gratings two standard double decimeters which have been determined with almost unprecedented care by Professor W. A. Rogers. They are upon speculum metal: were graduated and determined by Professor Rogers early in 1885 and were purchased by the university late in the same year. They are designated respectively S_1^d and S_2^d and are discussed at length in the Proceedings of the American Society of Microscopists for 1885.

The bar S_1 is 23^{cm} in length. Near one edge is the double decimeter S_1^d divided to centimeters, the 5^{cm} lines being triple. S_2 is 27^{cm} in length and graduated in the same way. The defining lines in both are fine and sharp and the surfaces are accurately plane. They are standard at 16°·67 C. and from an elaborate series of comparisons with four different standards the coefficient of expansion was found to be,

$$17{\cdot}946\ \mu \text{ per meter per degree C.}$$

S_1^d and S_2^d depend for their accuracy on a long series of independent comparisons with Professor Rogers' bronze yard and meter R_2 and steel standards whose relation to R_2 was very exactly known. R_2 has been determined by elaborate comparisons with various standard meters and yards, and is described and discussed at length in the Proceedings of the American Academy, vol. xviii. The length of the meter was determined both directly and through the yard, by comparison with the following standards:

I. The meter designated T_2 copper with platinum plugs, traced and standardized by Tresca in 1880 from the Conservatoire line metre No. 19, which bears a very exactly known relation to the Metre des Archives.

II. The yard and meter designated C. S., brass with silver plugs, belonging to the Stevens Institute. The yard was compared with the Imperial Yard in 1880 so that it is directly and exactly known. It was afterwards sent to Breteuil and the meter was determined with great exactness by elaborate comparisons with type I of the International Bureau of Weights and Measures.

III. "Bronze 11" a primary copy of the Imperial Yard presented to the United States in 1856. It was taken to England in 1878 and finally determined by direct comparison with the Imperial Yard, Bronze Yard No. 6, and Cast Iron Yards B No. 62 and C No. 63.

The subdivisions of R_2 have been determined with very great care, and thus S^q_1 and S^q_2, whose lengths relative to R_2 are accurately known, may finally be referred to the ultimate standard Type I of the International Bureau.

Only the 5^{cm} spaces of S^q_1 and S^q_2 were investigated by Prof. Rogers, but these were determined by various methods under widely different conditions, and their relations to the standards with which they were compared may be regarded as definitely known. From a combination of all results the subdivisions of S^q_1 have the following lengths at the standard temperature.

$$\text{Standard } S^q_1 = 199{\cdot}99918^{cm}$$
$$^{dm}_{1}S^q_1 = 99{\cdot}99985$$
$$^{dm}_{2}S^q_1 = 99{\cdot}99924$$
$$5^{cm}_{1}S^q_1 = 50{\cdot}00010$$
$$5^{cm}_{2}S^q_1 = 49{\cdot}99985$$
$$5^{cm}_{3}S^q_1 = 49{\cdot}99961$$
$$5^{cm}_{4}S^q_1 = 50{\cdot}00022$$

Similarly the following values were derived for S^q_2.

$$\text{Standard } S^q_2 = 199{\cdot}99985^{cm}$$
$$^{dm}_{1}S^q_2 = 100{\cdot}00001$$
$$^{dm}_{2}S^q_2 = 99{\cdot}99967$$
$$5^{cm}_{1}S^q_2 = 50{\cdot}00020$$
$$5^{cm}_{2}S^q_2 = 49{\cdot}99981$$
$$5^{cm}_{3}S^q_2 = 49{\cdot}99964$$
$$5^{cm}_{4}S^q_2 = 50{\cdot}00042$$

As to the degree of accuracy attained in determining S^q_1 and S^q_2, Prof. Rogers says that including all sources of uncertainty either standard may have an error of $\pm 0{\cdot}3\mu$, but the mean of the two, since the determinations were independent ought to be even more reliable. Taking all things into consideration it seems very improbable that the mean value of S^q_1 and S^q_2 can be in error by as much as one part in half a million.

