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How should they treat a consumptive Englishman? Mr. Venn has made an error in answering the question, but the illustration puts in a clear light the advantage of ceasing to speak of probability, and of speaking only of the relative frequency of this event to that.*

Five hundred and eighty-first Meeting.

April 9, 1867. — MONTHLY MEETING.

The PRESIDENT in the chair.

The following paper was presented.

On the Natural Classification of Arguments. By C. S. PERCE.

PART I. § 1. *Essential Parts of an Argument.*

In this paper, the term "argument" will denote a body of premises considered as such. The term "premise" will refer exclusively to something laid down, (whether in any enduring and communicable form of expression, or only in some imagined sign,) and not to anything only *virtually* contained in what is said or thought, and also exclusively to that part of what is laid down which is (or is supposed to be) relevant to the conclusion.

Every inference involves the judgment that, if *such* propositions as the premises are are true, then a proposition related to them, as the conclusion is, must be, or is likely to be, true. The principle implied in this judgment, respecting a genus of argument, is termed the *leading principle* of the argument.

A *valid* argument is one whose leading principle is true.

In order that an argument should determine the necessary or probable truth of its conclusion, both the premises and leading principle must be true.

§ 2. *Relations between the Premises and Leading Principle.*

The leading principle contains, by definition, whatever is considered requisite besides the premises to determine the necessary or probable truth of the conclusion. And as it does not contain in itself the subsumption of anything under it, each premise must, in fact, be equivalent to a subsumption under the leading principle.

* See a notice, Venn's *Logic of Chance*, in the North American Review for July, 1867.

The leading principle can contain nothing irrelevant or superfluous.

No fact, not superfluous, can be omitted from the premises without being thereby added to the leading principle, and nothing can be eliminated from the leading principle except by being expressed in the premises. Matter may thus be transferred from the premises to the leading principle, and *vice versa*.

There is no argument without premises, nor is there any without a leading principle.

It can be shown that there are arguments no part of whose leading principle can be transferred to the premises, and that every argument can be reduced to such an argument by addition to its premises. For, let the premises of any argument be denoted by *P*, the conclusion by *C*, and the leading principle by *L*. Then, if the whole of the leading principle be expressed as a premise, the argument will become

L and *P*

∴ *C*.

But this new argument must also have its leading principle, which may be denoted by *L'*. Now, as *L* and *P* (supposing them to be true) contain all that is requisite to determine the probable or necessary truth of *C*, they contain *L'*. Thus *L'* must be contained in the leading principle, whether expressed in the premise or not. Hence every argument has, as portion of its leading principle, a certain principle which cannot be eliminated from its leading principle. Such a principle may be termed a *logical principle*.

An argument whose leading principle contains nothing which can be eliminated is termed a *complete*, in opposition to an *incomplete*, *rhetorical*, or *enthymematic* argument.*

* Neither of these terms is quite satisfactory. Enthymeme is usually defined as a syllogism with a premise suppressed. This seems to determine the same sphere as the definition I have given; but the doctrine of a suppressed premise is objectionable. The sense of a premise which is said to be suppressed is either conveyed in some way, or it is not. If it is, the premise is not suppressed in any sense which concerns the logician; if it is not, it ceases to be a premise altogether. What I mean by the distinction is this. He who is convinced that Sortes is mortal because he is a man (the latter belief not only being the cause of the former, but also being felt to be so) necessarily says to himself that all *such* arguments are valid. This genus of argument is either clearly or obscurely recognized. In the former case, the judgment amounts to another premise, because the proposition (for example), "All reasoning from humanity to mortality is certain," only says in other words

Since it can never be requisite that a fact stated should also be implied in order to justify a conclusion, every *logical principle* considered as a proposition will be found to be quite empty. Considered as regulating the procedure of inference, it is determinate; but considered as expressing truth, it is nothing. It is on this account that that method of investigating logic which works upon syllogistic forms is preferable to that other, which is too often confounded with it, which undertakes to enunciate logical principles.

§ 4. *Decomposition of Argument.*

Since a statement is not an argument for itself, no fact concluded can be stated in any one premise. Thus it is no argument to say All *A* is *B*; ergo Some *A* is *B*.

If one fact has such a relation to another that, if the former is true, the latter is necessarily or probably true, this relation constitutes a determinate fact; and therefore, since the leading principle of a complete argument involves no matter of fact, every complete argument has at least two premises.

Every conclusion may be regarded as a statement substituted for either of its premises, the substitution being justified by the other premises. Nothing is relevant to the other premises, except what is requisite to justify this substitution. Either, therefore, these other premises will by themselves yield a conclusion which, taken as a premise along with the first premise, justifies the final conclusion; or else some part of them, taken with the first premise, will yield a conclusion

that every man is mortal. But if the judgment amounts merely to this, that the argument in question belongs to some genus all under which are valid, then in one sense it does, and in another it does not, contain a premise. It does in this sense, that by an act of attention such a proposition may be shown to have been virtually involved in it; it does not in this sense, that the person making the judgment did not *actually* understand this premise to be contained in it. This I express by saying that this proposition is contained in the leading principle, but is not *laid down*. This manner of stating the matter frees us at once from all psychological perplexities; and at the same time we lose nothing, since all that we know of thought is but a reflection of what we know of its expression.

