MIND

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as appendix) are the less to be overlooked, because here M. Picavet does what he can, in other way than by the much-missed index, to bring together the multiplex threads of his whole inquiry. In the last paragraphs of all, there is a striking imagination of the state of mind of an Ideologist transported from the beginning of the century, when he worked so confidently for human enlightenment and progress, to the century's end with its vast increase of scientific knowledge but also increasing sense of the limits set to positive science and its ever-growing burden of social difficulties and perils. The Ideologist, it is allowed, would have to abate much of his practical optimism, and could no longer deal so lightly as he did with philosophical questionings because they had failed of decision. None the less he might truly claim to have done a real stroke of work in his day. He had broken ground in every one of the lines upon which psychology has since advanced,—an effort only partially recognised in the foregoing notice but admirably shown in the book itself. He had also had his own measure of philosophic insight when he proclaimed that all other human search and all human striving should own the sway of a science of " Ideas ".

<div style="text-align:right">G. Croom Robertson.</div>

Vorlesungen über die Algebra der Logik (Exakte Logik). By Dr. Ernst Schröder. Leipzig: B. G. Teubner. Vol. I. Pp. 717.

The appearance of the first volume—a very bulky one—of Dr. Schröder's great work marks an important stage in the progress of Exact Logic. With the exception of the brief former paper of the same writer (*Der Operationskreis des Logikkalkuls*) the subject has hitherto received no presentation in Germany ; and, for the purpose of making it accessible to the reader who approaches it for the first time, this presentation is practically the only thing that yet exists in any language.

Mr. Charles S. Peirce, to whom Symbolic Logic owes its present state of development, wrote his papers with the brevity and abstractness that befit a scientific journal. Dr. Schröder's book will be objected to on the ground that it is unnecessarily diffuse ; but it should be remembered that the subject has had hard work to get itself recognised, and that it is a principle of psychology that a certain degree of voluminousness in a sensation is essential to the producing of a lasting impression. It must be admitted that the book is discursive to the last degree. On the other hand, it is not undesirable that everything that can be said, by way of elucidation and reinforcement, should once be said ; coming books can be written with all the greater conciseness. It goes without saying that Dr. Schröder's book is a work of true German thoroughness, and patience with teasing details ; it will

be impossible hereafter for any one to write upon the subject without having made himself familiar with the views set forth in this volume.

The plan of Dr. Schröder in his book follows closely upon that of Mr. Peirce as set forth in Vol. III. of the *American Journal of Mathematics* ; that is to say, all the formulæ are established by analytical proofs based upon the definitions of sum, of product, and of the negative, and upon the axiom of identity and that of the syllogism. (Later it is found necessary to add another axiom to cover one of the two parts of the distribution law.) The proofs are, for the most part, the same as those given by Prof. Peirce, but frequently alternative proofs are given in addition, and occasionally the method of treatment varies. Dr. Schröder considers it an important difference between his treatment and that of Mr. Peirce that with him (in this first volume) the letters stand for classes (p. 290), while with Mr. Peirce they stand for statements. This is not a strictly correct account of Mr. Peirce's treatment. The great effect which that writer has had if at once simplifying and extending the whole body of logical doctrine (not merely its symbolic exposition) is based upon his *identification* of the proposition with the relation of illation. It is plain that (provided universal propositions are taken as not implying the existence of their terms) there is no difference between

The statement P implies the statement P_1, or, if P then P_1, and

The term t implies the term t_1, or, every t is a t_1,

as far as the part they can play in a logical structure is concerned. The relation between P and P_1 and the relation between t and t_1 are both sufficiently defined by saying that they are transitive relations, in the sense in which the term is used by De Morgan ; that is (if we use a common sign $\leqslant$ to express the common relation), we shall have for a (dual) definition of the relation

$$s \leqslant p$$

(whether s and p stand for terms or for propositions), *whatever* p *is, that* s *shall also be ; or, whatever is* s, *that shall also be* p. Expressed symbolically, this will be—

