

PROCEEDINGS

OF THE

ROYAL SOCIETY OF LONDON.

From January 6, 1887, to June 16, 1887.

VOL. XLII.

TEXAS TECHNOLOGICAL COLLEGE LIBRARY
LUBBOCK, TEXAS

LONDON:
HARRISON AND SONS, ST. MARTIN'S LANE,
Printers in Ordinary to Her Majesty.
MDCCCLXXXVII.

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was written I have discovered a method of making fibres with a perfect elasticity, and so have avoided the inconvenience of the shifting zero of the glass fibre, and with a torsion, if required, ten million times less than that of spun glass. This will be described at the next meeting of the Physical Society.

II. "Note to a Memoir on the Theory of Mathematical Form ('Phil. Trans.' 1886 (vol. 177), p. 1)." By A. B. KEMPE, M.A., F.R.S. Received February 26, 1887.

An interesting letter of criticism from Professor C. S. Peirce on my recently published Memoir on the Theory of Mathematical Form has led me to reconsider certain paragraphs therein, relating to the definition of what I have termed "aspects," and I am anxious to make the following amendments.

For Section 5 substitute—

5. In like manner some *pairs* of units are distinguished from each other, while others are not. Pairs may in some cases be distinguished even though the units composing them are not. Thus the angular points of a square are undistinguishable from each other, and a pair of such points lying at the extremities of a side are undistinguished from the three other like pairs, but are distinguished from each of the two pairs arrived at by taking the angular points at the extremities of the diagonals, which pairs again are undistinguishable from each other. Further, though two units a and b are undistinguishable from each other, an absence of symmetry may cause ab to be distinguished from ba . Thus, if we put aside differences arising from their positions on the paper, and the use of reference letters (Secs. 41 and 42), the three black spots, a, b, c , shown in fig. 1, are undistinguished from each other; but ab is distinguished from ba , for when we take the spots a, b , in the order ab an arrow proceeds from the first spot to the second, but when we take them in the order ba an arrow proceeds to the first spot from the second.

For Section 7 substitute—

7. Again, there are distinguished and undistinguished triads, tetrads, . . . m -ads, . . . n -ads, . . . ; every m -ad being of course distinguished from every n -ad. Just as we may have ab distinguished from ba , though a is undistinguished from b , so we may have $pqrst . . . uv$ distinguished from $quvst . . . rp$, though the units $p, q, r, s, t, . . . u, v$, are all undistinguished from each other, and further, though their pairs are also undistinguished, as likewise their triads, &c. Here $pqrst . . . uv$ and $quvst . . . rp$ will be termed, as in the case of pairs, different *aspects* of the collection $p, q, r, s, t, . . . u, v$. An aspect will be fully defined and con-

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sidered in Secs. 73ff, and we shall there see that an aspect of a collection of n units, i.e., of an n -ad, is itself an n -ad. It will be convenient to speak of the aspects of the pairs, triads, &c., contained in a collection, as if they were themselves pairs, triads, &c., of the collection.

For Sections 73 to 77 substitute—

73. If with each unit of a collection of n units a, b, c, \dots of any form, we mentally associate one of a given discrete heap (Sec. 37) of n marks, so that each unit is thereby distinguished from each of the others, and from all other units, we obtain what I have, in Secs. 6 and 7, termed an aspect of a, b, c, \dots .

It may assist in arriving at a proper idea of the use and importance of aspects, and may justify the use of the expression "aspect," to point out that when, in the consideration of a collection of units, we give different degrees of prominence in our minds to different units of the collection, we are mentally affixing distinctive marks to those units, and have "a particular aspect" of the collection before our mind's eye. It may be, in some cases, that we give a distinctive degree of prominence to one unit only of the collection, and equal degrees of prominence to the rest: thus, when we consider the relations of a number of things to a particular thing, we affix a distinctive mental mark to the latter thing, and marks undistinguished from each other to the former things. It is convenient, however, for our present purposes to confine the use of the word "aspect" to the case in which a different degree of prominence, or other distinctive mental mark, is affixed to each unit of the collection.

The n marks may be associated with the n units, a, b, c, \dots in n different ways, giving rise to n aspects of a, b, c, \dots ; each aspect being derivable from the others by transpositions of the marks. When we regard a unit a as associated with a particular mark g in a particular aspect of a, b, c, \dots , we deal with a unit A , which is a different unit from a , and may be called an aspect of a . Thus an aspect of the n -ad, a, b, c, \dots , is an n -ad composed of n unit aspects, A, B, C, \dots .

The unit aspects composing one aspect of a, b, c, \dots are each distinct units from those composing another aspect, and thus the n aspects of a, b, c, \dots derived by employing a particular collection of n marks, furnish in all n unit aspects such as A .