These vague arguments are just such as alone are suitable to oratory or popular discourse, and they are appropriate to no other; and this fact justifies the appellation, "rhetorical argument." There is also authority for this use of the term. "Complete" and "incomplete" are adjectives which I have preferred to "perfect" and "imperfect," as being less misleading when applied to argument, although the latter are the best when syllogism is the noun to be limited.

which, taken as a premise along with all the others, will again justify the final conclusion. In either case, it follows that every argument of more than two premises can be resolved into a series of arguments of two premises each. This justifies the distinction of *simple* and *complex* arguments.

§ 5. *Of a General Type of Syllogistic Arguments.*

A valid, complete, simple argument will be designated as a *syllogistic* argument.

Every proposition may, in at least one way, be put into the form,

$$S \text{ is } P;$$

the import of which is, that the objects to which *S* or the *total subject* applies have the characteristics attributed to every object to which *P* or the *total predicate* applies.

Every term has two powers or significations, according as it is subject or predicate. The former, which will here be termed its *breadth*, comprises the objects to which it is applied; while the latter, which will here be termed its *depth*, comprises the characters which are attributed to every one of the objects to which it can be applied. This breadth and depth must not be confounded with logical extension and comprehension, as these terms are usually taken.

Every substitution of one proposition for another must consist in the substitution of term for term. Such substitution can be justified only so far as the first term represents what is represented by the second. Hence the only possible substitutions are—

1st. The substitution for a term fulfilling the function of a subject of another whose breadth is included in that of the former; and

2d. The substitution for a term fulfilling the function of a predicate of another whose depth is included in that of the former.

If, therefore, in either premise a term appears as subject which does not appear in the conclusion as subject, then the other premise must declare that the breadth of that term includes the breadth of the term which replaces it in the conclusion. But this is to declare that every object of the latter term has every character of the former. The eliminated term, therefore, if it does not fulfil the function of predicate in one premise, does so in the other. But if the eliminated term fulfils the function of predicate in one premise, the other premise must declare that its depth includes that of the term which replaces it in the conclusion. Now, this is to declare that every character of

the latter term belongs to every object of the former. Hence, in the other premise, it must fulfil the function of a subject. Hence the general formula of all argument must be

$$M \text{ is } P$$

$$S \text{ is } M$$

$$\therefore S \text{ is } P;$$

which is to be understood in this sense, — that the terms of every syllogistic argument fulfil functions of subject and predicate as here indicated, but not that the argument can be grammatically expressed in this way.

PART II. § 1. *Of Apagogical Forms.*

If *C* is true when *P* is, then *P* is false when *C* is. Hence it is always possible to substitute for any premise the denial of the conclusion, provided the denial of that premise be at the same time substituted for the conclusion.* Hence, corresponding to every syllogistic argument in the general form,

$$S \text{ is } M; M \text{ is } P;$$

$$S \text{ is } P.$$

There are two others:—

$$\text{It is false that } S \text{ is } P; M \text{ is } P; \quad S \text{ is } M; \text{ it is false that } S \text{ is } P;$$

$$\text{It is false that } S \text{ is } M; \quad \text{It is false that } M \text{ is } P.$$

§ 2. *Of Contradiction.*

The apagogical forms make it necessary to consider in what way propositions deny one another.

If a proposition be put into the general form,

$$S \text{ is } P,$$

its contradictory has, 1st, as its subject, instead of *S*, "the *S* now

* This operation will be termed a *contraposition* of the premise and conclusion.

meant" * or "some S "; and has, 2d, as its predicate, instead of P , that which differs from P or "not P ."

From these relations of contradictories, from the necessities of the logic of apagogically related arguments, therefore, arises the need of the two divisions of propositions into affirmative and negative on the one hand, and into universal and particular on the other. The contradictory of a universal proposition is particular, and the contradictory of an affirmative proposition is negative. Contradiction is a reciprocal relation, and therefore the contradictory of a particular proposition is universal, and that of a negative proposition is affirmative. The contradiction of particular and negative propositions could not be brought under the general formula, were the distinctions of affirmative and negative absolute and not merely relative; but, in fact, not-not- P is the same as P . And, if it is said that "what is now meant of the part of S meant at another time, is P ," since the part of S meant at another time is left to be determined in whatever way the proposition made at another time may determine it, this can only be true if All S is P . Therefore, if one man says "some S is not P ," and another replies, "some of that same S is P ," this second person, since he allows the first man's some S , which has not been defined, to remain undefined, in effect says that All S is P .

Whether contradictories differ in other respects than these well-known ones is an open question.

§ 3. Of Barbara.

Since some S means "the part now meant of S ," a particular proposition is equivalent to a universal proposition with another subject; and in the same way a negative proposition is equivalent to an affirmative proposition with another predicate.

The form, S is P ,

therefore, as well as representing propositions in general, particularly represents Universal Affirmative propositions; and thus the general form of syllogism

M is P ; S is M ;

S is P ,

represents specially the syllogisms of the mood *Barbara*.

