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Tros Tyriusque mihi nullo discrimine agetur.

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11.— *The Logic of Chance. An Essay on the Foundations and Province of the Theory of Probability, with especial Reference to its Application to Moral and Social Science.* By JOHN VENN, M. A. London and Cambridge. 1866. 16mo. pp. 370.

HERE is a book which should be read by every thinking man. Great changes have taken place of late years in the philosophy of chances. Mr. Venn remarks, with great ingenuity and penetration, that this doctrine has had its realistic, conceptualistic, and nominalistic stages. The logic of the Middle Ages is almost coextensive with demonstrative logic; but our age of science opened with a discussion of probable argument (in the *Novum Organum*), and this part of the subject has given the chief interest to modern studies of logic. What is called the doctrine of chances is, to be sure, but a small part of this field of inquiry :—but it is a part where the varieties in the conceptions of probability have been most evident. When this doctrine was first studied, probability seems to have been regarded as something inhering in the singular events, so that it was possible for Bernouilli to enounce it as a *theorem* (and not merely as an identical proposition), that events happen with frequencies proportional to their probabilities. That was a realistic view. Afterwards it was said that probability does not exist in the singular events, but consists in the degree of credence which ought to be reposed in the occurrence of an event. This is conceptualistic. Finally, probability is regarded as the ratio of the number of events in a certain part of an aggregate of them to the number in the whole aggregate. This is the nominalistic view.

This last is the position of Mr. Venn and of the most advanced writers on the subject. The theory was perhaps first put forth by Mr. Stuart Mill; but his head became involved in clouds, and he relapsed into the conceptualistic opinion. Yet the arguments upon the modern side are overwhelming. The question is by no means one of words; but if we were to inquire into the manner in which the terms *probable, likely,* and so forth, have been used, we should find that they always refer to a determination of a genus of argument. See, for example, Locke on the Understanding, Book IV. ch. 14, § 1. There we find it stated that a thing is probable when it is supported by reasons *such as* lead to a true conclusion. These words *such as* plainly refer to a genus of argument. Now, what constitutes the validity of a genus of argument? The necessity of thinking the conclusion, say the conceptualists. But a madman may be under a necessity of thinking fallaciously, and (as Bacon suggests) all mankind may be mad after one uniform fashion. Hence the nominalist answers the question thus: A genus of argument is valid when from true premises it will yield a true

conclusion, — invariably if demonstrative, generally if probable. The conceptualist says, that probability is the degree of credence which *ought* to be placed in the occurrence of an event. Here is an allusion to an entry on the debtor side of man's ledger. What is this entry? What is the meaning of this *ought*? Since probability is not an affair of morals, the *ought* must refer to an alternative to be avoided. Now the reasoner has nothing to fear but error. Probability will accordingly be the degree of credence which it is necessary to repose in a proposition in order to escape error. Conceptualists have not undertaken to say what is meant by "degree of credence." They would probably pronounce it indefinable and indescribable. Their philosophy deals much with the indefinable and indescribable. But propositions are either absolutely true or absolutely false. There is nothing *in the facts* which corresponds at all to a degree of credence, except that a genus of argument may yield a certain proportion of true conclusions from true premises. Thus, the following form of argument would, in the long run, yield (from true premises) a true conclusion two thirds of the time: —

A is taken at random from among the B's;
⅔ of the B's are C;
∴ A is C.

Truth being, then, the agreement of a representation with its object, and there being nothing *in re* answering to a degree of credence, a modification of a judgment in that respect cannot make it more true, although it may indicate the proportion of *such* judgments which are true *in the long run.* That is, indeed, the precise and only use or significance of these fractions termed probabilities: they give security in the long run. Now, in order that the degree of credence should correspond to any truth in the long run, it must be the representation of a general statistical fact, — a real, objective fact. And then, as it is the fact which is said to be probable, and not the belief, the introduction of "degree of credence" at all into the definition of probability is as superfluous as the introduction of a reflection upon a mental process into any other definition would be, — as though we were to define man as "that which (if the essence of the name is to be apprehended) ought to be conceived as a rational animal."

To say that the conceptualistic and nominalistic theories are both true at once, is mere ignorance, because their numerical results conflict. A conceptualist might hesitate, perhaps, to say that the probability of a proposition of which he knows absolutely nothing is ½, although this would be, in one sense, justifiable for the nominalist, inasmuch as one half of all possible propositions (being contradictions of the other half) are true; but he does not hesitate to assume events

to be equally probable when he does not know anything about their probabilities, and this is for the nominalist an utterly unwarrantable procedure. A probability is a statistical fact, and cannot be assumed arbitrarily. Boole first did away with this absurdity, and thereby brought the mathematical doctrine of probabilities into harmony with the modern logical doctrine of probable inference. But Boole (owing to the *needs* of his calculus) admitted the assumption that simple events whose probabilities are given are independent, — an assumption of the same vicious character. Mr. Venn strikes down this last remnant of conceptualism with a very vigorous hand.

