SOLUTION OF THE EQUATIONS.

The solution of these equations by the method of least squares gives the following values of the coefficients of the final equations:

By B. P.	C. S. P.—B. P.	By B. P.	C. S. P.—B. P.
$[a^2] = 4.1177$	.0003	$[bc] = -.6027$	—.0035
$[ab] = -.3598$	—.0003	$[bm] = -.7815$	.3427
$[ac] = 1.3060$	.0013	$[c^2] = .7058$	—.0004
$[am] = 7.506$	— .863	$[cm] = 2.3336$	—.3433
$[b^2] = .8826$	—.0002	$[m^2] = 16.507$	—3.236

The solution of these equations gives:

By B. P., $\delta b = .5741\ \delta\pi + 0''.12$.
By C. S. P., $\delta b = .5781\ \delta\pi + 0''.20$.
By B. P., $\delta l = -.2670\ \delta\pi - .5600\ \delta\lambda + 1''.833$.
By C. S. P., $\delta l = -.2669\ \delta\pi - .5600\ \delta\lambda + 1''.631$.

The substitution of these values in the individual equations leaves for the residual coefficients the following values:

Star.	B. P. and C. S. P.		δp	δp
	$D_\pi p$	$D_\pi \lambda$	B. P.	C. S. P.
			"	"
2	—.036	0	—0.22	—0.19
1	.050	0	.12	.24
4	.0	0	—1.14	—1.15
11	.016	0	—.01	—.02
14	—.040	0	1.19	1.04

The computations are contained in the accompanying sheets, marked B. P. $_{1-4}$, C. S. P. $_{1-7}$.

Very respectfully,

BENJAMIN PEIRCE.

A. D. BACHE, LL.D.,
Superintendent United States Coast Survey.

APPENDIX No. 18.

REPORT OF DR. B. A. GOULD, ON THE COMPUTATIONS CONNECTED WITH OBSERVATIONS BY THE TELEGRAPHIC METHOD FOR DIFFERENCE OF LONGITUDE.

CAMBRIDGE, *November*, 1863.

DEAR SIR: The work under my direction during the past year has been of the same character as during the year preceding, consisting in great measure of computations and reductions of the field-work of former years. The determinations of six differences of longitude have been completed, and the former reductions repeated, wherever the new and more accurate data now at our disposal promised any appreciable improvement in the accuracy of the main result, or of any important collateral ones.

The subsidiary results continue to offer new inducements to further research, and but for the smallness of the force at my disposal many of the yet unexplained indications would have been further investigated. But my primary duty was manifestly to complete the definite determinations of longitude, and the greater part of our labor has been directed exclusively to this end.

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Memoranda

CONCERNING

THE ARISTOTELEAN SYLLOGISM.

Distributed at the Lowell Institute, Nov., 1866, by Charles S. Peirce, of Cambridge, Mass.

The QUANTITY of Propositions is the respect in which *Universal* and *Particular* Propositions differ. The QUALITY of Propositions is the respect in which *Affirmative* and *Negative* Propositions differ.

NAMES AND SIGNS FOR PROPOSITIONS.

Universal Affirmative:	**A**:	Any S is P.
Particular Affirmative:	**I**:	Some S is P.
Universal Negative:	**E**:	Any S is not P.
Particular Negative:	**O**:	Some S is not P.

Terms occupying the places of S and P in the above, are called the logical *Subject* and *Predicate*.

RELATIONS OF PROPOSITIONS.

In the following diagram, the different propositions are supposed to have the same logical Subject and Predicate. The lines connecting A with O, and E with I, are meant to indicate that these connected propositions contradict one another. The sign ⟶ has its broad end towards a proposition which implies another, and its point toward the proposition implied.

A syllogism in the first figure argues from a *Rule*, and the subsumption of a *Case*, to the *Result* of that rule in that case.

Rule: Any man is mortal,
Case: Napoleon III. is a man;
Result: Napoleon III. is mortal.

The Rule must be universal; and the Case affirmative. And the subject of the Rule must be the predicate of the Case. The Result has the quality of the rule and the quantity of the case; and has for its subject the subject of the case, and for its predicate the predicate of the rule.

THE THREE FIGURES.

Figure 1.