* What S is meant being generally undetermined.

§ 4. Of the First Figure.

Since, in the general form, S may be any subject and P any predicate, it is possible to modify Barbara by making the major premise and conclusion negative, or by making the minor premise and conclusion particular, or in both these ways at once. Thus we obtain all the modes of the first figure.

It is also possible to have such arguments as these:—

Some M is P ,

S has all the common characters of that part of M (whatever that part may be, and therefore of each and every M),

$\therefore S$ is P ,

and

All not- M is P ,

S is not M ,

$\therefore S$ is P ;

but as the theory of apagogical argument has not obliged us to take account of these peculiar modifications of subject and predicate, these arguments must be considered as belonging to Barbara. In this sense the major premise must always be universal, and the minor affirmative.

Three propositions which are related to one another as though major premise, minor premise, and conclusion of a syllogism of the first figure will be termed respectively *Rule*, *Case*, and *Result*.

§ 5. Second and Third Figures.

Let the first figure be written thus:—

Fig. 1.

Any M is P

Any S is M

Any S is P

Then its two apagogical modifications are the second and third figures.

Fig. 2.

Any **M** is **P**
 Some **S** is **P**
 Some **S** is not **M**

Fig. 3.

Some **S** is not **P**
 Any **S** is **M**
 Some **M** is not **P**

It is customary to enumerate six moods of the third figure instead of four, and the moods Darapti and Felapton appear to be omitted. But a particular proposition is asserted (actually and not merely virtually) by the universal proposition which does not otherwise differ from it; and therefore Darapti is included both under Disamis and Datisi, and Felapton both under Bocardo and Ferison. (De Morgan.)

The second figure, from the assertion of the rule and the denial of the result, infers the denial of the case; the third figure, from the denial of the result and assertion of the case, infers the denial of the rule. Hence we write the moods as follows, by allowing inferences only on the straight lines:—

Fig. 1.

Assertion of Rule,
 Assertion of Case;
 Assertion of Result.

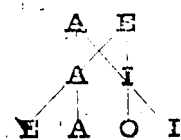


Fig. 2.

Assertion of Rule,
 Denial of Result;
 Denial of Case.

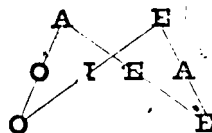
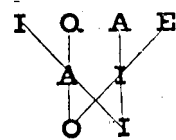


Fig. 3.

Denial of Result,
 Assertion of Case;
 Denial of Rule.



The symmetry of the system of moods of the three figures is also exhibited in the following table.

Enter at the top the proposition asserting or denying the rule; enter at the side the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by italics belong to the first figure, those by black-letter to the second figure, and those by script to the third figure.

	I	A	E	O
E	<i>E</i>	E	A	
A	<i>A</i>	A	E	<i>O</i>
I	<i>A</i>	I	O	<i>E</i>
O		O	I	

If, as the denial of the result in the second and third figures, we put the form "Any *N* is *N*," we have —

Fig. 2.

No *M* is *N*
 Any *N* is *N*
 ∴ No *N* is *M*.

Fig. 3.

Any *N* is *N*
 Some *N* is *M*
 ∴ Some *M* is *N*.

These are the formulæ of the two simple conversions. Neither can be expressed syllogistically except in the figures in which they are here put (or in what is called the fourth figure, which we shall consider hereafter). If, for the denial of the result in the second figure, we put "No not-*N* is *N*" (where "not-*N*" has not as yet been defined) we obtain

All *M* is *N*,
 No not-*N* is *N*;
 ∴ No not-*N* is *M*.

In the same way, if we put "Some *N* is some-*N*" (where some-*N* has not been defined) for the denial of the result in the third figure, we have

Some *N* is some-*N*
 All *N* is *M*
 ∴ Some *M* is some-*N*.

These are the two ways of contraposing the Universal Affirmative.

There are two ostensive reductions of each mood of the second and third figures. I shall distinguish them as the short reduction and the long reduction. The short reduction is effected by converting or contraposing that premise which is not the denial of the result. The long reduction is effected by transposing the premises, contraposing or converting the denial of the result, and contraposing or converting the conclusion. The alteration thus produced in the order of the terms is shown in the following figure:—

	Short Reduction.	Long Reduction.
$N \circ M$	$M \ N$	$M \ E$
$E \ M$	$E \ M$	$N \ M$
$E \ N$	$E \ N$	$N \ E$
$\Sigma \ \Pi$	$\Sigma \ \Pi$	$\Sigma \ P$
$\Sigma \ P$	$P \ \Sigma$	$\Pi \ \Sigma$
$P \ \Pi$	$P \ \Pi$	$\Pi \ P$

The names bestowed by Shyreswood, or Petrus Hispanus, upon the moods indicate the possibility of the short reduction in the case of Cesare and Festino of the second figure, and of Datisi and Ferison of the third figure; also the possibility of the long reduction of Camestres of the second figure and of Disamis of the third.