So much for the standards of length. The comparator used in the measurements was a very efficient instrument, particularly suited for the purpose. It consisted essentially of a long carriage ($\frac{1}{2}$ meter) running on V-shaped ways and carrying the microscope. This carriage slides against adjustable stops, and

is pressed against them with perfect uniformity by means of weights. An adjustable platform below carries the standards and objects to be measured. The ways of both carriage and platform had been ground till they were perfectly uniform and true and the working of the instrument left little to be desired in the way of accuracy. Throughout a long series of measurements the stops would not be displaced by so much as $0{\cdot}1\mu$ if proper care were used in moving the carriage. The microscope was attached so firmly as to avoid all shaking, and was armed with a half-inch objective and an excellent eyepiece micrometer. The objective was made specially for micrometric work and was fitted with a Tolles' opaque illuminator. Measurements were made as follows: The standard bar, and the grating mounted on a polished block of speculum metal, were placed side by side,—or sometimes end to end,—on the platform and very accurately leveled. The stops were set very nearly three centimeters apart, one end of the grating brought under the microscope resting against one of the stops, and the micrometer set on the terminal line. Then the carriage was brought against the other stop and the micrometer again set. The same process was then gone through with on three centimeters of the standard, and then going back to the grating it was compared in the same manner with succeeding triple centimeters till the fifteen centimeter line was reached, thus eliminating the errors of the single centimeters and making the determination rest only on the fifteen centimeter line. The temperature was given by a thermometer placed against the standard bar or the block that carried the grating. In this manner each grating was repeatedly compared with the first 15 centimeters of each bar, at or near 20°, the temperature at which the gratings had been used. The micrometer constant was determined by measuring tenths of millimeters ruled on Prof. Rowland's engine, but in practice the stops were so adjusted that it was almost eliminated. Each division of the micrometer head equalled $0{\cdot}28\mu$ and the probable error of setting was less than half that amount.

All measurements were reduced to 20° C. as in case of the angular determinations. The line along which the linear measures were made was that formed by the terminations of the rulings. It therefore was necessary to know very exactly the angle between this line and the direction of the individual rulings, in other words the angle between the line of motion of the grating and the direction of the diamond stroke in the dividing engine. This was ascertained by means of two test plates each some twelve cm. long ruled in cms. and then superimposed line for line. By measuring the minute distances between each end of a pair of superimposed lines, the length of the lines and the amount by which their ends overlapped at each end of the test plate, the required angle could be deduced with great exactness. It differed so little from 90° however, that the correction produced, barely one part in a million, was entirely negligible.

After all reductions and corrections, the following series of values were obtained for the grating spaces of gratings I and II.

Series.	Grating I.	Standard.
1	0·00250023mm.	S_2'
2	0·00250016	"
3	0·00250013	"
4	0·00250015	"
5	0·00250018	"
6	0·00250021	S_1'
7	0·00250023	"
8	0·00250023	"
9	0·00250023	"

Mean value adopted after weighting and combining the above observations was:

0·002500194mm ±10

The probable error thus appears to be not far from one part in two hundred and fifty thousand. The difference in the results obtained from the two standards seems to be purely accidental as appears from the measurements on grating II.

Series.	Grating II.	Standard.
1	0·00351888mm	S_1'
2	0·00351883	"
3	0·00351885	"
4	0·00351886	"
5	0·00351883	"
6	0·00351893	S_2'
7	0·00351888	"
8	0·00351888	"
9	0·00351888	"

Mean adopted, 0·003518870±10

The probable error appears to be rather less than in the measurements of grating I. As however the angular determinations made with I are the better, so far as probable errors of observation are concerned the result from the two gratings are about equal in value.