He has, however, fallen into some conceptualistic errors of his own; and these are specially manifest in his "applications to moral and social science." The most important of these is contained in the chapter "on the credibility of extraordinary stories"; but it is defended with so much ingenuity as almost to give it the value of a real contribution to science. It is maintained that the credibility of an extraordinary story depends either entirely upon the veracity of the witness, or, in more extraordinary cases, entirely upon the *a priori* credibility of the story; but that these considerations cannot, under any circumstances, be combined, unless arbitrarily. In order to support this opinion, the author invents an illustration. He supposes that statistics were to have shown that nine out of ten consumptives who go to the island of Madeira live through the first year, and that nine out of ten Englishmen who go to the same island die the first year; what, then, would be the just rate of insurance for the first year of a consumptive Englishman who is about to go to that island? There are no certain data for the least approximation to the proportion of consumptive Englishmen who die in Madeira during the first year. But it is certain that an insurance company which insured only Englishmen in Madeira during the first year, or only consumptives under the same circumstances, would be warranted (a certain moral fact being neglected) in taking the consumptive Englishman at its ordinary rate. Hence, Mr. Venn thinks that an insurance company which insured all sorts of men could with safety and fairness insure the consumptive Englishman either as Englishman or as consumptive.[*] Now, the case of

* This is an error. For supposing every man to be insured for the same amount, which we may take as our unit of value, and adopting the notation,

(c,e) = number of consumptive Englishmen insured,
(c,ē) =   "   consumptives not English   "
(c̄,e) =   "   not consumptive English   "
x = unknown ratio of consumptive English who *do not die* in the first year. The amount paid out yearly by the company would be, in the long run,

$$\tfrac{1}{10}(c,\bar{e}) + \tfrac{9}{10}(\bar{c},e) + x(c,e),$$

and x is unknown. This objection to Venn's theory may, however, be waived.

an extraordinary story is parallel to this: for such a story is, 1st, told by a certain person, who tells a known proportion of true stories,— say nine out of ten; and, 2d, is of a certain sort (as a fish story), of which a known proportion are true,—say one in ten. Then, as much as before, we come out right, in the long run, by considering such a story under either of the two classes to which it belongs. Hence, says Mr. Venn, we must repose such belief in the story as the veracity of the witness alone, or the antecedent probability alone, requires, or else arbitrarily modify one or other of these degrees of credence. In examining this theory, let us first remark, that there are two principal phrases in which the word probability occurs: for, first, we may speak of the probability of an event or proposition, and then we express ourselves incompletely, inasmuch as we refer to the frequency of true conclusions in the genus of arguments by which the event or proposition in question may have been inferred, without indicating what genus of argument that is; and, secondly, we may speak of the probability that any individual of a certain class has a certain character, when we mean the ratio of the number of those of that class that have that character to the total number in the class. Now it is this latter phrase which we use when we speak of the probability that a story of a certain sort, told by a certain man, is true. And since there is nothing in the data to show what this ratio is, the probability in question is unknown. But a "degree of credence" or "credibility," to be logically determined, must, as we have seen, be an expression of probability in the nominalistic sense; and therefore this "degree of credence" (supposing it to exist) is unknown. "We know not what to believe," is the ordinary and logically correct expression in such cases of perplexity.

Credence and expectation cannot be represented by single numbers. Probability is not always known; and then the probability of each degree of probability must enter into the credence. Perhaps this again is not known; then there will be a probability of each degree of probability of each degree of probability; and so on. In the same way, when a risk is run, the expectation is composed of the probabilities of each possible issue, but is not a single number, as the Petersburg problem shows. Suppose the capitalists of the world were to owe me a hundred dollars, and were to offer to pay in either of the following ways: 1st, a coin should be pitched up until it turned up heads (or else a hundred times, if it did not come up heads sooner), and I should be paid two dollars if the head came up the first time, four if the second time, eight if the third time, &c.; or, 2d, a coin should be turned up a hundred times, and I should receive two dollars for every head. Each of these offers would be worth a hundred dollars, *in the*

long run; that is to say, if repeated often enough, I should receive on the average a hundred dollars at each trial. But if the trial were to be made but once, I should infinitely prefer the second alternative, on account of its greater security. Mere certainty is worth a great deal. We wish to know our fate. How much it is worth is a question of political economy. It must go into the market, where its worth is what it will fetch. And since security may be of many kinds (according to the distribution of the probabilities of each sum of money and of each loss, in prospect), the value of the various kinds will fluctuate among one another with the ratio of demand and supply,—the demand varying with the moral and intellectual state of the community,—and thus no single and constant number can represent the value of any kind.

12.—*France and England in North America, a Series of Historical Narratives.* By FRANCIS PARKMAN, Author of "History of the Conspiracy of Pontiac," "Prairie and Rocky-Mountain Life," &c. Part Second. *The Jesuits in North America in the Seventeenth Century.* Boston: Little, Brown, and Company. 1867. pp. lxix., 463.

IT is not always that the word "Series" on a title-page, any more than the "To be continued" at the end of the monthly instalment of a story, may be said to produce cheerful emotions in the breast of the conscientious reader. As boys are taught at boarding-schools, with abiding thoroughness, the perhaps unnecessary lesson to leave nothing on their plates, so there are certain readers, certain reviewers even, who have learned somewhere, or are driven by a fatal tyranny of nature, to make clean work of whatever book they fall foul of, down to the last consoling crumb of *Finis* at the end of the appendix. That solemn word which the printer, with a kind of grim humor, sets up like the headstone of many a book departed from the m... soon as read, and never to return except in nightmare, gives an assurance of present safety that is delicious. The albatross has at last dropped from the neck. But a colophon which has not that definite brevity, which merely indicates the end of a particular volume, leaves readers of the turn we have mentioned in a dreadful condition of doubt and apprehension,— of doubt as to how much may yet remain, of apprehension lest their task may survive them. We know some who have become involved in the endless coils of the "Rebellion Record," and are wellnigh desperate in consequence. They began it full of hope in 1861, and 1867 finds them still climbing that Piranesi staircase whose landing is lost in abysmal space. They cannot read in the paper that there is promise