The short reduction of Camestres and Baroco is effected by introducing the term not- P , and defining it as that which S is when it is not P . Hence for the second premise (Any or some S is not P) we substitute "Any or some S is not- P "; and as the first premise, Any M is P , gives by contraposition Any not- P is not M , the moods

Any M is P ,
Any or some S is not P ;
 \therefore Any or some S is not M ,

are reduced to

No not- P is M ,
Any or some S is not- P ;
 \therefore Any or some S is not M .

The short reduction of Disamis and Bocardo is effected by introducing the term some- S , defining it as that part of S which is or is not P when some S is or is not P . We can therefore substitute for the first premise, Some S is or is not P , All some- S is or is not P ; while, for the second premise, All S is M , can be contraposed into "Some M is some- S "; and thus the forms

Some S is (or is not) P ,
Any S is M ;
 \therefore Some M is (or is not) P ,

are reduced to the following:—

Any some- S is (or is not) P ,
Some M is some- S ;
 \therefore Some M is (or is not) P .

To reduce Cesare, Festino, and Baroco in the long way, it is necessary to introduce the terms not- P and some- S . Not- P is defined as that class to which any M belongs which is not P . Hence for the first premise of Cesare and Festino we can substitute "Any M is not- P ." Some- S is defined as that class of S which is (or is not) P , when some S is (or is not) P . Hence for the second premises of Festino and Baroco we can first substitute "Any some- S is (or is not) P "; and then, by contraposition or conversion, we obtain "Any P (or not- P) is not some- S ." Then, by the transposition of the premises, we obtain from Cesare, which is

No M is P Any not- P is not S
Any S is P Any M is not- P
(\therefore No S is M). \therefore Any M is not- S .

And from the conclusion of this reduced form we obtain the conclusion of Cesare by simple conversion. So Festino and its long reduction are

Any M is not P , Any not- P is not some- S ,
 Some S is P ; Any M is not- P ;
 (\therefore Some S is not M) \therefore Any M is not some- S ;

and the conclusion of Festino is obtained from that of the reduced form by a substitution which may be made syllogistically thus:—

Any M is not some- S ,
 Some S is some- S ;
 \therefore Some S is not M .

Baroco and its long reduction are

Any M is P , Any P is not some- S ,
 Some S is not P ; Any M is P ;
 (\therefore Some S is not M) \therefore Any M is not some- S ;

and the conclusion of Baroco is obtained from the conclusion of the reduction in the same way as that of Festino.

In order to reduce Datisi, Bocardo, and Ferison in the long way, we must define Some- S as that S which is M when some S is M , and Not- P as that which some (or any) S is when it is not P . Hence for "Some S is M " we can substitute "Any some- S is M "; and for "Some (or any) S is not P ," "Some (or any) S is not- P ." "Some S is not- P " may be converted simply; and "Any S is not- P " may be contraposed so as to become "Some not- P is some- S ." Then Datisi and its long reduction are

Any S is P , Any some- S is M ,
 Some S is M ; Some P is some- S ;
 (\therefore Some M is P) \therefore Some P is M .

And from the conclusion of the reduction, the conclusion of Datisi is

obtained by simple conversion. Ferison and its long reduction are

Any S is not P , Any some- S is M ,
 Some S is M ; Some not- P is some- S ;
 (\therefore Some M is not P) \therefore Some not- P is M .

And from the conclusion of the reduction, the conclusion of Ferison may be obtained by a substitution whose possibility is expressed syllogistically thus:—

Any not- P is not P ,
 Some not- P is M ;
 \therefore Some M is not P .

Bocardo and its long reduction are

Some S is not P , Any S is M ,
 Any S is M ; Some not- P is S ;
 (\therefore Some M is not P) \therefore Some not- P is M .

And the conclusion of Bocardo is obtained from that of its reduction in the same way as the conclusion of Ferison.

The ostensive reduction of the indirect or apagogical figures may be considered as the exhibition of them under the general form of syllogism,

S is M ; M is P ;
 $\therefore S$ is P .

But in this sense, it is not truly a reduction if the substitutions made in the process are inferences. But although the possibility of the conversions and contrapositions can be expressed syllogistically, yet this can be done only by taking as one of the premises,

"All N is N ,"
 "Any not- N is not N ,"

or "Some N is some- N ."

Now, these are properly not premises, for they express no facts; they are merely forms of words without meaning. Hence, as no complete argument has less than two premises, the conversions and contrapositions are not inferences. The only other substitutions which have been made have been of not-*P* and some-*S* for their definitions. These also can be put into syllogistic form; but a mere modification of language is not an inference. Hence no inferences have been employed in reducing the arguments of the second and third figures to such forms that they are readily perceived to come under the general form of syllogism.

There is, however, an intention in which these substitutions are inferential. For, although the passage from holding for true a fact expressed in the form "No *A* is *B*," to holding its converse, is not an inference, because, these facts being identical, the relation between them is not a fact; yet the passage from one of these forms taken merely as having some meaning, but not this or that meaning, to another, since these forms are not identical and their logical relation is a fact, is an inference. This distinction may be expressed by saying that they are not inferences, but substitutions having the form of inferences.

Thus the reduction of the second and third figures, considered as mere forms, is inferential; but when we consider only what is meant by any particular argument in an indirect figure, the reduction is a mere change of wording.

