"You will perceive, however, that the atomic weights seem to arrange themselves on the diagram in parallel shelving lines. Also, that there is a correspondence between the series of artiads and perissads which have the highest atomic weight—that is to say, Na, K, Rb, Cs, Tl, on the one hand, and Mg, Ca, Sr, Ba, Pb, on the other, inasmuch as they form strong bases and peroxides, but no suboxides or acids. A correspondence is also to be traced between the two series having the next highest atomic weights; that is to say, Fl, Cl, Br, I, and O, S, Se, Te, inasmuch as they have a strong tendency to unite in simple proportions with the members of the two groups just mentioned, forming definite and distinct compounds, and also form strong acids with oxygen, but never bases. There is also some correspondence between the next highest groups on each side, that is between N, P, As, Sb, Bi, and C, Si, Ti, Zr, Sn, Pt, inasmuch as they form acids which unite in the most complicated proportions with bases, putting types at defiance, and also (as far as the higher members of each series are concerned) have some tendency to form weak bases.

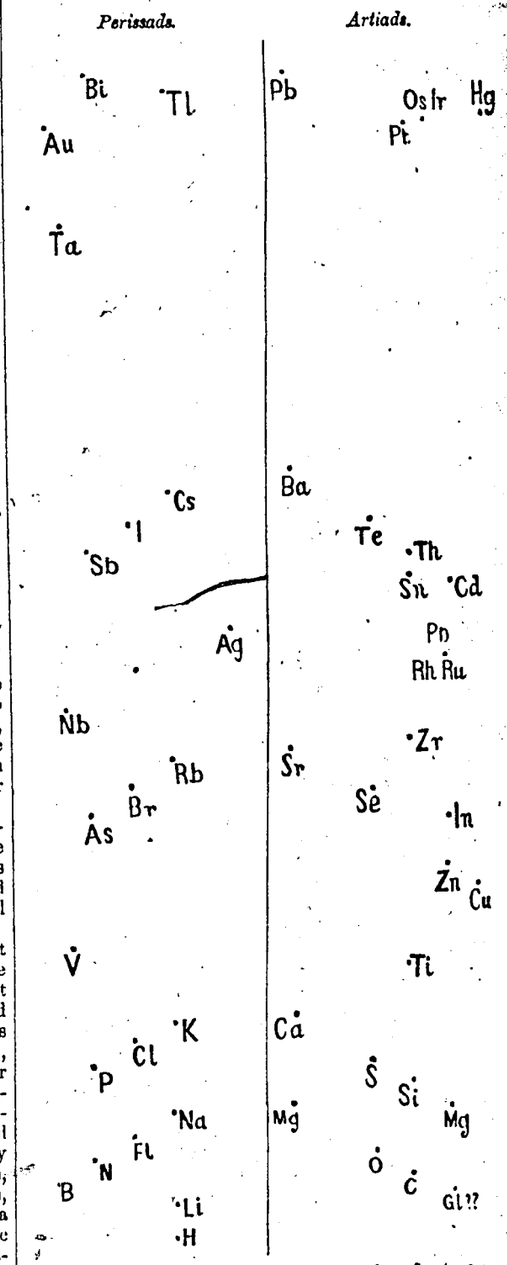
"I also notice that there is a much greater difference in chemical characters between K and Na (not isomorphous), between Fl and Cl (the fluorides being very insoluble), between N and P (N_2O_5 being monobasic), between O and S, between C and Si, and between Gl and Mg, than between any other two adjacent members of their respective series. Also that the maximum of resemblance is between As and Sb, Br and I, Rb and Cs, Sr and Ba, Se and Te, Zn and Cd.

"Finally, there is, as you point out, special resemblance between elements occupying corresponding places in the series, as between C and B, O and Fl, S and P, Ca and K, Se and As, Sr and Rb, Ag and Pd, Au and Os, Tl and Pb, &c."

The table sent to us was drawn on engineers' profile paper, and is somewhat easier to be understood and to be verified than the engraving. The engraving is, however, a faithful copy of the original, excepting the ruled lines. The profile paper and the cross-section paper of the engineers is exceedingly convenient for preparing similar tables and charts.

Our correspondent's table, besides being a very admirable exposition of the facts of pairing, at the same time illustrates almost everything of value which has been written on the classification of the elements and the numerical relations of the atomic weights. It will well repay a careful study.

There is one point on which we suggest a different view from that of our correspondent: the probable number of the elements. Is not our classification at present—of perissads and artiads, paired groups and paired individuals—about as perfect and satisfactory as any classification of natural history? Our classification, so far as it goes, is good; and is there plainly room for new classes or new individuals? In our last we suggested that it might be agreed that some of the unpaired elements are endowed with both atomicities, and thus be paired with themselves. The suggestion (by no means a new one) is applicable especially to iron, manganese, chromium, copper, and mercury. Iron, in FeS_2, is an artiad; in Fe_2O_3, may it not be a perissad? Dr. Odling says, "The ferrous and ferric atoms have distinct chemical properties, and form distinct series of compounds, which differ more from one another than do the salts of ferrum from those of aluminium and bismuth. So great indeed is the difference that, had we been unacquainted with the methods of converting ferrous and ferric compounds into one another, we should never have suspected them to have contained the same metal, or even similar metals." If



it be determined that the pairing is an order of nature, and that the known elements are nearly all paired, and that the known groups appear filled, we ought not to expect many new elements.

table by CSP

THE JOURNAL
OF
SPECULATIVE PHILOSOPHY.

Vol. II. 1869. No. 4.

GROUNDS OF VALIDITY OF THE LAWS OF LOGIC:

FURTHER CONSEQUENCES OF FOUR INCAPACITIES.

[By C. S. Peirce.]

