SCIENCE

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MSS. intended for publication and books, etc., intended for review should be sent to the responsible editor, Prof. J. McKeen Cattell, Garrison-on-Hudson, N. Y.

SYLVESTER.

ON Monday, March 15, 1897, in London, where, September 3, 1814, he was born, died the most extraordinary personage for half a century in the mathematical world.

James Joseph Sylvester was second wrangler at Cambridge in 1837. When we recall that Sylvester, Wm. Thomson, Maxwell, Clifford, J. J. Thomson were all second wranglers, we involuntarily wonder if any senior wrangler except Cayley can be ranked with them.

Yet it was characteristic of Sylvester that not to have been first was always bitter to him.

The man who beat him, Wm. N. Griffin, also a Johnian, afterwards a modest clergyman, was tremendously impressed by Sylvester, and honored him in a treatise on optics where he used Sylvester's first published paper, 'Analytical development of Fresnel's optical theory of crystals,' *Philosophical Magazine*, 1837.

Sylvester could not be equally generous, and explicitly rated above Griffin the fourth wrangler George Green, justly celebrated, who died in 1841.

Sylvester's second paper, 'On the motion and rest of fluids,' *Phil. Mag.*, 1838 and 1839, also seemed to point to physics.

In 1838 he succeeded the Rev. Wm. Ritchie as professor of natural philosophy in University College, London.

His unwillingness to submit to the re-

ligious tests then enforced at Cambridge and to sign the 39 articles not only debarred him from his degree and from competing for the Smith's prizes, but, what was far worse, deprived him of the Fellowship morally his due. He keenly felt the injustice.

In his celebrated address at the Johns Hopkins University his denunciation of the narrowness, bigotry and intense selfishness exhibited in these compulsory creed tests, made a wonderful burst of oratory. These opinions were fully shared by De Morgan, his colleague at University College. Copies I possess of the five examination papers set by Sylvester at the June examination, session of 1839–40, show him striving as a physicist, but it was all a false start. Even his first paper shows he was always the Sylvester we knew. To the 'Index of Contents' he appends the characteristic note: "Since writing this index I have made many additions more interesting than any of the propositions here cited, which will appear toward the conclusion." Ever he is borne along helpless but ecstatic in the ungovernable flood of his thought.

A physical experiment never suggests itself to the great mental experimenter. Cayley once asked for his box of drawing instruments. Sylvester answered, "I never had one." Something of this irksomeness of the outside world, the world of matter, may have made him accept, in 1841, the professorship offered him in the University of Virginia.

On his way to America he visited Rowan Hamilton at Dublin in that observatory where the maker of quaternions was as out of place as Sylvester himself would have been. The Virginians so utterly failed to understand Sylvester, his character, his aspirations, his powers, that the Rev. Dr. Dabney, of Virginia, has seriously assured me that Sylvester was actually deficient in intellect, a sort of semi-idiotic calculating boy. For the sake of the contrast, and to show the sort of civilization in which this genius had risked himself, two letters from Sylvester's tutors at Cambridge may here be of interest.

The great Colenso, Bishop of Natal, previously Fellow and Tutor of St. John's College, writes: "Having been informed that my friend and former pupil, Mr. J. J. Sylvester, is a candidate for the office of professor of mathematics, I beg to state my high opinion of his character both as a mathematician and a gentleman.

"On the former point, indeed, his degree of Second Wrangler at the University of Cambridge would be, in itself, a sufficient testimonial. But I beg to add that his powers are of a far higher order than even that degree would certify."

Philip Kelland, himself a Senior Wrangler, and then professor of mathematics in the University of Edinburgh, writes: "I have been requested to express my opinion of the qualifications of Mr. J. J. Sylvester, as a mathematician."

"Mr. Sylvester was one of my private pupils in the University of Cambridge, where he took the degree of Second Wrangler. My opinion of Mr. Sylvester then was that in originality of thought and acuteness of perception he had never been surpassed, and I predicted for him an eminent position among the mathematicians of Europe. My anticipations have been verified. Mr. Sylvester's published papers manifest a depth and originality which entitles them to the high position they occupy in the field of scientific discovery. They prove him to be a man able to grapple with the most difficult mathematical questions and are satisfactory evidence of the extent of his attainments and the vigor of his mental powers."

The five papers produced in this year, 1841, before Sylvester's departure for Virginia, show that now his key-note is really struck. They adumbrate some of his greatest discoveries.