The substitutions made use of in the ostensive reductions are shown in the following table. Where

e, denotes simple conversion of *E*;

i, denotes simple conversion of *I*;

a₂, contraposition of *A* into *E*;

a₃, contraposition *A* into *I*;

o₂, the substitution of "Some *S* is not *M*" for "Any *M* is not some-*S*";

o₃, the substitution of "Some *M* is not *P*" for "Some not-*P* is *M*";

e'', introduction of not-*P* by definition;

i'', introduction of some-*S* by definition.

Reduction of Second Figure.

Name of Mood.	Short Reduction.	Long Reduction.
<i>Cesare</i>	<i>e</i>	<i>e'' a₂ e</i>
<i>Camestres</i>	<i>a₂ e''</i>	<i>e e</i>
<i>Festino</i>	<i>e</i>	<i>e'' i'' a₂ o₂</i>
<i>Baroco</i>	<i>a₂ e''</i>	<i>i'' e o₂</i>

Reduction of Third Figure.

Name of Mood.	Short Reduction.	Long Reduction.
<i>Disamis</i>	<i>a₃ i''</i>	<i>i i'</i>
<i>Datisi</i>	<i>i</i>	<i>i'' a₃ i</i>
<i>Bocardo</i>	<i>a₃ i''</i>	<i>e'' i o₃</i>
<i>Ferison</i>	<i>i</i>	<i>i'' e'' a₃ o₃</i>

With the exception of the substitutions *i''* and *e''*, which will be considered hereafter, all those which are used in the reduction of the moods of either oblique figure have the form of inferences in the same figure.

The so-called *reductio per impossibile* is the repetition or inversion of that contraposition of propositions by which the indirect figures have been obtained. Now, contradiction arises from a difference both in quantity and quality; but it is to be observed that, in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure a change of the *quantity* alone, of the contraposed propositions, is sufficient. This shows that the two

contrapositions are of essentially different kinds, and that the reductions *per impossibile* of the second and third figures respectively involve the following formal inferences.*

FIGURE 2.

The Result follows from the Case;

∴ The Negative of the Case follows from the Negative of the Result.

FIGURE 3.

The Result follows from the Rule;

∴ The Rule changed in Quantity follows from the Result changed in Quantity.

But these inferences may also be expressed as follows:—

FIGURE 2.

Whatever (*S*) is \checkmark is $\overset{P}{\text{not } P}$;

∴ Whatever (*S*) is $\overset{\text{not } P}{P}$ is not *M*.

FIGURE 3.

Any $\overset{S}{\text{some } S}$ is whatever (*P* or not-*P*) *M* is;

∴ Some *M* is whatever (*P* or not-*P*) $\overset{\text{some } S}{S}$ is.

Now, the limitations in parentheses do not affect the essential nature of the inferences; and omitting them we have,

FIGURE 2.

Any *M* is $\overset{P}{\text{not } P}$;

∴ Any $\overset{\text{not } P}{P}$ is not *M*.

FIGURE 3.

Any $\overset{S}{\text{some } S}$ is *M*;

∴ Some *M* is $\overset{\text{some } S}{S}$.

* A formal inference is a substitution having the form of an inference.

We have already seen that the former of these is of the form of the second figure, and the latter of the form of the third figure of syllogism.

Hence it appears that no syllogism of an indirect figure can be reduced to the first figure without a substitution which has the form of the very figure from which the syllogism is reduced. In other words, the indirect syllogisms are of an essentially different form from that of the first figure, although in a more general sense they come under that form.

§ 6. *The Theophrastean Moods.*

It is now necessary to consider the five moods of Theophrastus, viz. *Baralippton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesomorum*. *Baralippton* is included in *Dabitis*, and *Fapesmo* in *Frisesomorum*, in the same way in which *Darapti* is included in *Disamis* and *Datisi*, and *Felapton* in *Bocardo* and *Ferison*. The Theophrastean moods are thus reduced to three, viz.:—

No *X* is *Y*, No *X* is *Y*, Some *Y* is *Z*,
All *Z* is *X*; Some *Y* is *Z*; All *Z* is *X*,
∴ Any *Y* is not *Z*. ∴ Some *Z* is not *X*. ∴ Some *X* is *Y*.

Suppose we have, 1st, a Rule; 2d, a Case under that rule, which is itself a Rule; and, 3d, a Case under this second rule, which conflicts with the first rule. Then it would be easy to prove that these three propositions must be of the form,

1. No *X* is *Y*.
2. All *Z* is *X*.
3. Some *Y* is *Z*.

These three propositions cannot all be true at once; if, then, any two are asserted, the third must be denied, which is what is done in the three Theophrastean moods.

These moods are resolved into one another by the contraposition of propositions, and therefore should be considered as belonging to different figures.

They can be ostensibly reduced to the first Aristotelian figure in two ways; thus,

	Short Reduction.	Long Reduction.
$B \ A$	$B \ A$	$B \ I$
$I \ B$	$I \ B$	$A \ B$
$A \ I$	$I \ A$	$A \ I$

The verses of Stryeswood show how Celantes and Dabitis are to be reduced in the short way, and Frisesomorum in the long way. Celantes and its long reduction are as follows:—

Any X is not Y , Any not- X is not Z ,
 Any Z is X ; Any Y is not- X ;
 \therefore Any Y is not Z . \therefore Any Y is not Z .