If, as I maintained in an article in the last number of this Journal, every judgment results from inference, to doubt every inference is to doubt everything. It has often been argued that absolute scepticism is self-contradictory; but this is a mistake: and even if it were not so, it would be no argument against the absolute sceptic, inasmuch as he does not admit that no contradictory propositions are true. Indeed, it would be impossible to move such a man, for his scepticism consists in considering every argument and never deciding upon its validity; he would, therefore, act in this way in reference to the arguments brought against him.

But then there are no such beings as absolute sceptics. Every exercise of the mind consists in inference, and so, though there are inanimate objects without beliefs, there are no intelligent beings in that condition.

Yet it is quite possible that a person should doubt every principle of inference. He may not have studied logic, and though a logical formula may sound very obviously true to him, he may feel a little uncertain whether some subtile deception may not lurk in it. Indeed, I certainly shall have, among the most cultivated and respected of my readers, those who deny that those laws of logic which men generally admit have universal validity. But I address myself, also, to those who have no such doubts, for even to them it may be interesting to consider how it is that these principles come to be true. Finally, having put forth in former numbers of this Journal some rather heretical principles of philosophical research, one of which is that nothing can be admitted to be absolutely inexplicable, it behooves me to take up a challenge which has been given me to show how upon my principles the validity of the laws of logic can be other than inexplicable.

I shall be arrested, at the outset, by a sweeping objection to my whole undertaking. It will be said that my deduction of logical principles, being itself an argument, depends for its whole virtue upon the truth of the very principles in question; so that whatever my proof may be, it must take for granted the very things to be proved. But to this I reply, that I am neither addressing absolute sceptics, nor men in any state of fictitious doubt whatever. I require the reader to be candid; and if he becomes convinced of a conclusion, to admit it. There is nothing to prevent a man's perceiving the force of certain special arguments, although he does not yet know that a certain general law of arguments holds good; for the general rule may hold good in some cases and not in others. A man may reason well without understanding the principles of reasoning, just as he may play billiards well without understanding analytical mechanics. If you, the reader, actually find that my arguments have a convincing force with you, it is a mere pretence to call them illogical.

That if one sign denotes generally everything denoted by a second, and this second denotes generally everything denoted by a third, then the first denotes generally everything denoted by the third, is not doubted by anybody who distinctly apprehends the meaning of these words. The deduction of the general form of syllogism, therefore, will consist only of an explanation of the *suppositio communis*.* Now, what the formal logician means by an expression of the form, "Every M is P," is that anything of which M is predicable is P; thus, if S is M, that S is P. The premise that "Every M is P" may, therefore, be denied; but to admit it, unambiguously, in the sense intended, is to admit that the inference is good that S is P if S is M. He, therefore, who does not deny that S is P—M, S, P, being any terms such that S is M and every M is P—denies nothing that the formal logician maintains in reference to this matter; and he who does deny this, simply is deceived by an ambiguity of language. How we come to make any judgments in the sense of the above "Every M is P," may be understood from the theory of reality put forth in the article in the last number. It was there shown that real things are of a cognitive and therefore significative nature, so that the real is that which signifies something real. Consequently, to predicate anything of anything real is to predicate it of that of which that subject [the real] is itself predicated; for to predicate one thing of another is to state that the former is a sign of the latter.

These considerations show the reason of the validity of the formula,

S is M; M is P:
$\therefore$ S is P.

They hold good whatever S and P may be, provided that they be such that any middle term between them can be found. That P should be a negative term, therefore, or that S should be a particular term, would not interfere at all with the validity of this formula. Hence, the following formulæ are also valid:

S is M; M is not P:
$\therefore$ S is not P.

Some S is M; M is P:
$\therefore$ Some S is P.

Some S is M; M is not P;
$\therefore$ Some S is not P.

Moreover, as all that class of inferences which depend upon the introduction of relative terms can be reduced to the general form, they also are shown to be valid. Thus, it is proved to be correct to reason thus:

Every relation of a subject to its predicate is a relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please.

Every relation of "man" to "animal" is a relation of a subject to its predicate.

∴ Every relation of "man" to "animal" is a relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please.

Every relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please, is a relation of the relative "not *headed*, except by the *head* of some," to its correlate.

∴ Every relation of "man" to "animal" is a relation of the relative "not headed, except by the head of some," to its correlate.*

At the same time, as will be seen from this example, the proof of the validity of

* The word *suppositio* is one of the useful technical terms of the middle ages which was condemned by the purists of the *renaissance* as incorrect. The early logicians made a distinction between *significatio* and *suppositio*. *Significatio* is defined as "rei per vocem secundum placitum representatio." It is a mere affair of lexicography, and depends on a special convention (*secundum placitum*), and not on a general principle. *Suppositio* belongs, not directly to the vox, but to the vox as having this or that *significatio*. "Unde significatio prior est suppositione et differunt in hoc, quia significatio est vocis, suppositio vero est termini jam compositi ex voce et significatione." The various *suppositiones* which may belong to one word with one *significatio* are the different senses in which the word may be taken, according to the general principles of the language or of logic. Thus, the word *table* has different *significationes* in the expressions "table of logarithms" and "writing-table"; but the word *man* has one and the same *significatio*, and only different *suppositiones*, in the following sentences: "A man is an animal," "a butcher is a man," "man cooks his food," "man appeared upon the earth at such a date," &c. Some later writers have endeavored to make "*acceptio*" do service for "*suppositio*"; but it seems to me better, now that scientific terminology is no longer forbidden, to revive *supposition*. I should add that as the principles of logic and language for the different uses of the different parts of speech are different, *supposition* must be restricted to the acceptation of a *substantive*. The term *copulatio* was used for the acceptation of an adjective or verb.

* "If any one will by ordinary syllogism prove that because every man is an animal, therefore every head of a man is a head of an animal, I shall be ready to —— set him another question."—*De Morgan*: On the Syllogism No. IV. and on the Logic of Relations.

these inferences depends upon the assumption of the truth of certain general statements concerning relatives. These formulæ can all be deduced from the principle, that in a system of signs in which no sign is taken in two different senses, two signs which differ only in their manner of representing their object, but which are equivalent in meaning, can always be substituted for one another. Any case of the falsification of this principle would be a case of the dependence of the mode of existence of the thing represented upon the mode of this or that representation of it, which, as has been shown in the article in the last number, is contrary to the nature of reality.