They are: 'On the relation of Sturm's auxiliary functions to the roots of an algebraic equation,' British Assoc. Rep. (pt. 2), 1841; 'Examples of the dialytic method of elimination as applied to ternary systems of equations,' Camb. M. Jour. II., 1841; 'On the amount and distribution of multiplicity in an algebraic equation,' Phil. Mag. XVII., 1841; 'On a new and more general theory of multiple roots,' Phil. Mag. XVIII., 1841; 'On a linear method of eliminating between double, treble and other systems of algebraic equations,' Phil. Mag. XVIII., 1841; 'On the dialytic method of elimination,' Phil. Mag. XXI., Irish Acad. Proc. II.

This was left behind in Ireland, on the way to Virginia. Then suddenly occurs a complete stoppage in this wonderful productivity. Not one paper, not one word, is dated from the University of Virginia. Not until 1844 does the wounded bird begin again feebly to chirp, and indeed it is a whole decade before the song pours forth again with mellow vigor that wins a waiting world.

Disheartening was the whole experience; but the final cause of his sudden abandonment of the University of Virginia I gave in an address entitled 'Original Research and Creative Authorship the Essence of University Teaching,' printed in Science, N. S., Vol. I., pp. 203–7, February 22, 1895.

On the return to England with heavy heart and dampened ardor he takes up for his support the work of an actuary and then begins the study of law. In 1847 we find him at 26 Lincoln's Inn Fields, 'eating his terms.' On November 22, 1850, he is called to the bar and practices conveyancing.

But already in his paper dated August 12, 1850, we meet the significant names Boole, Cayley, and harvest is at hand.

The very words which must now be used to say what had already happened and what was now to happen were not then in existence. They were afterward made by Sylvester and constitute in themselves a tremendous contribution. As he himself says: "Names are, of course, all important to the progress of thought, and the invention of a really good name, of which the want, not previously perceived, is recognized, when supplied, as having ought to be felt, is entitled to rank on a level in importance with the discovery of a new scientific theory."

Elsewhere he says of himself: "Perhaps I may without immodesty lay claim to the appellation of the Mathematical Adam, as I believe that I have given more names (passed into general circulation) to the creatures of the mathematical reason than all the other mathematicians of the age combined."

In one year, 1851, Sylvester created a whole new continent, a new world in the universe of mathematics. Demonstration of its creation is given by the Glossary of new Terms which he gives in the *Philosophical Transactions*, Vol. 143, pp. 543–548.

Says Dr. W. Franz Meyer in his exceedingly valuable Bericht über die Fortschritte der projectiven Invariantentheorie, the best history of the subject (1892):

"Als äusseres Zeichen für den Umfang der vorgeschrittenen Entwickelung mag die ausgedehnte, grösstenteils von *Sylvester* selbst herrührende Terminologie dienen, die sich am Ende seiner grossen Abhandlung über Sturm'sche Functionen (1853) zusammengestellt findet."

Using then this new language, let us briefly say what had happened in the decade when Sylvester's genius was suffering from its Virginia wound. The birth-day of the giant *Theory of Invariants* is April 28, 1841, the date attached by George Boole to a paper in the Cambridge *Mathematical Journal* where he not only proved the invariantive property of discriminants generally, but also gave a simple principle to

form simultaneous invariants of a system of two functions. The paper appeared in November, 1841, and shortly after, in February, 1842, Boole showed that the polars of a Form lead to a broad class of covariants. Here he extended the results of the first article to more than two Forms. Boole's papers led Cayley, nearly three years later (1845), to propose to himself the problem to determine *a priori* what functions of the coefficients of an equation possess this property of invariance, and he discovered its possession by other functions besides discriminants, for example the quadrinvariants of binary quantics, and in particular the invariant S of a quartic.

Boole next discovered the other invariant T of a quartic and the expression of the discriminant in terms of S and T. Cayley next (1846) published a symbolic method of finding invariants. Early in 1851 Boole reproduced, with additions, his paper on Linear Transformations; then at last began Sylvester. He always mourned what he called 'the years he lost fighting the world'; but, after all, it was he who made the Theory of Invariants.

Says Meyer: "sehen wir in dem Cyklus *Sylvester*'scher Publicationen (1851–1854) bereits die Grundzüge einer allgemeinen Theorie erstehen, welche die Elemente von den verschiedenartigsten Zweigen der späteren Disciplin umfasst." "Sylvester beginnt damit, die Ergebnisse seiner Vorgänger unter einem einzigen Gesichtspunkte zuvereinigen."

With deepest foresight Sylvester introduced, together with the original variables, those dual to them, and created the theory of contravariants and intermediate forms. He introduced, with many other processes for producing invariantive forms, the principle of mutual differentiation.