"Any X is not I ," becomes, by conversion, "Any Y is not X ." The term "not- X " is then introduced, being defined as that which Y is when it is not X . Then " Z is X " becomes "Any not- X is not Z "; and, the premises being transposed, the reduction is effected.

Dabitis and its long reduction are as follows:—

Any Z is X , Any some- Z is Y ,
 Some Y is Z ; Some X is some- Z ;
 \therefore Some X is Y . \therefore Some X is Y .

"Some Y is Z " becomes, by conversion, "Some Z is Y ." Then the term "some- Z " is introduced, being defined as that Z which is Y if "some Z is Y ." Then "Any Z is X " becomes "Some X is some- Z ," and, the premises being transposed, the reduction is effected.

Frisesomorum is,

Some Y is Z ,
 Any X is not Y ;
 \therefore Some Z is not X .

Let some- Y be that Y which is Z when some Y is Z ; and then we have,

Some Y is some- Y ,
 Any X is not Y ;
 \therefore Some some- Y is not X .

Then let not- X be that which any Y is when some Y is not X , and we have,

Some some- Y is not- X ,

which yields by conversion,

Some not- X is some- Y ;

and we thus obtain the reduction,

Any some- Y is Z ,
 Some not- X is some- Y ;
 \therefore Some not- X is Z .

From the conclusion of this reduction, the conclusion of Frisesomorum is justified as follows:—

Some not- X is Z ,
 Any X is not not- X ;
 \therefore Some Z is not X .

Another mode of effecting the short reduction of Frisesomorum is this: Let not- Y be that which any X is when no X is Y , and we have

Some Y is Z ,
 Any not- Y is not Y ;
 \therefore Some Z is not not- Y .

Let some- Z be that Z which is not not- Y when some Z is not- Y , and we have,

Any some- Z is not not- Y ,
 and by conversion
 Any not- Y is not some- Z .

Thus we obtain as the reduced form,

Any not- Y is not some- Z ,

Any X is not- Y ;

\therefore Any X is some- Z .

From the conclusion of this reduction, we get that of Frisesomorum thus:—

Some some- Z is Z ,

Any X is not some- Z ;

\therefore Some Z is not X .

In either reduction of Celantes, if we neglect the substitution of terms for their definitions, the substitutions are all of the second syllogistic figure. This of itself shows that Celantes belongs to that figure, and this is confirmed by the fact that it concludes the denial of a Case. In the same way, the reductions of Dabitis involve only substitutions in the third figure, and it concludes the denial of a Rule. Frisesomorum concludes a proposition which is at once the denial of a rule and the denial of a case: its long reduction involves one conversion in the second figure and another in the third, and its short reductions involve conversions in Frisesomorum itself. It therefore belongs to a figure which unites the characters of the second and third, and which may be termed the second-third figure in Theophrastean syllogism.

There are, then, two kinds of syllogism, — the Aristotelian and Theophrastean. In the Aristotelian occur the 1st, 2d, and 3d figures, with four moods of each. In the Theophrastean occur the 2d, 3d, and 2d-3d figures, with one mood of each. The first figure is the fundamental or typical one, and Barbara is the typical mood. There is a strong analogy between the figures of syllogism and the four forms of proposition. A is the fundamental form of proposition, just as the first figure is the fundamental form of syllogism. The second and third figures are derived from the first by the contraposition of propositions, and E and I are derived from A by the contraposition of terms; thus:—

Any S is P .

Any not- P is not S . Some P is some- S .

O combines the modifications of E and I , just as the 2d-3d figure combines the 2d and 3d. In the second-third figure, only O can be concluded, in the third only I and O , in the second only E and O , in the first either A E I O . Thus A is the first figure of proposition, E the second, I the third, O the second-third.*

§ 7. Mathematical Syllogisms.

A kind of argument very common in mathematics may be exemplified as follows:—

Every part is less than that of which it is a part,

Boston is a part of the Universe;

\therefore Boston is less than the Universe.

This may be reduced to syllogistic form thus:—

Any relation of part to whole is a relation of less to greater,

The relation of Boston to the Universe is a relation of part to whole;

\therefore The relation of Boston to the Universe is a relation of less to greater.

If logic is to take account of the peculiarities of such syllogisms, it would be necessary to consider some propositions as having three terms, subject, predicate, and object; and such propositions would be divided into *active* and *passive*. The varieties in them would be endless.

PART III. § 1. Induction and Hypothesis.

In the syllogism,

Any M is P ,

$\Sigma' S$ is M ;

$\therefore \Sigma' S$ is P ;

where $\Sigma' S$ denotes the sum of all the classes which come under M , if the second premise and conclusion are known to be true, the first

* Hypotheticals have not been considered above, the well-known opinion having been adopted that, "If A , then B ," means the same as "Every state of things in which A is true is a state of things in which B is (or will be) true."

premise is, by enumeration, true. Whence we have, as a valid demonstrative form of inference,

$$\Sigma' S' \text{ is } P,$$

$$\Sigma' S' \text{ is } M;$$

$$\therefore M \text{ is } P.$$

This is called perfect induction. It would be better to call it formal induction.