Assertion of Rule,		A	E	
Assertion of Case;		A	I	
Assertion of Result.	E	A	O	I

Figure 2.

Assertion of Rule,		A	E	
Denial of Result;	O	I	E	A
Denial of Case.	O			E

Figure 3.

Denial of Result,	I	O	A	E
Assertion of Case;		A	I	
Denial of Rule.		O	I	

The letters A, E, I, O, in the above diagram are so arranged that inferences can be made along the straight lines.

It is important to observe that the second and third figures are *apagogical*, that is, infer a thing to be false in order to avoid a false result which would follow from it. That which is thus reduced to an absurdity is a Case in the second figure, and a Rule in the third.

To *contrapose* two terms or propositions is to transpose them, and at the same time substitute for each its contradictory. The second figure is derived from the first by the contraposition of the Case and Result, the third by the contraposition of the Rule and Result. The Rule and Case of the first figure cannot be contraposed, because they already occupy the same logical position, namely, that of a *premiss*; their contraposition in either of the other figures converts these figures into one another.

Let F, S, T, denote syllogisms of the first, second, and third figures, respectively. And let s, t, f, denote the processes of contraposition of the case and result, rule and result, and rule and case, respectively. Then

$$sF=S \quad sS=F$$
$$tF=T \quad tT=F$$
$$fS=T \quad fT=S$$
$$s^2=t^2=f^2=1$$
$$f=st=ts \quad s=ft=tf \quad t=fs=sf$$

The following table exhibits all the moods of Aristotelean syllogism (varieties resulting from variations of the Quantity and Quality of the propositions). Enter at the top, the proposition asserting or denying the rule; enter at the side, the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by Italics belong to the first figure, those by black letter to the second figure, and those by script to the third figure.

	I	**A**	**E**	**O**
E		𝔈	𝔄	
A	𝒥	*A*	*E*	𝒪
I	𝒜	*I*	*O*	ℰ
O		𝔒	ℑ	

Two moods of the third figure, namely, A A I and E A O, are omitted, for two reasons. The first is that they correspond by contraposition to two moods in the first figure, A A I and E A O, never given by logicians, who, therefore, act inconsistently in admitting these. The second reason is, that, like those moods in the first figure, they are virtually enumerated already, if the change of a proposition from universal to particular be not an inference; but if it be, then, again like those moods of the first figure, the argument they embody may be analyzed into a syllogism and an inference from universal to particular.

The celebrated lines of William Shyreswood (?) are here given. The vowels of the first three syllables of each word indicate the three propositions of the syllogisms. He enumerates, along with the moods of the first figure, the Theophrastean moods (two of which we omit for the same reason that we do those two in the third figure).

Barbara: Celarent: Darii: Ferio: Baralipton:
Celantes: Dabitis: Fapesmo: Frisesomorum.
Cesare: Camestres: Festino: Baroco. Darapti:
Felapton: Disamis: Datisi: Bocardo: Ferison.

The diagram upon the opposite page shows the relations in which the second and third figures stand to the first. In order to understand the seven syllogistic formulas there set down, it is necessary to notice that propositions may be divided into four parts: 1st the *Any* or *Some*, 2d the Subject, 3d the *is* or *is not*, and 4th the predicate. When a proposition admits of varieties in either of these parts, they are shown in the diagram by two words or letters, one above the other, as is / is not in the rule of the first figure. Two independent variations may occur in one formula, and the variations of different parts are independent, but in the same part either the upper or lower line must always be read, in any one syllogysm.

For example, the result in the first figure has four forms; Any *or* some S is *or* is not P; but if *Some* has been read in the case, some must also be read in the result. So, in the second figure, where a variation is possible in the quality of either premiss; but the same line of the third part of both propositions must be taken.

Fig. 1.	Fig. 2.	Fig. 3.
Any M is / is not P	Any M is / is not P	Some / Any S is not / is P
Any / Some S is M	Some / Any S is not / is P	Any / Some S is M
Any / Some S is / is not P	Some / Any S is not M	Some M is not / is P
	Any not P / P is not M	Any some S / S is not / is P
	Some / Any S is not P / P	Some M is some S / S
	Some / Any S is not M	Some M is not / is P
	Any P / not P is not some S / S	Any S / some S is M
	Any M is P / not P	Some not P / P is S / some S
	Any M is not some S / S	Some not P / P is M

At the top of the diagram are given the formulas of the first figure, and of the second and third, as derived from that of the first by contraposition of the propositions. Under the second and third figures, respectively, are given forms expressing the same arguments in the first figure. It is necessary to study carefully the manner in which this reduction to the first figure is effected.