The next formula of syllogism to be considered is the following:

S is other than *P*; *M* is *P*:
∴ *S* is other than *M*.

The meaning of "not" or "other than" seems to have greatly perplexed the German logicians, and it may be, therefore, that it is used in different senses. If so, I propose to defend the validity of the above formula only when *other than* is used in a particular sense. By saying that one thing or class is other than a second, I mean that any third whatever is identical with the class which is composed of that third and of whatever is, at once, the first and second. For example, if I say that rats are not mice, I mean that any third class as dogs is identical with dogs and rats-which-are-mice; that is to say, the addition of rats-which-are-mice, to anything, leaves the latter just what it was before. This being all that I mean by *S* is other than *P*, I mean absolutely the same thing when I say that *S* is other than *P*, that I do when I say that *P* is other than *S*; and the same when I say that *S* is other than *M*, that I do when I say that *M* is other than *S*. Hence the above formula is only another way of writing the following:

M is *P*; *S* is not *S*:
∴ *M* is not *S*.

But we have already seen that this is valid.

A very similar formula to the above is the following:

S is *M*; some *S* is *P*:
∴ Some *M* is *P*.

By saying that some of a class is of any character, I mean simply that no statement which implies that none of that class is of that character is true. But to say that none of that class is of that character, is, as I take the word "not," to say that nothing of that character is of that class. Consequently, to say that some of *A* is *B*, is, as I understand words and in the only sense in which I defend this formula, to say that some *B* is *A*. In this way the formula is reduced to the following, which has already been shown to be valid:

Some *P* is *S*; *S* is *M*:
∴ Some *P* is *M*.

The only demonstrative syllogisms which are not included among the above forms are the Theophrastean moods, which are all easily reduced by means of simple conversions.

Let us now consider what can be said against all this, and let us take up the objections which have actually been made to the syllogistic formulæ, beginning with those which are of a general nature and then examining those sophisms which have been pronounced irresolvable by the rules of ordinary logic.

It is a very ancient notion that no proof can be of any value, because it rests on premises which themselves equally require proof, which again must rest on other premises, and so back to infinity. This really does show that nothing can be proved beyond the possibility of a doubt; that no argument could be legitimately used against an absolute sceptic; and that inference is only a transition from one cognition to another, and not the creation of a cognition. But the objection is intended to go much further than this, and to show (as it certainly seems to do) that inference not only cannot produce *infallible* cognition, but that it cannot *produce* cognition at all. It is true, that since some judgment precedes every judgment inferred, either the first premises were not inferred, or there have been no first premises. But it does not follow that because there has been no first in a series, therefore that series has had no beginning in time; for the series may be *continuous*, and may have begun gradually, as was shown in an article in No. 3 of this volume, where this difficulty has already been resolved.

A somewhat similar objection has been made by Locke and others, to the effect that the ordinary demonstrative syllogism is a *petitio principii*, inasmuch as the conclusion is already implicitly stated in the major premise. Take, for example, the syllogism,

All men are mortal;
Socrates is a man:
∴ Socrates is mortal.

This attempt to prove that Socrates is mortal begs the question, it is said, since if the conclusion is denied by any one, he thereby denies that all men are mortal. But what such considerations really prove is that the syllogism is demonstrative. To call it a *petitio principii* is a mere confusion of language. It is strange that philosophers, who are so suspicious of the words *virtual* and *potential*, should have allowed this "implicit" to pass unchallenged. A *petitio principii* consists in reasoning from the unknown to the unknown. Hence, a logician who is simply engaged in stating what general forms of argument are valid, can, at most, have nothing more to do with the consideration of this fallacy than to note those cases in which from logical principles a premise of a certain form cannot be better known than a conclusion of the corresponding form. But it is plainly beyond the province of the logician, who has only proposed to state what forms of facts involve what others, to inquire whether man can have a knowledge of universal propositions without a knowledge of every particular contained under them, by means of natural insight, divine revelation, induction, or testimony. The only *petitio principii*, therefore, which he can notice is the assumption of the conclusion itself in the premise; and this, no doubt, those who call the syllogism a *petitio principii* believe is done in that formula. But the proposition "All men are mortal" does not in itself involve the statement that Socrates is mortal, but only that "whatever has man truly predicated of it is mortal." In other words, the *conclusion* is not involved in the meaning of the premise, but only the *validity of the syllogism*. So that this objection merely amounts to arguing that the syllogism is not valid, because it is demonstrative.*

A much more interesting objection is that a syllogism is a purely mechanical process. It proceeds according to a bare rule or formula; and a machine might be constructed which would so transpose the terms of premises. This being so (and it is so), it is argued that this cannot be *thought*; that there is no life in it. Swift has ridiculed the syllogism in the "Voyage to Laputa," by describing a machine for making science:

"By this contrivance, the most ignorant person, at a reasonable charge, and with little bodily labor, might write books in philosophy, poetry, politics, laws, mathematics, and theology, without the least assistance from genius or study."