$$s \leqslant p$$

is-the-same-thing-as

$$(p \leqslant x) \leqslant (s \leqslant x) \qquad\qquad D_1$$

and is-the-same-thing-as

$$(x \leqslant s) \leqslant (x \leqslant p), \qquad\qquad D_1$$

where x stands for anything whatever. This is, as it happens, in strict accordance with Mill's account of the proposition ; he says (*Logic*, eighth edition, p. 135) that it asserts that "*all things which have* a certain attribute have along with it a certain other

attribute," which is exactly what is asserted in D. Either D or D, amounts to a statement of the *dictum de omni* (in one the $s < p$ plays the part of a major premise, in the other of a minor premise);[1] and Mill agrees with De Morgan that to give any real meaning to the *dictum de omni*, we must consider it not as an axiom but as a definition. In speaking of the relation $s < p$ in words, it is necessary to use the language either of the term or of the proposition; but everything that has just been said of subject and predicate must be taken as having also been said in terms of premise and conclusion, or of antecedent and consequent (for it makes no difference for this purpose whether, in 'S-is-followed-by P,' the following is of a logical or of an extralogical nature).

While this definition gives all the marks of "all . . . is," or of 'is-always-followed-by' that are essential to the building up of the logical discipline, it does not (nor is it necessary to) distinguish them from other transitive relations, such as, for instance, is-an-ancestor-of. It has, I believe, not been noticed that the non-symmetrical negative copula, 'none but . . . is,' is also included in the same definition. The proposition " none but the brave deserve the fair," considered as a statement concerning "the brave," has a distinctive copula, which I have proposed to symbolise thus : $b \lessgtr d$. Now the syllogism (easy in real life but without the pale of the ordinary Logic)—

> None but the brave deserve the fair,
> None but those who deserve the fair are happy,
> ∴ None but the brave are happy—

exhibits exactly the same transitiveness as the syllogism in Barbara. Symbolically expressed, it is—

$$b \lessgtr f, \ f \lessgtr h, \ \therefore b \lessgtr h.$$

That is to say, the character of transitiveness is possessed by the negative non-symmetrical copula as well as by the copula "all . . . is".

To return to Dr. Schröder, it is hence not strictly correct to say that in the development of the subject by Mr. Peirce the letters in $x < y$ represent statements. After it has been shown that, for the purposes of Logic, there is no difference between the transitive relation for terms and the transitive relation for propositions, it is assumed by Mr. Peirce that in $x < y$ the letters stand for *either* terms or propositions at pleasure. Dr. Schröder,

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[1] It must be noticed that the dictum as ordinarily stated is a very insufficient description of the syllogism in Barbara, inasmuch as it leaves out the part played by the minor premise altogether. As it stands, it covers only immediate inference from the universal to the particular; to cover syllogism it should read: "Whatever can be affirmed of the whole can be affirmed of *whatever can be shown to be* a part of that whole," *i.e.*, of what the minor has affirmed to be a part of that whole.

in his second volume (the advance sheets of part of which lie before us), develops the transitive relation for propositions, after having done it in the first volume for terms. . There are marks of difference between the two owing to his assumption that every *proposition* can have solely the values 0 and 1 (p. 256); that is, that every proposition is (during the limits of the discussion) either always true or always false. But this is a most unfortunate restriction. Why exclude from an Algebra which is intended to cover all possible instances of (non-relative) reasoning such propositions as ' sometimes when it rains I am pleased and sometimes when it rains I am indifferent'? This restriction is the cause of a distinct error on the part of Dr. Schröder. He considers that

$$x < y + z$$

is of a different content, according as the letters stand for terms or for propositions. It is true that if y or else z is said to be a *logical* consequence of x, then the logical consequence of x is either always y or always z (or both); and it is also true that, on the other hand, 'men are all either honest or else unhappy' is satisfied by some individuals being honest and other individuals being unhappy. But so also any *material* propositional sequence, such as 'If it rains, either I stay in or else I take an umbrella,' is satisfied by some *instances* of its raining being followed by my staying in and all other instances being followed by my taking an umbrella. Dr. Schröder, in fact, seems to pay too little attention to material following. Logical following has its exact parallel in the proposition in the case of the singular subject. ' She is either a queen or a fairy' does not admit of part of her being a queen and part of her being a fairy. There seems, in fact, to be a close relationship between the logical sequence between propositions, and the sequence between terms when the subject is singular. Again, Dr. Schröder, after showing that, for propositi