Hilbert attributes the sudden growth of the theory to these processes for producing and handling invariantive creatures. "Die Theorie dieser Gebilde erhob sich, von speciellen Aufgaben ausgehend, rasch zu grosser Allgemeinheit—dank vor Allem dem Umstande, dass es gelang, eine Reihe von besonderen der Invariantentheorie eigenthümlichen Prozessen zu entdecken, deren Anwendung die Aufstellung und Behandlung invarianter Bildungen beträchtlich erleichterte."

"Was die Theorie der algebraischen Invarianten anbetrifft so sind die ersten Begründer derselben, *Cayley* und *Sylvester*, zugleich auch als die Vertreter der naiven Periode anzusehen: an der Aufstellung der einfachsten Invariantenbildungen und an den eleganten Anwendungen auf die Auflösung der Gleichungen der ersten 4 Grade hatten sie die unmittelbare Freude der ersten Entdeckung." It was Sylvester alone who created the theory of canonic forms and proceeded to apply it with astonishing power. What marvelous mass of brand new being he now brought forth!

Moreover he trumpeted abroad the eruption. He called for communications to himself in English, French, Italian, Latin or German, so only the 'Latin character' were used.

From 1851 to 1854 he produces forty-six different memoirs. Then comes a dead silence of a whole year, broken in 1856 by a feeble chirp called 'A Trifle on Projectiles."

What has happened? Some more 'fighting the world.' Sylvester declared himself a candidate for the vacant professorship of geometry in Gresham College, delivered a probationary lecture on the 4th of December, 1854, and was ignominiously 'turned down.' Let us save a couple of sentences from this lecture:

"He who would know what geometry is must venture boldly into its depths and learn to think and feel as a geometer. I believe that it is impossible to do this, to study geometry as it admits of being studied, and I am conscious it can be taught, without finding the reasoning invigorated, the invention quickened, the sentiment of the orderly and beautiful awakened and enhanced, and reverence for truth, the foundation of all integrity of character, converted into a fixed principle of the mental and moral constitution, according to the old and expressive adage '*abeunt studia in mores.*'"

But this silent year concealed still another stunning blow of precisely the same sort, as bears witness the following letter from Lord Brougham to The Lord Panmure:

"Brougham,

Private. 28 Aug. 1855.

My dear P.

My learned excellent friend and brother mathematician Mr. Sylvester is again a candidate for the professorship at Woolwich on the death of Mr. O'Brian who carried it against him last year.

I entreat once more your favorable consideration of this eminent man who has already to thank you for your great kindness.

Yours sincerely

H. Brougham.

On this third trial, backed by such an array of credentials as no man ever presented before, he barely scraped through, was appointed professor of mathematics at the Royal Military Academy, and served at Woolwich exactly 14 years, 10 months and 15 days.

A single sentence of his will best express his greatest achievement there and his manner of exit thence:

"If Her Most Gracious Majesty should ever be moved to recognize the palmary exploit of the writer of this note in the field of English science as having been the one successfully to resolve a question and conquer an algebraical difficulty which had exercised in vain for two centuries past, since the time of Newton, the highest mathematical intellects in Europe (Euler Lagrange, Maclaurin, Waring among the number), by conferring upon him some honorary distinction in commemoration of the deed, he will crave the privilege of being allowed to enter the royal presence, not covered, like De Courcy, but barefooted, with rope around his waist, and a *goose*-quill behind his ear, in token of repentant humility, and as an emblem of convicted simplicity in having once supposed that on such kind of success he could found any additional title to receive fair and just consideration at the hands of Her Majesty's Government when quitting his appointment as public professor at Woolwich under the coercive operation of a non-Parliamentary retrospective and utterly unprecedented War Office enactment." Athenæum Club, January 31, 1871. Of course this means a row of barren years, 1870, 1871, 1872, 1873.

The fortunate accident of a visit paid Sylvester in the autumn of 1873 by Pafnuti Lvovich Chebyshev, of the University of St. Petersburg, reawakened our genius to produce in a single burst of enthusiasm a new branch of science.

On Friday evening, January 23, 1874, Sylvester delivered at the Royal Institution a lecture entitled 'On Recent Discoveries in Mechanical Conversion of Motion," whose ideas, carried on by two of his hearers, H. Hart and A. B. Kempe, have made themselves a permanent place even in the elements of geometry and kinematics. A synopsis of this lecture was published, but so curtailed and twisted into the third person that the life and flavor are quite gone from it. I possess the unique manuscript of this epoch-making lecture as actually delivered. A few sentences will show how characteristic and inimitable was the original form:

"The air of Russia seems no less favorable to mathematical acumen than to a genius for fable and song. Lobacheffsky, the first to mitigate the severity of the Euclidean code and to beat down the bars

of a supposed adamantine necessity, was born (a Russian of Russians), in the government of Nijni Novgorod; Tchebicheff [Chebyshev], the prince and conqueror of prime numbers, able to cope with their refractory character and to confine the stream of their erratic flow, their progression, within algebraic limits, in the adjacent circumscription of Moscow; and our own Cayley was cradled amidst the snows of St. Petersburg." [Sylvester himself contracted Chebyshev's limits for the distribution of primes.] "I think I may fairly affirm that a simple direct solution of the problem of the duplication of the cube by mechanical means was never accomplished down to this day. I will not say but that, by a merciful interpretation of his oracle, Apollo may have put up with the solution which the ancient geometers obtained by means of drawing two parabolic curves; but of this I feel assured that had I been then alive, and could have shown my solution, which I am about to exhibit to you, Apollo would have leaped for joy and danced (like David before the ark), with my triple cell in hand, in place of his lyre, before his own duplicated altar."

That in the very next year Sylvester was taking a more active part than has hitherto been known in the organization of the incipient Johns Hopkins University is seen from the following letter to him in London from the great Joseph Henry:

SMITHSONIAN INSTITUTION,
August 25, 1875.

MY DEAR SIR:

Your letter of the 13th inst. has just been received and in reply I have to say that I have written to President Gilman of the Hopkins University giving my views as to what it ought to be and have stated that if properly managed it may do more for the advance of literature and science in this country than any other institution ever established; it is entirely independent of public favor and may lead instead of following popular opinion.

I have advised that liberal salaries be paid to the occupants of the principal chairs and that to fill them the best men in the world who can be obtained should be secured.

I have mentioned your name prominently as one of the very first mathematicians of the day; what the result will be, however, I can not say.

The Trustees are all citizens of Baltimore and among them I have some personal friends; the President, Mr. Gilman, and one of them, came to Washington a few weeks ago to get from me any suggestions that I might have to offer.

It is to be regretted that in this country the Trustees, who control the management of bequests of this character, think it important to produce a palpable manifestation of of the institution to be established by spending a large amount of the bequest in architectural displays. Against this custom I have protested and have asserted that if the proper men and the necessary implements of instruction are provided, the teaching may be done in log cabins.

It would give me great pleasure to have you again as my guest, and I will do what I can to secure your election. Very truly your friend,

JOSEPH HENRY.

We know the result.

Sylvester was offered the place; demanded a higher salary; won; came.

I was his first pupil, his first class, and he always insisted that it was I who brought him back to the Theory of Invariative Forms. In a letter to me of September 24, 1882, he writes: "Nor can I ever be oblivious, of the advantage which I derived from your well-grounded persistence in inducing me to lecture on the Modern Algebra, which had the effect of bringing my mind back to this subject, from which it had for some time previously been withdrawn, and in which I have been laboring, with a success which has considerably exceeded my anticipations, ever since."

He made this same statement at greater length in his celebrated address at the Johns Hopkins on February 22, 1877: "At this moment I happen to be engaged in a research of fascinating interest to myself, and which, if the day only responds to the promise of its dawn, will meet, I believe, a sympathetic response from the professors of our divine algebraical art wherever scattered through the world.

"There are things called Algebraical Forms; Professor Cayley calls them Quantics. These are not, properly speaking, Geometrical Forms, although capable, to some extent, of being embodied in them, but rather schemes of processes, or of operations for forming, for calling into existence, as it were, algebraic quantities.

"To every such Quantic is associated an infinite variety of other forms that may be regarded as engendered from and floating, like an atmosphere, around it; but infinite in number as are these derived existences, these emanations from the parent form, it is found that they admit of being obtained by composition, by mixture, so to say, of a certain limited number of fundamental forms, standard rays, as they might be termed, in the Algebraic Spectrum of the Quantic to which they belong; and, as it is a leading pursuit of the physicists of the present day to ascertain the fixed lines in the spectrum of every chemical substance, so it is the aim and object of a great school of mathematicians to make out the fundamental derived forms, the Covariants and Invariants, as they are called, of these Quantics.

"This is the kind of investigation in which I have, for the last month or two, been immersed, and which I entertain great hopes of bringing to a successful issue.

"Why do I mention it here? It is to illustrate my opinion as to the invaluable aid of teaching to the teacher, in throwing him back upon his own thoughts and leading him to evolve new results from ideas that would have otherwise remained passive or dormant in his mind.