It will be perceived that the arrangements of the terms in the three figures, as determined by the rules given on the second page, are as follows: where the first letter of each pair indicates the subject of a proposition of the syllogism and the second its predicate.

	Fig. 1.	*Fig.* 2.	*Fig.* 3.
1st.	*Β Α*	*Ν Μ*	*Σ Π*
2d.	*Γ Β*	*Ξ Μ*	*Σ Ρ*
3d.	*Γ Α*	*Ξ Ν*	*Ρ Π*

It is plain that there are two ways of transposing the arrangements of the terms of the second and third figures without removing a term from the conclusion, so as to give the term the same arrangement as that of the first figure. This is shown in the following table, where the columns headed *s* show the propositions whose terms are to be transposed, while those headed *m* show the propositions to be transposed.*

	Fig. 2.		Fig. 3.	
	s	*m*	*s*	*m*
Short Reduction	1st		2d	
Long Reduction	2d 3d	2d 1st	1st 3d	1st 2d

The effect of these transpositions is here shown.

SECOND FIGURE.

	Short Red.	*Long Red.*
Ν Μ	*Μ Ν*	*Μ Ξ*
Ξ Μ	*Ξ Μ*	*Ν Μ*
Ξ Ν	*Ξ Ν*	*Ν Ξ*

* "Ubicunque ponitur *s* significatur quod propositio . . . debet converti simpliciter . . . et ubicunque ponitur *m* debet fieri transpositio in præmissis." — *Petrus Hisp.*

THIRD FIGURE.

	Short Red.	Long Red.
Σ Π	Σ Π	Σ P
Σ P	P Σ	Π Σ
P Π	P Π	Π P

It must next be shown how these transpositions may be made, in syllogisms themselves.

The short reduction of the second figure is shown in the second syllogism of that column of the large diagram headed Fig. 2. The term not-P is introduced. This we define as that class to which some or any S belongs, when it is not P. Accordingly, for 'some or any S is not P,' we can substitute 'some or any S is not-P,' and this substitution is made in the reduction. But we cannot, on that account, substitute 'any M is not-P' for 'any M is not P.' For 'any M is not P,' is substituted, in the reduction, 'any P is not M;' and for 'any M is P' is substituted 'any not-P is not M.' The only syllogisms by which these substitutions can be justified are these:—

Any M is not P,
Any P is P;
∴ Any P is not M.

Any M is P,
Any not-P is not P;
∴ Any not-P is not M.

Both these are syllogisms in the second figure.

The short reduction of the third figure is shown in the second syllogism of the column headed Fig. 3. The term some-S is introduced. The definition of this term is that it is that part of S which is or is not P when some S is or is not P. Hence, we can and do substitute 'Any some-S is or is not P' for 'Some S is or is not P,' though we could not substitute 'Any some-S is M' for 'Some S is M.' For 'Some S is M' we substitute 'Some M is S;' and for 'Any S is M' we substitute 'Some M is some-S;' and these substitutions are justified by inferences which can be expressed syllogistically, only thus:—

Any S is S,
Some S is M;
∴ Some M is S.

Some S is some-S,
Any S is M;
∴ Some M is some-S.

These are both syllogisms in the third figure.

The long reduction of the second syllogism is shown in the third syllogism of the column headed Fig. 2. Here not-P is defined as that class to which any M belongs which is not P. Hence we can substitute 'Any M is not-P' for 'Any M is not P.' Some-S is defined as in the short reduction of the third figure. Hence, for 'Some S is or is not P,' we can say 'Any Some-S is or is not P.' Then, we use the inferences which are expressed syllogistically, thus:—

Any $\overset{\text{some S}}{\text{S}}$ is not P,
Any P is P;
∴ Any P is not $\overset{\text{some S.}}{\text{S.}}$

Any $\overset{\text{some S}}{\text{S}}$ is P,
Any not P is not P;
∴ Any not P is not $\overset{\text{some S.}}{\text{S.}}$

These are both syllogisms of the second figure. Substituting their conclusions for the second premiss of the second figure and transposing the premisses we obtain the premisses of the reduction. The conclusion of the reduction justifies that of the second figure, by inferences which are expressed syllogistically, as follows:—

Any M is not some-S,
Some S is some-S;
∴ Some S is not M.