The idea involved in this objection seems to be that it requires mind to apply any formula or use any machine. If, then, this mind is itself only another formula, it requires another mind behind it to set it into operation, and so on *ad infinitum*. This objection fails in much the same way that the first one which we considered failed. It is as though a man should address a land surveyor as follows:—"You do not make a true representation of the land; you only measure lengths from point to point—that is to say, lines. If you observe angles, it is only to solve triangles and obtain the lengths of their sides. And when you come to make your map, you use a pencil which can only make lines, again. So, you have to do solely with lines. But the land is a surface; and no number of lines, however great, will make any surface, however small. You, therefore, fail entirely to represent the land." The surveyor, I think, would reply, "Sir, you have proved that my lines cannot make up the land, and that, therefore, my map *is not* the land. I never pretended that it was. But that does not prevent it from truly representing the land, as far as it goes. It cannot, indeed, represent every blade of grass; but it does not represent that there is not a blade of grass where there is. To abstract from a circumstance is not to deny it." Suppose the objector were, at this point, to say, "To abstract from a circumstance *is* to deny it. Wherever your map does not represent a blade of grass, it represents there is no blade of grass. Let us take things on their own valuation." Would not the surveyor reply: "This map is my description of the country. Its own valuation can be nothing but what I say, and all the world understands, that I mean by it. Is it very unreasonable that I should demand to be taken as I mean, especially when I succeed in making myself understood?" What the objector's reply to this

* Mr. Mill thinks the syllogism is merely a formula for recalling forgotten facts. Whether he means to deny, what all logicians since Kant have held, that the syllogism serves to render confused thoughts distinct, or whether he does not know that this is the usual doctrine, does not appear.

question would be, I leave it to any one to say who thinks his position well taken. Now this fine of objection is parallel to that which is made against the syllogism. It is shown that no number of syllogisms can constitute the sum total of any mental action, however restricted. This may be freely granted, and yet it will not follow that the syllogism does not truly represent the mental action, as far as it purports to represent it at all. There is reason to believe that the action of the mind is, as it were, a continuous movement. Now the doctrine embodied in syllogistic formulæ (so far as it applies to the mind at all) is, that if two successive positions, occupied by the mind in this movement, be taken, they will be found to have certain relations. It is true that no number of successions of positions can make up a continuous movement; and this, I suppose, is what is meant by saying that a syllogism is a dead formula, while thinking is a living process. But the reply is that the syllogism is not intended to represent the mind as to its life or deadness, but only as to the relation of its different judgments concerning the same thing. And it should be added that the relation between syllogism and thought does not spring from considerations of formal logic, but from those of psychology. All that the formal logician has to say is, that if facts capable of expression in such and such forms of words are true, another fact whose expression is related in a certain way to the expression of these others is also true.

Hegel taught that ordinary reasoning is "one-sided." A part of what he meant was that by such inference a part only of all that is true of an object can be learned, owing to the generality or abstractedness of the predicates inferred. This objection is, therefore, somewhat similar to the last; for the point of it is that no number of syllogisms would give a complete knowledge of the object. This, however, presents a difficulty which the other did not; namely, that if nothing incognizable exists, and all knowledge is by mental action, by mental action everything is cognizable. So that if by syllogism everything is not cognizable, syllogism does not exhaust the modes of mental action, But grant the validity of this argument and it proves too much; for it makes, not the syllogism particularly, but all finite knowledge to be worthless. However much we know, more may come to be found out. Hence, all can never be known. This seems to contradict the fact that nothing is absolutely incognizable; and it would really do so *if our knowledge* were something absolutely limited. For, to say that all can never be known, means that information may increase beyond any assignable point; that is, that an absolute termination of all increase of knowledge is absolutely incognizable, and therefore does not exist. In other words, the proposition merely means that the sum of all that will be known up to any time, however advanced, into the future, has a ratio less than any assignable ratio to all that may be known at a time still more advanced. This does not contradict the fact that everything is cognizable; it only contradicts a proposition, which no one can maintain, that everything will be known at some time some number of years into the future. It may, however, very justly be said that the difficulty still remains, how at every future time, however late, there can be something yet to happen. It is no longer a contradiction, but it is a difficulty; that is to say, *lengths of time* are shown not to afford an adequate conception of futurity in general; and the question arises, in what other way we are to conceive of it. I might indeed, perhaps, fairly drop the question here, and say that the difficulty had become so entirely removed from the syllogism in particular, that the formal logician need not feel himself specially called on to consider it. The solution, however, is very simple. It is that we conceive of the future, as a whole, by considering that this *word*, like any other general term, as "inhabitant of St. Louis," may be taken distributively or collectively. We conceive of the infinite, therefore, not directly or on the side of its infinity, but by means of a consideration concerning words or a second intention.

Another objection to the syllogism is that its "therefore" is merely subjective; that, because a certain conclusion syllogistically follows from a premise, it does not follow that the fact denoted by the conclusion really depends upon the fact denoted by the premise, so that the syllogism does not represent things as they really are. But it has been fully shown that if the facts are as the premises represent, they are also as the conclusion represents. Now this is a purely objective statement: therefore, there is a real connection between the facts stated as

premises and those stated as conclusion. It is true that there is often an appearance of reasoning deductively from effects to causes. Thus we may reason as follows: "There is smoke; there is never smoke without fire: hence, there has been fire." Yet smoke is not the cause of fire, but the effect of it. Indeed, it is evident, that in many cases an event is a demonstrative sign of a certain previous event having occurred. Hence, we can reason deductively from relatively future to relatively past, whereas causation really determines events in the direct order of time. Nevertheless, if we can thus reason against the stream of time, it is because there really are such facts as that "If there is smoke, there has been fire," in which the following event is the antecedent. Indeed, if we consider the manner in which such a proposition became known to us, we shall find that what it really means is that "If we find smoke, we *shall* find evidence on the whole that there has been fire"; and this, if reality consists in the agreement that the whole community would eventually come to, is the very same thing as to say that there really has been fire. In short, the whole present difficulty is resolved instantly by this theory of reality, because it makes all reality something which is constituted by an event indefinitely future.