Computing now the wave-length corresponding to the given values of φ and δ for each grating, we have finally for the wave-length of D_1 at 20° C. and 760mm pressure:

From Grating I uncorrected, 5896·11 tenth meters.
From Grating II " 5895·95 "

The difference in the above results is by no means large compared with the results obtained from different gratings by other investigators, but it certainly is enormously great compared with the experimental errors alone.

As nearly as can be judged these ought not in either grating to exceed one part in two hundred thousand, while the above discrepancy is about one part in thirty-five thousand.

Its cause must be sought in the individual peculiarities of the gratings, rather than in the method of using them.

All gratings are subject to irregularities of ruling, and the effects of these is various, according to the nature and magnitude of the defects. Linear or periodic errors in ruling, unless very small, will make themselves apparent by changing the focus of the spectra or producing ghosts, respectively; and if such errors are large, render the grating totally unfit for exact measurement. Accidental errors, such as a flaw or break in the ruling, are also serious, but are easily detected and may be approximately corrected, as was done by Ångström in the case of one of his gratings. Any marked and extensive irregularities of spacing will produce bad definition or false lines, and in most cases both. If, then, a grating on microscopical examinations is free from flaws and on the spectrometer gives sharply defined spectra, alike in focus and free from ghosts, it is safe to conclude that it is tolerably free from the errors above mentioned, but unfortunately there is one fault that does not at once become visible, while it introduces a very serious error in the measurements. This is a rather sudden change in the grating space through a portion of the grating, usually at one end. Such an error is usually due to abnormal running of the screw when the dividing engine is first started, and may in this case be avoided by letting the engine run for some time before beginning to rule. Thus Grating I, ruled with this precaution, is nearly free from this error. Sometimes, however, it is the terminal or an intermediate portion of the grating that is thus affected in which case the error may be due to a change of temperature or to a fault in the screw. If an error of this kind is extensive, it will produce the effect of two contiguous gratings of different grating space, injuring the definition and widening or reduplicating the lines. When, however, the abnormal spacing is confined to a few hundred lines it produ[illegible] no visible effect when the whole grating is used, but simply diffuses a small portion of the light and increases or decreases the average grating space. For it is evident that, such a portion of the grating must possess little brilliancy and less resolving power, and the more its spacing differs from that of the rest of the grating, the less chance of visible effect and the greater error introduced. Such a fault is compatible with the sharpest definition, but can be detected by cutting down the aperture of the grating till the spectrum from the abnormal portion is relatively bright and distinct enough to be seen. The effective grating space, producing the spectra on which measurements are made is, of course, that of the normal portion only. Both the gratings used in these experiments were affected by the above error, No. I, very slightly, No. II, somewhat more seriously. Not only the discrepancies between different gratings, but those between different orders of spectra in the same grating are due to this cause. For while in one order, where the effect due to the abnormal portion is imperceptible, the spectrum as measured is produced by the effective grating space alone, in another order there may be produced a slight shading off of the lines so that their apparent centers may correspond approximately to the average grating space. In any case, it is quite clear that a combination of the results from different orders of spectra will not eliminate the error.

The remedy lies either in stopping out the imperfect portion of the grating, or measuring it and introducing a correction. As the work of angular measurements was nearly finished before the study of the gratings was begun owing to a delay in getting apparatus, the latter course was adopted in these experiments. Each grating was examined in detail, and the relation of the grating spaces in the various portions of it carefully determined. From these data a simple graphical method gave the correction to be applied to the wave length. In each grating the fault was confined to a small portion, and as the order of spectrum employed in each was selected on account of its good definition and freedom from anything like haziness or shading of the lines, it seems safe to assume that the abnormal portion produced no visible effect and that consequently the correction above mentioned counteracts the error quite effectually. In grating I the correction was one part in 300,000, and in grating II one part in 60,000. Applying these to the wave lengths we have for grating I,

Wave length	5896·11
Correction	—·02
Corrected w. l.	5896·09

And for grating II,

Wave length	5895·95
Correction	+·10
Corrected w. l.	5896·05

Combining these and giving to Grating I the greater weight

on account of its very small error of ruling, we have finally for the wave length of D_1 at 20° C. and 760mm pressure,

5896·08 or *in vacuo*, 5897·71

It is no easy matter to give any well founded estimate of the probable error of the above result. So far as experimental errors are concerned the result with either grating should be correct to one part in two hundred and fifty thousand, but the error in the gratings introduces a complication by no means easy to estimate. As nearly as the writer can judge, however, it seems probable that the error of the final result does not exceed one part in two hundred thousand. For comparison, the values deduced from the work of Peirce and of Ångström are subjoined.