Since the marks employed are all distinguished from each other, the n unit aspects A, B, C, \dots composing a particular aspect of a, b, c, \dots are all distinguished from each other.

We may associate the same collection of n marks with any number of collections of n units a, b, c, \dots ; p, q, r, \dots ; &c., and obtain n aspects of each collection.

In future when we compare aspects of a collection, or of a number of collections of the same number of units, unless the contrary is manifestly the case, it will be supposed that the same collection of n marks is employed in each case.

We may choose as our discrete heap of n marks a collection of n "relative positions," or sorts of places, which are distinguished from each other, and from all other relative positions: e.g., we may choose the sorts of places—first, second, third, &c., in a row. In such a case the units a, b, c, \dots may be regarded as occupying the sorts of places which serve as marks.

74. If two collections of units a, b, c, \dots and p, q, r, \dots are undistinguished from each other, they may be regarded as corresponding to each other in one or more ways, in each of which correspondences to each unit, pair, triad, &c., of one collection there corresponds in the other a counterpart unit, pair, triad, &c., undistinguished from the former in any circumstance. In any one of those correspondences two corresponding units may be regarded as occupying corresponding places, or, as we may express it, places of the same sort; and we may if we please regard these sorts of places as distinguished from each other, and from all other sorts of places; i.e., we may regard the correspondence as giving rise to two aspects, A, B, C, \dots and P, Q, R, \dots , the former an aspect of a, b, c, \dots , the latter of p, q, r, \dots . The two aspects A, B, C, \dots and P, Q, R, \dots are clearly undistinguished from each other; and, as the units of each are distinguished from each other, to each unit of one there corresponds one unit, and one only, in the other which is undistinguished from it.

75. Two aspects will not be undistinguished unless they can be regarded as derived in the manner indicated in the last section; and therefore the undistinguishableness of two aspects A, B, C, \dots and P, Q, R, \dots indicates the existence of a definite correspondence between the two undistinguished collections a, b, c, \dots and p, q, r, \dots in which correspondence to each unit, pair, triad, &c., of the one there corresponds in the other a counterpart unit, pair, triad, &c., undistinguished from the former in any circumstance.

Similarly, the undistinguishableness of two aspects of the same collection a, b, c, \dots may be said to indicate a self-correspondence of the collection, i.e., a correspondence in which to each unit, pair, triad, &c., of the collection a, b, c, \dots there corresponds the same, or another unit, pair, triad, &c., of a, b, c, \dots undistinguished from the former in any circumstance.

76. In future, when a correspondence of two collections, or a self-correspondence of a collection, are spoken of, correspondences such as those described in the preceding sections are intended to be referred to, unless the contrary is expressly stated, as in Sections 162 to 169.

77. We may regard an aspect A, B, C, . . . in the aggregate as a single unit V, which may be termed a *unified aspect* of a, b, c, . . .

Section 167 is erroneous, and should be omitted.

I may add the following errata:—

In Sec. 69, line 3, for "undistinguished" read "distinguished."

" " 122, " 1, " "aspects" " "collections."

" page 43, footnote, " "Grassman" " "Grassmann."

" " 56 " "Pierce" " "Peirce."

" Sec. 385, line 8, " "m units" " "r units."

III. "On Ellipsoidal Current Sheets." By HORACE LAMB, M.A., F.R.S., Professor of Pure Mathematics in the Owens College, Victoria University, Manchester. Received March 2, 1887.

(Abstract.)

This paper treats of the induction of electric currents in an ellipsoidal sheet of conducting matter whose conductivity per unit area varies as the perpendicular from the centre on the tangent plane, or (say) in a thin shell of uniform material bounded by similar and coaxial ellipsoids. The method followed is to determine in the first instance the normal types of free currents. In any normal type the currents decay according to the law $e^{-t/\tau}$; the time-constant τ may be conveniently called the "modulus of decay," or the "persistency" of the type.

When the normal types and their persistencies have been found, it is an easy matter to find the currents induced by given varying electromotive forces, assuming these to be resolved by Fourier's theorem, as regards the time, into a series of simple harmonic terms. Supposing then that we have an external magnetic system whose potential varies as e^{ipt} , we can determine a fictitious distribution of current over the shell, which shall produce the same field in the interior. If $\bar{\phi}$ denote the current-function for that part of this distribution which is of any specified normal type, ϕ that of the induced currents of this type, it is shown that

$$\phi = -\frac{ip\tau}{1+ip\tau}\bar{\phi},$$

where τ is the corresponding persistency of free currents. When $p\tau$ is very great this becomes

$$\phi = -\bar{\phi},$$

in accordance with a well-known principle.

This method can be applied to find the currents induced by rota-

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