Another objection, for which I am quite willing to allow a great German philosopher the whole credit, is that sometimes the conclusion is false, although both the premises and the syllogistic form are correct.* Of this he gives the following examples. From the middle term that a wall has been painted blue, it may correctly be concluded that it is blue; but notwithstanding this syllogism it may be green if it has also received a coat of yellow, from which last circumstance by itself it would follow that it is yellow. If from the middle term of the sensuous faculty it be concluded that man is neither good nor bad, since neither can be predicated of the sensuous, the syllogism is correct; but the conclusion is false, since of man in the concrete, spirituality is equally true, and may serve as middle term in an opposite syllogism. From the middle term of the gravitation of the planets, satellites, and comets, towards the sun, it follows correctly that these bodies fall into the sun; but they do not fall into it, because (!) they equally gravitate to their own centres, or, in other words (!!), they are supported by centrifugal force. Now, does Hegel mean to say that these syllogisms satisfy the rules for syllogism given by those who defend syllogism? or does he mean to grant that they do not satisfy *those* rules, but to set up some rules of his own for syllogism which shall insure its yielding false conclusions from true premises? If the latter, he ignores the real issue, which is whether the syllogism as defined by the rules of formal logic is correct, and not whether the syllogism as represented by Hegel is correct. But if he means that the above examples satisfy the usual definition of a true syllogism, he is mistaken. The first, stated in form, is as follows:

> Whatever has been painted blue is blue;
> This wall has been painted blue:
> ∴ This wall is blue.

Now "painted blue" may mean painted with blue paint, or painted so as to be blue. If, in the example, the former were meant, the major premise would be false. As he has stated that it is true, the latter meaning of "painted blue" must be the one intended. Again, "blue" may mean blue at some time, or blue at this time. If the latter be meant, the major premise is plainly false; therefore, the former is meant. But the conclusion is said to contradict the statement that the wall is yellow. If blue were here taken in the more general sense, there would be no such contradiction. Hence, he means in the conclusion that this wall is now blue; that is to say, he reasons thus:

> Whatever has been made blue has been blue;
> This has been made blue:
> ∴ This is blue now.

Now substituting letters for the subjects and predicates, we get the form,

> *M* is *P*;
> *S* is *M*:
> ∴ *S* is *Q*.

This is not a syllogism in the ordinary sense of that term, or in any sense in which anybody maintains that the syllogism is valid.

The second example given by Hegel, when written out in full, is as follows:

> Sensuality is neither good nor bad;
> Man *has* (not *is*) sensuality:
> ∴ Man is neither good nor bad.

* "So zeigt sich jener Schlussatz dadurch als falsch, obgleich für sich dessen Prämissen und ebenso dessen Consequenz ganz richtig sind." —Hegel's Werke, vol. v., p. 124.

Or, the same argument may be stated as follows:

The sensuous, *as such*, is neither good nor bad;
Man is sensuous:
∴ Man is neither good nor bad.

When letters are substituted for subject and predicate in either of these arguments, it takes the form,

M is *P*;
S is *M*:
∴ *S* is *P*.

This, again, bears but a very slight resemblance to a syllogism.

The third example, when stated at full length, is as follows:

Whatever tends towards the sun, *on the whole*, falls into the sun;
The planets tend toward the sun:
∴ The planets fall into the sun.

This is a fallacy similar to the last.

I wonder that this eminent logician did not add to his list of examples of correct syllogism the following:

It either rains, or it does not rain:
It does not rain:
∴ It rains.

This is fully as deserving of serious consideration as any of those which he has brought forward. The rainy day and the pleasant day are both, in the first place, day. Secondly, each is the negation of a day. It is indifferent which be regarded as the positive. The pleasant is Other to the rainy, and the rainy is in like manner Other to the pleasant. Thus, both are equally Others. Both are Others of each other, or each is Other for itself. So this day being other than rainy, that to which it is Other is itself. But it is Other than itself. Hence, it is itself Rainy.

Some sophisms have, however, been adduced, mostly by the Eleatics and Sophists, which really are extremely difficult to resolve by syllogistic rules; and according to some modern authors this is actually impossible. These sophisms fall into three classes: *1st*, those which relate to continuity; *2d*, those which relate to consequences of supposing things to be other than they are; *3d*, those which relate to propositions which imply their own falsity. Of the first class, the most celebrated are Zeno's arguments concerning motion. One of these is, that if Achilles overtakes a tortoise in any finite time, and the tortoise has the start of him by a distance which may be called *a*, then Achilles has to pass over the sum of distances represented by the polynomial

$$\tfrac{1}{2}a + \tfrac{1}{4}a + \tfrac{1}{8}a + \tfrac{1}{16}a + \tfrac{1}{32}a \;\&c.$$

up to infinity. Every term of this polynomial is finite, and it has an infinite number of terms; consequently, Achilles must in a finite time pass over a distance equal to the sum of an infinite number of finite distances. Now this distance must be infinite, because no finite distance, however small, an be multiplied by an infinite number without giving an infinite distance. So that even if none of these finite distances were larger than the smallest, (which is finite since all are finite,) the sum of the whole would be infinite. But Achilles cannot pass over an infinite distance in a finite time; therefore, he cannot overtake the tortoise in any time, however great.

The solution of this fallacy is as follows: The conclusion is dependent on the fact that Achilles cannot overtake the tortoise without passing over an infinite number of terms of that series of finite distances. That is, no case of his overtaking the tortoise would be a case of his not passing over a non-finite number of terms; that is (by simple conversion), no case of his not passing over a non-finite number of terms would be a case of his overtaking the tortoise. But if he does not pass over a non-finite number of terms, he either passes over a finite number, or he passes over none; and conversely. Consequently, nothing more has been said than that every case of his passing over only a finite number of terms, or of his not passing over any, is a case of his not overtaking the tortoise. Consequently, nothing more can be concluded than that he passes over a distance greater than the sum of any finite number of the above series of terms. But because a quantity is greater than any quantity of a certain series, it does not follow that it is greater than any quantity.

In fact, the reasoning in this sophism may be exhibited as follows:—We start with the series of numbers,

$$\tfrac{1}{2}a$$
$$\tfrac{1}{2}a + \tfrac{1}{4}a$$
$$\tfrac{1}{2}a + \tfrac{1}{4}a + \tfrac{1}{8}a$$
$$\tfrac{1}{2}a + \tfrac{1}{4}a + \tfrac{1}{8}a + \tfrac{1}{16}a$$
$$\&c. \;\&c. \;\&c.$$

Then, the implied argument is

Any number of this series is less than *a*;
But any number you please is less than the number of terms of this series:
Hence, any number you please is less than *a*.