Micrometer measure by Rowland, from Peirce's preliminary result	5896·22
Thalen's correction of Ångström	5895·89

both being in air at ordinary temperature and 760mm.

As neither result was corrected for errors in the gratings the cause of the discrepancy is obvious.

Two determinations of absolute wave length have been published since this work was undertaken by the writer. One is a very elaborate one by Müller and Kempf, who employed four gratings by Wanschaff and used the method of minimum deviation. Their results were as follows:

Grating	(2151)	(5001)	(8001)	(80014)
Wave length	5896·46	5896·14	5895·97	5896·33

By a correction founded on the unwarrantable assumption that the mean value was correct the above results are brought into apparent agreement. Nothing, however, short of a study in detail of each grating can furnish data for obtaining anything like an accurate result from the above figures. It would seem that (5001), which had the smallest probable error, should show but a trifling error of ruling, while one would expect to find a portion or portions of (2151), in which the grating space is abnormally large. Corresponding errors of ruling should appear in (8001) and (80014). A similar study of the gratings used by Ångström would be of no little interest.

The other determination alluded to is one by M. [illegible]e Lépinay, using a quartz plate and Talbot's bands. Without discussing the method it is sufficient to say that the result obtained depends on the relation of the liter to the decimeter, a ratio not at present exactly determined.

The results detailed in this paper are in a certain sense preliminary. The writer hopes that in the near future, experiments with metallic gratings will enable him to lessen the probable error very materially and therefore defers, for the present, further discussion of the problem.

Through the courtesy of Mr. Peirce the writer has been enabled to test the legitimacy of the above correction and at the same time check his own results. Mr. Peirce kindly forwarded his gratings and standard of length for examination and comparison, and the results were decidedly instructive.

Grating "H," with which a large part of the work was done, showed, as was suspected, a local error, equivalent to a correction of one part in 55000 in the resulting wave length. Tested in the spectrometer, the portion including the error showed a grating space distinctly greater than that of the grating taken as a whole, showing thus both the necessity and the algebraic sign of the correction. The other gratings showed similar errors varying in amount, but the same in sign, the correction requiring in every case a reduction in the wave length. The abnormal portion was invariably at one end or the other of the grating concerned, never in the middle.

The standard of length used by Mr. Peirce—"No. 3" a glass decimeter—was compared with S_1^a and S_2^a and the preliminary results show that the length assigned to it was too great by very nearly 2μ, 1 part in 50000. Now the wave length of D_1 as deduced from grating H was,

	5896·26
Less error of ruling..	—·10
Less error of "No. 3"	—·12
Corrected value.....	5896·04 in air at 30 in. pressure and 70° F;

which shows a tolerably close correspondence with the results obtained by the writer. A more complete discussion of Peirce's results is reserved until the relation between "No. 3" and S_1^a and S_2^a shall be more exactly known. The latter standards would appear to be the more trustworthy, since they are based on various independent determinations, while "No. 3" is based on an indirect comparison with meter "No. 49," a standard concerning the exact length of which there seems to be some little doubt.

ART. XXI.—*On the Relative Wave-length of the Lines of the Solar Spectrum*; by HENRY A. ROWLAND.

FOR several years past I have been engaged in making a photographic map of the solar spectrum to replace the ordinary engraved maps and I have now finished the map from the extreme ultra violet, wave length 3200, down to wave length 5790. In order to place the scale correctly on this map, I have

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