"But for the persistence of a student of this university in urging upon me his desire to study with me the modern algebra I should never have been led into this investigation; and the new facts and principles which I have discovered in regard to it (important facts, I believe) would, so far as I am concerned, have remained still hidden in the womb of time." In vain I represented to this inquisitive student that he would do better to take up some other subject lying less off the beaten track of study, such as the higher parts of the Calculus or Elliptic Functions, or the theory of Substitutions, or I wot not what besides. He stuck with perfect respectfulness, but with invincible pertinacity, to his point. He would have the New Algebra (Heaven knows where he had heard about it, for it is almost unknown on this continent), that or nothing. I was obliged to yield, and what was the consequence? In trying to throw light upon an obscure explanation in our text-book my brain took fire; I plunged with requickened zeal into a subject which I had for years abandoned, and found food for thoughts which have engaged my attention for a considerable time past, and will probably occupy all my powers of contemplation advantageously for several months to come."

Another specific instance of the same thing he mentions in his paper, 'Proof of the Hitherto Undemonstrated Fundamental Theorem of Invariants,' dated November 13, 1877:

"I am about to demonstrate a theorem which has been waiting proof for the last quarter of a century and upwards. It is the more necessary that this should be done, because the theorem has been supposed to lead to false conclusions, and its correctness has consequently been impugned. Thus in Professor Faà de Bruno's valuable *Théorie des formes binaires*, Turin, 1876, at the foot of page 150 occurs the following passage: "Cela suppose essentiellement que les équations de condition *soient toutes indépendantes entr'elles, ce qui n'est pas toujours le cas*, ainsi qu'il résulte des recherches du Professor Gordan sur les nombres des covariants des formes quintique et sextique."

The reader is cautioned against supposing that the consequence alleged above does

result from Gordan's researches, which are indubitably correct. This supposed consequence must have arisen from a misapprehension, on the part of M. de Bruno, of the nature of Professor Cayley's rectification of the error of reasoning contained in his second memoir on Quantics, which had led to results discordant with Gordan's. Thus error breeds error, unless and until the pernicious brood is stamped out for good and all under the iron heel of rigid demonstration. In the early part of this year Mr. Halsted, a fellow of John's Hopkins University, called my attention to this passage in M. de Bruno's book; and all I could say in reply was that 'the extrinsic evidence in support of the independence of the equations which had been impugned rendered it in my mind as certain as any fact in nature could be, but that to reduce it to an exact demonstration transcended, I thought, the powers of the human understanding.'"

In 1883 Sylvester was made Savilian professor of geometry at Oxford, the first Cambridge man so honored since the appointment of Wallis in 1649.

To greet the new environment, he created a new subject for his researches—Reciprocants, which has inspired, among others, J. Hammond, of Oxford; McMahon, of Woolwich; A. R. Forsyth, of Cambridge; Leudesdorf, Elliott and Halphen.

Sylvester never solved exercise problems such as are proposed in the *Educational Times*, though he made them all his life long down to his latest years. For example, *unsolved* problems by him will be found even in Vol. LXII. and Vol. LXIII. of the *Educational Times* reprints (1895). If at the time of meeting his own problem he met also a neat solution he would communicate them together, but he never solved any. In the meagre notices that have been given of Sylvester the strangest errors abound. Thus C. S. Pierce, in the *Post*, March 16th, speaks of his accepting, 'with much diffidence,' a word whose meaning he never knew; and gives 1862 as the date of his retirement from Woolwich, which is eight years wrong, as this forced retirement was July 31, 1870, after his 55th birthday. Cajori, in his inadequate account (History of Mathematics, p. 326), puts the studying of law before the professorship at University College and the professorship at the University of Virginia, both of which it followed. Effect must follow cause. And strange, that of the few things he ascribes to Sylvester, he should have hit upon something not his, "the discovery of the partial differential equations satisfied by the invariants and covariants of binary quantics." But Sylvester has explicitly said in Section VI. of his 'Calculus of Forms:' "I alluded to the partial differential equations by which every invariant may be defined. M. Aronhold, as I collect from private information, was the first to think of the application of this method to the subject; but it was Mr. Cayley who communicated to me the equations which define the invariants of functions of two variables."

Surely he needs nothing but his very own, this marvellous man who gave so lavishly to every one devoted to mathematics, or, indeed, to the highest advance of human thought in any form.

George Bruce Halsted.
University of Texas.

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*THE GREAT FAULT AND ACCOMPANYING SANDSTONE DIKES OF UTE PASS, COLORADO.**

Three year years ago Whitman Cross first directed the attention of geologists to the fact that dike-like masses of sandstone occur in the granite of the Pike's Peak massif, forming a belt about one mile wide extending north-northwest from the vicinity of Green Mountain Falls, in Ute Pass,

*Abstract of a paper read before the Boston Society of Na al History, January 20, 1897.