This involves an obvious confusion between the number of terms and the value of the greatest term.

Another argument by Zeno against motion, is that a body fills a space no larger than itself. In that place there is no room for motion. Hence, while in the place where it is, it does not move. But it never is other than in the place where it is. Hence, it never moves. Putting this into form, it will read:

No body in a place no larger than itself is moving;
But every body is a body in a place no larger than itself:
∴ No body is moving.

The error of this consists in the fact that the minor premise is only true in the sense that during a time sufficiently short the space occupied by a body is as little larger than itself as you please. All that can be inferred from this is, that during no time a body will move no distance.

All the arguments of Zeno depend on supposing that a *continuum* has ultimate parts. But a *continuum* is precisely that, every part of which has parts, in the same sense. Hence, he makes out his contradictions only by making a self-contradictory supposition. In ordinary and mathematical language, we allow ourselves to speak of such parts—*points*—and whenever we are led into contradiction thereby, we have simply to express ourselves more accurately to resolve the difficulty.

Suppose a piece of glass to be laid on a sheet of paper so as to cover half of it. Then, every part of the paper is *covered*, or *not covered*; for "not" means merely outside of, or other than. But is the line under the edge of the glass covered or not? It is no more on one side of the edge than it is on the other. Therefore, it is either on both sides, or neither side. It is not on neither side; for if it were it would be *not* on either side, therefore not on the covered side, therefore not covered, therefore on the uncovered side. It is not partly on one side and partly on the other, because it has no width. Hence, it is wholly on both sides, or both covered and not covered.

The solution of this is, that we have supposed a part too narrow to be partly uncovered and partly covered; that is to say, a part which has no parts in a continuous surface, which by definition has no such parts. The reasoning, therefore, simply serves to reduce this supposition to an absurdity.

It may be said that there really is such a thing as a line. If a shadow falls on a surface, there really is a division between the light and the darkness. That is true. But it does not follow that because we attach a definite meaning to the part of a surface being covered, therefore we know what we mean when we say that a line is covered. We may define a covered line as one which separates two surfaces both of which are covered, or as one which separates two surfaces *either* of which is covered. In the former case, the line under the edge is uncovered; in the latter case, it is covered.

In the sophisms thus far considered, the appearance of contradiction depends mostly upon an ambiguity; in those which we are now to consider, two true propositions really do in form conflict with one another. We are apt to think that formal logic forbids this, whereas a familiar argument, the *reductio ad absurdum*, depends on showing that contrary predicates are true of a subject, and *that therefore that subject does not exist*. Many logicians, it is true, make affirmative propositions assert the existence of their subjects.* The objection to this is that it cannot be extended to hypotheticals. The proposition

If *A* then *B*

may conveniently be regarded as equivalent to

Every case of the truth of *A* is a case of the truth of *B*.

But this cannot be done if the latter proposition asserts the existence of its subject; that is, asserts that *A* really happens. If, however, a categorical affirmative be regarded as asserting the existence of its subject, the principle of the *reductio ad absurdum* is that two propositions of the forms,

If *A* were true, *B* would not be true,

and

If *A* were true, *B* would be true,

may both be true at once; and that if they are so, *A* is not true. It will be well, perhaps, to illustrate this point. No man of common sense would deliberately upset his inkstand if there were ink in it; that is, if any ink would run out. Hence, by simple conversion,

If he were deliberately to upset his inkstand, no ink would be spilt.

* The usage of ordinary language has no relevancy in the matter.

But suppose there is ink in it. Then, it is also true, that

> If he were deliberately to upset his inkstand, the ink would be spilt.

These propositions are both true, and the law of contradiction is not violated which asserts only that nothing has contradictory predicates: only, it follows from these propositions that the man will not deliberately overturn his inkstand.

There are two ways in which deceptive sophisms may result from this circumstance. In the first place, contradictory propositions are never both true. Now, as a universal proposition may be true when the subject does not exist, it follows that the contradictory of a universal—that is, a particular—cannot be taken in such a sense as to be true when the subject does not exist. But a particular simply asserts a part of what is asserted in the universal over it; therefore, the universal over it asserts the subject to exist. Consequently, there are two kinds of universals, those which do not assert the subject to exist, and these have no particular propositions under them, and those which do assert that the subject exists, and these strictly speaking have no contradictories. For example, there is no use of such a form of proposition as "Some griffins would be dreadful animals," as particular under the useful form "The griffin would be a dreadful animal"; and the apparent contradictories "All of John Smith's family are ill," and "Some of John Smith's family are not ill," are both false at once if John Smith has no family. Here, though an inference from a universal to the particular under it is always valid, yet a procedure which greatly resembles this would be sophistical if the universal were one of those propositions which does not assert the existence of its subject. The following sophism depends upon this; I call it the True Gorgias:

Gorgias. What say you, Socrates, of black? Is any black, white?

Socrates. No, by Zeus!

Gor. Do you say, then, that no black is white? *Soc.* None at all.

Gor. But is everything either black or non-black? *Soc.* Of course.

Gor. And everything either white or non-white? *Soc.* Yes.

Gor. And everything either rough or smooth? *Soc.* Yes.

Gor. And everything either real or unreal? *Soc.* Oh, bother! yes.

Gor. Do you say, then, that all black is either rough black or smooth black? *Soc.* Yes.

Gor. And that all white is either real white or unreal white? *Soc.* Yes.

Gor. And yet is no black, white? *Soc.* None at all.

Gor. Nor no white, black? *Soc.* By no means.

Gor. What? Is no smooth black, white? *Soc.* No; you cannot prove that, Gorgias.

Gor. Nor no rough black, white? *Soc.* Neither.

Gor. Nor no real white, black? *Soc.* No.

Gor. Nor no unreal white, black? *Soc.* No, I say. No white at all is black.