Any M is not S,
Any S is S;
∴ Any S is not M.

Both these are syllogisms of the second figure.

The long reduction of the third figure is shown in the third syllogism of the column headed Fig 3. Some S is here defined as that part of S which is M when some S is M. Hence, for 'Some S is M,' we can substitute 'Any Some-S is M.' Not-P is defined as in the short reduction of the second figure. Hence, in place of 'Some or any S is not P,' we can put 'Some or any S is not-

P.' In place of 'Some S is P or not-P' we again substitute 'Some P or not-P is S,' and in place of 'Any S is P or not-P' we substitute 'Some P or not-P is some-S,' in virtue of inferences which are expressed syllogistically thus:—

Any	S	is	S,	Some	S	is	some-S,
Some	S	is	not P; P;	Any	S	is	not P; P;
∴ Some	not P P	is	S.	∴ Some	not P P	is	some S.

These are syllogisms of the third figure.

Then, the premisses being transposed, we have the premisses of the reduction. The conclusion of the reduction justifies that of the third figure by inferences which are expressed syllogistically, thus:—

Any not-P is	P,	Any P is P,	
Some not-P is	M;	Some P is M;	
∴ Some	M is not-P.	∴ Some M is P.	

These are syllogisms of the third figure.

The reduction called *reductio per impossibile*, is nothing more than the repetition or inverse repetition of that contraposition by which the second and third figures have been obtained. It is not *ostensive* (that is, does not yield an argument with essentially the same premisses and conclusion as that of the argument thus to be reduced), but apagogical, that is, shows by the first figure that the contradiction of the conclusion of the second or third leads to the contradiction of one of the premisses. Contradiction arises from a difference in both quantity and quality. But it is to be observed that in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure, a change of the *quantity* alone of the contraposed propositions is sufficient. This shows that the two contrapositions are of essentially different kinds. The reductions *per impossibile* of the second and third figures respectively involve, therefore, these inferences:—

FIG. 2.

The Result follows from the Case;
∴ The negative of the Case follows from the negative of the Result.

FIG. 3.

The Result follows from the Rule;
∴ The Rule changed in Quantity follows from the result changed in Quantity.

These inferences may also be expressed thus:—

FIG. 2.

Whatever (S) is	M	is	P; not P;	
∴ Whatever (S) is	not P P	is not	M.	

FIG. 3.

Any	S some S	is whatever (P or not-P)	M	is;
∴ Some	M	is whatever (P or not P)	some S S	is.

And if we omit the limitations in parentheses, which do not alter the essential nature of the inferences, we have

FIG. 2.

Any	M	is	P; ot P;
∴ Any	not P P	is not	M.

FIG. 3.

Any	S some S	is	M;
∴ Some	M	is	some S. S.

We have seen above that the former of these can only be reduced a syllogism in the second figure, and the latter only to one in the ird figure.

The ostensive reductions of each figure are also apagogical re-uctions of the other. There are also the following:—

Any	not-M	is	not S, / not some S,		Any	some-M	is	some S, / S,
Any	not P / P	is	not-M;		Any	some P / some not P	is	some-M;
Any	not P / P	is	not S. / not some S.		Any	some P / some not P	is	some S. / S.

But all these reductions involve the peculiar inferences we have und in those which have been examined, inasmuch as they are t complications of the latter.

Hence, it appears that no syllogism of the second or third figure n be reduced to the first, without taking for granted an inference hich can only be expressed syllogistically in that figure from hich it has been reduced. These inferences are not strictly syllo-stic, because one of the propositions taken as a premiss in the syl-gistic expression is a logical fact. But the fact that each can ly be expressed in the second or third figure of syllogism, as the se may be, shows that those figures alone involve the respective inciples of those inferences. Hence, it is proved that every fig-e involves the principle of the first figure, but the second and ird figures contain other principles, besides.

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