$$(a < b)' = \bar{a} + b,$$

that is, that

' If some are not wise, some will be unfortunate' is-equivalent-to

'Either all are wise, or else some are unfortunate,' asks, what could be the meaning of this if a and b stood for terms instead of for propositions? The answer is very easy. The last sentence is an abbreviated form—made possible by the accidents of language (see my paper on "Some Characteristics of Symbolic Logic," *Am. Jour. of Psychology*, 1889)—for the complete statement,

'All possible cases are included in cases of all being wise together with cases of some being unfortunate,' or,

' "The possible" implies that all are wise or else that some are unfortunate'. That is, the full expression for the equation written above is—

$$(a \leqslant b) = (\propto \leqslant \bar{a} + b).$$

When a and b are terms, this is—

'All a is b' is-the-same-thing-as 'everything is either non-a or else b,'

a transformation which is as valid and as simple for terms as it is for propositions.

In his treatment of the signification of the negative term,—a subject upon which very many logicians have gone astray,—Dr. Schröder virtually sets forth the correct doctrine (for instance, on p. 337), but not with quite sufficient constancy or clearness. It is true that there is not much difference between the presence of a quality and the absence of a quality, and hence that the signification of a negative term is of very much the same nature as that of a positive term, *so long as the quality which marks its signification is one and indivisible.* It makes no difference whether we divide numbers up into even and not-even or into odd and not-odd. But the case is very different when we come to complex qualities. We may set forth symbolically the two-fold force of a term in the following fashion: Since the aggregate of objects to which it applies is of the nature of a logical sum, and the congeries of qualities which it implies is of the nature of a logical product, the full import of a term, as civilisation, c, will be—

$$c = (C_1 + C_2 + \ldots\ldots\ldots) \gamma_1 \gamma_2 \gamma_3 \ldots\ldots$$

where $C_1, C_2 \ldots\ldots$ stand for all the different instances-of-its application (as the civilisation of the Assyrians, that of the Greeks, and so on), and $\gamma_1, \gamma_2, \gamma_3 \ldots\ldots$ stand for all the elements which are essential to its signification (as, being in the possession of good laws, ensuring the safety of the person and of property, securing a certain amount of happiness to a considerable number of individuals, &c.), and where each one of the instances has all of the essential qualities attached to it. What will then be the negative of the term civilisation? It will be, in accordance with the usual rule for taking the negative—

$$\bar{c} = \bar{C}_1 \bar{C}_2 \bar{C}_3 \ldots\ldots(\bar{\gamma}_1 + \bar{\gamma}_2 + \bar{\gamma}_3 \ldots\ldots);$$

that is, the non-civilisations are, at once, not any one of the civilisations, and at the same time they have the quality of *being deficient in some one, at least,* of the qualities that are essential to a thing's being a civilisation (the qualities, that is, in the absence of any one of which we should refuse to apply the name). The intent of the positive term and of the negative term are therefore extremely different; the one involves a *combination* of quality-elements, the other an *alternation* of absences of quality-elements. It is only in the case of terms of indivisible intent (as hot, cold, blue, heavy, parallel) that the difference between them becomes insignificant. When, therefore,

Lotze "wittily" says, as quoted by Dr. Schröder (p. 99), that it remains a for ever insoluble task to abstract the qualities of the *not-man,* he says what is true but unimportant. *Not-man* is not destitute of intent, as Lotze says it is, but its intent consists in *an alternation of deficiencies of some one, at least, of the elements of the intent of man.* This Dr. Schröder virtually says when he says that the characteristic group of marks of man do not occur in not-man, "or not completely" (p. 337). But he does not distinctly state the doctrine that the signification (intent) of a positive term is of the nature of a logical product, while that of a negative term is of the nature of a logical sum.