Gor. What if black is smooth, is it not white? *Soc.* Not in the least.

Gor. And if the last is false, is the first false? *Soc.* It follows.

Gor. If, then, black is white, does it follow, that black is not smooth? *Soc.* It does.

Gor. Black-white is not smooth? *Soc.* What do you mean?

Gor. Can any dead man speak? *Soc.* No, indeed.

Gor. And is any speaking man dead? *Soc.* I say, no.

Gor. And is any good king tyrannical? *Soc.* No.

Gor. And is any tyrannical king good? *Soc.* I just said no.

Gor. And you said, too, that no rough black is white, did you not? *Soc.* Yes.

Gor. Then, is any black-white, rough? *Soc.* No.

Gor. And is any unreal black, white? *Soc.* No.

Gor. Then, is any black-white unreal? *Soc.* No.

Gor. No black-white is rough? *Soc.* None.

Gor. All black-white, then, is non-rough? *Soc.* Yes.

Gor. And all black-white, non-unreal? *Soc.* Yes.

Gor. All black-white is then smooth? *Soc.* Yes.

Gor. And all real? *Soc.* Yes.

Gor. Some smooth, then, is black-white? *Soc.* Of course.

Gor. And some real is black-white? *Soc.* So it seems.

Gor. Some black-white smooth is black-white? *Soc.* Yes.

Gor. Some black smooth is black-white? *Soc.* Yes.

Gor. Some black smooth is white. *Soc.* Yes.

Gor. Some black real is black-white? *Soc.* Yes.

Gor. Some black real is white? *Soc.* Yes.

Gor. Some real black is white? *Soc.* Yes.

Gor. And some smooth black is white? *Soc.* Yes.

Gor. Then, some black is white? *Soc.* I think so myself.

The principle of the *reductio ad absurdum* also occasions deceptions in another way, owing to the fact that we have many words, such as *can*, *may*, *must*, &c., which imply more or less vaguely an otherwise unexpressed condition, so that these propositions are in fact hypotheticals. Accordingly, if the unexpressed condition is some state of things which does not actually come to pass, the two propositions may appear to be contrary to one another. Thus, the moralist says, "You ought to do this, and you can do it." This "You can do it" is principally hortatory in its force: so far as it is a statement of fact, it means merely, "If you try, you will do it." Now, if the act is an outward one and the act is not performed, the scientific man, in view of the fact that every event in the physical world depends exclusively on physical antecedents, says that in this case the laws of nature prevented the thing from being done, and that therefore, "Even if you had tried, you would not have done it." Yet the reproachful conscience still says you might have done it; that is, that "If you had tried, you would have done it." This is called the paradox of freedom and fate: and it is usually supposed that one of these propositions must be true and the other false. But since, in fact, you have not tried, there is no reason why the supposition that you have tried should not be reduced to an absurdity. In the same way, if you had tried and had performed the action, the conscience might say, "If you had not tried, you would not have done it"; while the understanding would say, "Even if you had not tried, you would have done it." These propositions are perfectly consistent, and only serve to reduce the supposition that you did not try to an absurdity.*

The third class of sophisms consists of the so-called *Insolubilia*. Here is an example of one of them with its resolution:

THIS PROPOSITION IS NOT TRUE.

IS IT TRUE OR NOT?

Suppose it true.
Then,
The proposition is true;
But, that it is not true is the proposition:
∴ That it is not true is true;
∴ It is not true.

Besides,
It is true.
∴ It is true that it is true,
∴ It is not true that it is not true;
But, the proposition is that it is not true,
∴ The proposition is not true.

Suppose it not true.
Then,
It is not true.
∴ It is true that it is not true.
But, the proposition is that it is not true.
∴ The proposition is true.

Besides,
The proposition is not true.
But that it is not true is the proposition.
∴ That it is not true, is not true.
∴ That it is true, is true.
∴ It is true.

∴ Whether it is true or not, it is both true and not.
∴ It is both true and not,
which is absurd.

* This seems to me to be the main difficulty of freedom and fate. But the question is overlaid with many others. The Necessitarians seem now to maintain less that every physical event is completely determined by physical causes, (which seems to me irrefragable,) than that every act of will is determined by the strongest motive. This has never been proved. Its advocates seem to think that it follows from universal causation, but why need the cause of an act lie within the consciousness at all? If I act from a reason at all, I act voluntarily; but which of two reasons shall appear strongest to me on a particular occasion may be owing to what I have eaten for dinner. Unless there is a perfect regularity as to what is the strongest motive with me, to say that I act from the strongest motive is mere tautology. If there is no calculating how a man will act except by taking into account external facts, the character of his motives does not determine how he acts. Mill and others have, therefore, not shown that a man always acts from the strongest motive. Hobbes maintained that a man always acts from a reflection upon what will please him most. This is a very crude opinion. Men are not always thinking of themselves.

Self-control seems to be the capacity for rising to an extended view of a practical subject instead of seeing only temporary urgency. This is the only freedom of which man has any reason to be proud; and it is because love of what is good for all on the whole, which is the widest possible consideration, is the essence of Christianity, that it is said that the service of Christ is perfect freedom.