In a similar way, from the syllogism,

$$\text{Any } M \text{ is } \Pi' P',$$

$$\text{Any } S \text{ is } M;$$

$$\therefore \text{Any } S \text{ is } \Pi' P';$$

where $\Pi' P'$ denotes the conjunction of all the characters of M , if the conclusion and first premise are true, the second premise is true by definition; so that we have the demonstrative form of argument,

$$\text{Any } M \text{ is } \Pi' P',$$

$$\text{Any } S \text{ is } \Pi' P';$$

$$\therefore \text{Any } S \text{ is } M.$$

This is reasoning from definition, or, as it may be termed, formal hypothesis.

One half of all possible propositions are true, because every proposition has its contradictory. Moreover, for every true particular proposition there is a true universal proposition, and for every true negative proposition there is a true affirmative proposition. This follows from the fact that the universal affirmative is the type of all propositions. Hence of all possible propositions in either of the forms,

$$\Sigma' S' \text{ is } M, \text{ and } M \text{ is } \Pi' P',$$

one half are true. In an untrue proposition of either of these forms, some finite ratio of the S 's or P 's are not true subjects or predicates. Hence, of all propositions of either of these forms which are partly true, some finite ratio more than one half are wholly true. Hence, if in the above formulæ for formal induction or hypothesis, we substitute

S' for $\Sigma' S'$ and P' for $\Pi' P'$ we obtain formulæ of probable inference. This reasoning gives no *determinate* probability to these modes of inference, but it is necessary to consider that, however weak synthetic inference might have been at first, yet if it had the least positive tendency to produce truth, it would continually become stronger, owing to the establishment of more and more secure premises.

The rules for valid induction and hypothesis deducible from this theory are as follows:—

1. The explaining syllogism, that is to say, the deductive syllogism one of whose premises is inductively or hypothetically inferred from the other and from its conclusion, must be valid.

2. The conclusion is not to be held as absolutely true, but only until it can be shown that, in the case of induction, S' was taken from some narrower class than M , or, in the case of hypothesis, that P' was taken from some higher class than M .

3. From the last rule it follows as a corollary that in the case of induction the subject of the premises must be a sum of subjects, and that in the case of hypothesis the predicate of the premises must be a conjunction of predicates.

4. Also, that this aggregate must be of different objects or qualities and not of mere names.

5. Also, that the only principle upon which the instanced subjects or predicates can be selected is that of belonging to M .*

* Positivism, apart from its theory of history and of the relations between the sciences, is distinguished from other doctrines by the manner in which it regards hypotheses. Almost all men think that metaphysical theories are valueless, because metaphysicians differ so much among themselves; but the positivists give another reason, namely, that these theories violate the sole condition of all legitimate hypothesis. This condition is that every good hypothesis must be such as is certainly capable of subsequent verification with the degree of certainty proper to the conclusions of the branch of science to which it belongs. There is, it seems to me, a confusion here between the probability of a hypothesis in itself, and its admissibility into any one of those bodies of doctrine which have received distinct names, or have been admitted into a scheme of the sciences, and which admit only conclusions which have a very high probability indeed. I have here to deal with the rule only so far as it is a general canon of the *legitimacy* of hypotheses, and not so far as it determines their *relevancy* to a particular science; and I shall, therefore, consider only another common statement of it; namely, "that no hypothesis is admissible which is not capable of verification by direct observation." The positivist regards an hypothesis, not as an inference, but as a device for stimulating and

Hence the formulæ are

Induction.

$S', S'', S''', \&c.$ are taken at random as M 's,

$S', S'', S''', \&c.$ are P ;

\therefore Any M is probably P .

directing observation. But I have shown above that certain premises will render an hypothesis probable, so that there is such a thing as legitimate hypothetic inference. It may be replied that such conclusions are not hypotheses, but inductions. That the sense in which I have used "hypothesis" is supported by good usage, I could prove by a hundred authorities. The following is from Kant: "An hypothesis is the holding for true of the judgment of the truth of a reason on account of the sufficiency of its consequents." Mill's definition (Logic, Book III. Ch. XIV. § 4) also nearly coincides with mine. Moreover, an hypothesis in every sense is an inference, because it is adopted for some reason, good or bad, and that reason, in being regarded as such, is regarded as lending the hypothesis some plausibility. The arguments which I term hypothetic are certainly not inductions, for induction is reasoning from particulars to generals, and this does not take place in these cases. The positivist canon for hypotheses is neither sufficient nor necessary. If it is granted that hypotheses are inferred, it will hardly be questioned that the observed facts must follow apodictically from the hypothesis without the aid of subsidiary hypotheses, and that the characters of that which is predicated in the hypothesis, and from which the inference is drawn, must be taken as they occur, and not be picked out in order to make a plausible argument. That the maxim of the positivists is superfluous or worse, is shown, first, by the fact that it is not implied in the proof that hypothetic inference is valid; and next, by the absurdities to which it gives rise when strictly applied to history, which is entirely hypothetical, and is absolutely incapable of verification by direct observation. To this last argument I know of but two answers: first, that this pushes the rule further than was intended, it being considered that history has already been so verified; and second, that the positivist does not pretend to know the world as it absolutely exists, but only the world which appears to him. To the first answer, the rejoinder is that a rule must be pushed to its logical consequences in all cases, until it can be shown that some of these cases differ in some material respect from the others. To the second answer, the rejoinder is double: first, that I mean, no more by "is" than the positivist by "appears" in the sense in which he uses it in saying that only what "appears" is known, so that the answer is irrelevant; second, that positivists, like the rest of the world, reject historic testimony sometimes, and in doing so distinguish hypothetically between what is and what in some other sense appears, and yet have no means of verifying the distinction by direct observation.