In Dr. Schröder's discussion, twenty pages long, of the import of negative judgments, there is a greater amount of error mixed up with a large amount of sound and much-needed doctrine. He shows, with justice, that it is a strange oversight on the part of logicians to say that 'A is not B' is the denial of 'A is B'. It is so only in case A is a singular term. 'All A is B' is denied either by 'not all A is B' or by 'some A is not B,' and not by 'all A is not B'. But it does not follow that the *not* in a negative sentence must always be attached to the predicate term. Schröder would discard from logic altogether such sentences as "geese are-not swans," and substitute for them "geese are not-swans"; that is, he would uniformly interpret the sentence as ordinarily printed "geese are not swans" (where the meaning is "no geese are swans"), in the latter sense and not in the former. While the mistake of ordinary logicians is due, as Dr. Schröder points out, to their forgetting, for the moment, the existence of other-than-singular subjects, he commits himself the corresponding error of neglecting the study of non-simple predicates, and of predicates separated by phrases from the copula *are-not.* Take the first negative sentence I come to on opening a volume of MIND: "Moral intuitions are not, any more than intellectual intuitions, simple and original". Here the effort to think the *not* an attachment to the predicate, simple-and-original, is quite futile. It is true that such sentences as "All A's are not B's" are ambiguous, and hence that a strict rhetoric requires us to avoid them; and that, moreover, when they do occur they are usually to be taken in the sense of the particular negative 'that is, with the *not* attached to the *all*), as in "All that glitters is not gold". Nevertheless they are of frequent occurrence when the *all* is not expressed but understood; and, moreover, a negative copula is needed for the expression of the proposition "no A is B". Far from presenting any difficulties in a symbolic treatment of logic, the copula "no . . . is" or "is-wholly-not" has two very important advantages over the copula "all . . . is" or "is-wholly". In the first place, it is not necessary, in solving problems, to transpose all the terms into the subject,—there is no (logical) difference between subject and predicate. In the second place, the number of theorems which constitute the body of the doctrine is re-

duced by one half,—a single statement with this copula is the representative of a statement together with its dual opposite in terms of the other copula. These are advantages which are possessed by both of the symmetrical copulas, 'no A is B' and 'all but A is B'[1]; and by neither of the unsymmetrical copulas, 'all A is B' and 'none but A is B'. 'All A is B' has, of course, a great superiority in point of naturalness, but the others ought not to be treated as if they were non-existent.

When it comes to the solution of problems, Dr. Schröder discards altogether Mr. Peirce's method, which consists in a consistent carrying out of the properties of the copula $\leqslant$, for the far simpler method of first reducing the second member of the statement to " zero," or " non-existent,"—that is, of transposing all the terms into the first member of the statement. His treatment of this part of his subject could not be improved upon.

A number of interesting points we have left ourselves no room to speak about. Dr. Schröder proves that subtraction and division are inexecutible operations, and that the words are pure nonsense-words in Logic. He also shows that only an historical interest attaches to the labours of Boole in the field of symbolic Logic. A particularly interesting passage is that in which he proves that the second subsumption of the distribution-law, viz.,

$$a\,(b+c) \leqslant ab + ac,$$

cannot be deduced from the other axioms and the definitions, by showing that in the logical calculus of groups all these other axioms and definitions hold but that this subsumption is not true. Into that calculus, however, the idea of the negative does not enter; hence it is only proved that the above subsumption cannot be deduced from the axioms and definitions exclusive of the definition of the negative.

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Spinoza's Erkenntnisslehre in ihrer Beziehung zur modernen Naturwissenschaft und Philosophie. Allgemein verständlich dargestellt von Dr. Martin Berendt und Dr. med. Julius Friedländer. Berlin: Mayer & Müller, 1891. Pp. xix., 315.

In spite of all that has been written about Spinoza, the authors of this work have contrived to say something new. There are important differences in the theory of knowledge as set forth in Spinoza's successive works—the *Short Treatise*, the *De Intellectus Emendatione*, and the *Ethica*—and, even in its final form, it is held to be far from clear by most of those who have expressly examined it. I know of no other discussion which can compare

[1] It is virtually in terms of this copula that Mr. Mitchell has developed his Algebra of Logic.

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