Another error in reference to hypothesis is, that the antecedent probability of what is testified to cannot affect the probability of the testimony of a good witness. This is as much as to say that probable arguments can neither support nor weaken one another. Mr. Venn goes so far as to maintain the impossibility of a conflict of probabilities. The difficulty is instantly removed by admitting indeterminate probabilities.

Hypothesis.

Any M is, for instance, $P', P'', P''', \&c.$,

S is $P', P'', P''', \&c.$;

$\therefore S$ is probably M .

§ 2. *Moods and Figures of Probable Inference.*

It is obvious that the explaining syllogism of an induction or hypothesis may be of any mood or figure.

It would also seem that the conclusion of an induction or hypothesis may be contraposed with one of the premises.

§ 3. *Analogy.*

The formula of analogy is as follows:—

$S', S'',$ and S''' are taken at random from such a class that their characters at random are such as P', P'', P''' .

t is $P', P'',$ and P''' .

$S', S'',$ and S''' are q .

$\therefore t$ is q .

Such an argument is double. It combines the two following:—

1.

S', S'', S''' are taken as being P', P'', P''' .

S', S'', S''' are q .

\therefore (By induction,) P', P'', P''' is q .

t is P', P'', P''' .

\therefore (Deductively,) t is q .

2.

S', S'', S''' are, for instance, P', P'', P''' .

t is P', P'', P''' .

\therefore (By hypothesis,) t has the common characters of S', S'', S''' .

S', S'', S''' are q .

\therefore (Deductively,) t is q .

Owing to its double character, analogy is very strong with only a moderate number of instances.

§ 4. *Formal Relations of the above Forms of Argument.*

If we take an identical proposition as the fact to be explained by induction and hypothesis, we obtain the following formulæ.

By Induction.

S, S', S'' are taken at random as being M ,

S, S', S'' have the characters common to S, S', S'' .

∴ Any M has the characters common to S, S', S'' .

By Hypothesis.

M is, for instance, P, P', P'' .

Whatever is at once P, P' , and P'' is P, P', P'' .

∴ Whatever is at once P, P' , and P'' is M .

By means of the substitution thus justified, Induction and Hypothesis can be reduced to the general type of syllogism, thus:—

Induction.

S, S', S'' are taken as M ,

S, S', S'' are P ;

∴ Any M is P .

Reduction.

S, S', S'' are P ;

Almost any M has the common characters of S, S', S'' .

∴ Almost any M is P .

Hypothesis.

M is, for instance, P, P', P'' ,

S is P, P', P'' ;

∴ S is M .

Reduction.

Whatever is, at once, P, P', P'' is like M ,

S is P, P', P'' ;

∴ S is like M .

Induction may, therefore, be defined as argument which assumes that a whole collection, from which a number of instances have been taken at random, has all the common characters of those instances; hypothesis, as an argument which assumes that a term which necessarily involves a certain number of characters, which have been lighted upon as they occurred, and have not been picked out, may be predicated of any object which has all these characters.

There is a resemblance between the transposition of propositions by which the forms of probable inference are derived and the contraposition by which the indirect figures are derived; in the latter case there is a *denial* or change of modal quality; while in the former there is reduction from certainty to probability, and from the sum of all results to some only, or a change in modal quantity. Thus probable inference is related to apagogical proof, somewhat as the third figure is to the second. Among probable inferences, it is obvious that hypothesis corresponds to the second figure, induction to the third, and analogy to the second-third.

Five hundred and eighty-second Meeting.

May 14, 1867. — MONTHLY MEETING.

The PRESIDENT in the chair.

The Corresponding Secretary read letters relative to exchanges.

The President read a letter from Dr. J. Mason Warren, presenting to the Academy a copy of his work on "Surgical Operations."

The following paper was presented:—

On a New List of Categories. By C. S. PEIRCE.

§ 1. This paper is based upon the theory already established, that the function of conceptions is to reduce the manifold of sensuous impressions to unity, and that the validity of a conception consists in the impossibility of reducing the content of consciousness to unity without the introduction of it.

§ 2. This theory gives rise to a conception of gradation among those conceptions which are universal. For one such conception may unite the manifold of sense and yet another may be required to unite the conception and the manifold to which it is applied; and so on.