Since the conclusion is false, the reasoning is bad, or the premises are not all true. But the reasoning is a dilemma; either, then, the disjunctive principle that it is either true or not is false, or the reasoning under one or the other branch is bad, or the reasoning is altogether valid. If the principle that it is either true or not is false, it is other than true and other than not true; that is, not true and not not true; that is, not true and true. But this is absurd. Hence, the disjunctive principle is valid. There are two arguments under each horn of the dilemma; both the arguments under one or the other branch must be false. But, in each case, the second argument involves all the premises and forms of inference involved in the first; hence, if the first is false, the second necessarily is so. We may, therefore, confine our attention to the first arguments in the two branches. The forms of argument contained in these are two: first, the simple syllogism in Barbara, and, second, the consequence from the truth of a proposition to the proposition itself. These are both correct. Hence, the whole form of reasoning is correct, and nothing remains to be false but a premise. But since the repetition of an alternative supposition is not a premise, there is, properly speaking, but one premise in the whole. This is that the proposition is the same as that that proposition is not true. This, then, must be false. Hence the proposition signifies either less or more than this. If it does not signify as much as this, it signifies nothing, and hence it is not true, and hence another proposition which says of it what it says of itself is true. But if the proposition in question signifies something more than that it is itself not true, then the premise that

> Whatever is said in the proposition is that it is not true,

is not true. And as a proposition is true only if whatever is said in it is true, but is false if anything said in it is false, the first argument on the second side of the dilemma contains a false premise, and the second an undistributed middle. But the first argument on the first side remains good. Hence, if the proposition means more than that it is not true, it is not true, and another proposition which repeats this of it is true. Hence, whether the proposition does or does not mean that it is not true, it is not true, and a proposition which repeats this of it is true.

Since this repeating proposition is true, it has a meaning. Now, a proposition has a meaning if any part of it has a meaning. Hence the original proposition (a part of which repeated has a meaning) has itself a meaning. Hence, it must imply something besides that which it explicitly states. But it has no particular determination to any further implication. Hence, what more it signifies it must signify by virtue of being a proposition at all. That is to say, every proposition must imply something analogous to what this implies. Now, the repetition of this proposition does not contain this implication, for otherwise it could not be true; hence, what every proposition implies must be something concerning itself. What every proposition implies concerning itself must be something which is false of the proposition now under discussion, for the whole falsity of this proposition lies therein, since all that it explicitly lays down is true. It must be something which would not be false if the proposition were true, for in that case some true proposition would be false. Hence, it must be that it is itself true. That is, *every proposition asserts its own truth.*

The proposition in question, therefore, is true in all other respects but its implication of its own truth.*

The difficulty of showing how the law of deductive reasoning is true depends upon our inability to conceive of its not being true. In the case of probable reasoning the difficulty is of quite another kind; here, where we see precisely what the procedure is, we wonder how such a process can have any validity at all. How magical it is that by examining a part of a class we can know what is true of the whole of the class, and by

* This is the principle which was most usually made the basis of the resolution of the *Insolubilia*. See, for example, *Pauli Venati Sophismata Aurea, Soph.* 30. The authority of Aristotle is claimed for this mode of solution. *Sophist. Elench., cap.* 25. The principal objection which was made to this mode of solution, viz., that the principle that every proposition implies its own truth, cannot be proved, I believe that I have removed. The only arguments against the truth of this principle were based on the imperfect doctrines of *modales* and *obligationes*. Other methods of solution suppose that a part of a proposition cannot denote the whole proposition, or that no intellection is a formal cognition of itself. A solution of this sort will be found in Occam's *Summa Totius Logices, 3d part of 3d part, cap.* 38. Such modern authors as think the solution "very easy" do not understand its difficulties. See Mansell's *Aldrich*, p. 135.

study of the past can know the future; in short, that we can know what we have not experienced!

Is not this an intellectual intuition! Is it not that besides ordinary experience which is dependent on there being a certain physical connection between our organs and the thing experienced, there is a second avenue of truth dependent only on there being a certain intellectual connection between our previous knowledge and what we learn in that way? Yes, this is true. Man has this faculty, just as opium has a somnific virtue; but some further questions may be asked, nevertheless. How is the existence of this faculty accounted for? In one sense, no doubt, by natural selection. Since it is absolutely essential to the preservation of so delicate an organism as man's, no race which had it not has been able to sustain itself. This accounts for the prevalence of this faculty, provided it was only a possible one. But how can it be possible? What could enable the mind to know physical things which do not physically influence it and which it does not influence? The question cannot be answered by any statement concerning the human mind, for it is equivalent to asking what makes the facts usually to be, as inductive and hypothetic conclusions from true premises represent them to be? Facts of a certain kind are usually true when facts having certain relations to them are true; what is the cause of this? That is the question.

The usual reply is that nature is everywhere regular; as things have been, so they will be; as one part of nature is, so is every other. But this explanation will not do. Nature is not regular. No disorder would be less orderly than the existing arrangement. It is true that the special laws and regularities are innumerable; but nobody thinks of the irregularities, which are infinitely more frequent. Every fact true of any one thing in the universe is related to every fact true of every other. But the immense majority of these relations are fortuitous and irregular. A man in China bought a cow three days and five minutes after a Greenlander had sneezed. Is that abstract circumstance connected with any regularity whatever? And are not such relations infinitely more frequent than those which are regular? But if a very large number of qualities were to be distributed among a very large number of things in almost any way, there would chance to be some few regularities. If, for example, upon a checker-board of an enormous number of squares, painted all sorts of colors, myriads of dice were to be thrown, it could hardly fail to happen, that upon some color, or shade of color, out of so many, some one of the six numbers should not be uppermost on any die. This would be a regularity; for, the universal proposition would be true that upon that color that number is never turned up. But suppose this regularity abolished, then a far more remarkable regularity would be created, namely, that on every color every number is turned up. Either way, therefore, a regularity must occur. Indeed, a little reflection will show that although we have here only variations of color and of the numbers of the dice, many regularities must occur. And the greater the number of objects, the more respects in which they vary, and the greater the number of varieties in each respect, the greater will be the number of regularities. Now, in the universe, all these numbers are infinite. Therefore, however disorderly the chaos, the *number* of regularities must be infinite. The orderliness of the universe, therefore, if it exists, must consist in the large *proportion* of relations which present a regularity to those which are quite irregular. But this proportion in the actual universe is, as we have seen, as small as it can be; and, therefore, the orderliness of the universe is as little as that of any arrangement whatever.

But even if there were such an orderliness in things, it never could be discovered. For it would belong to things either collectively or distributively. If it belonged to things collectively, that is to say, if things formed a system the difficulty would be that a system can only be known by seeing some considerable proportion of the whole. Now we never can know how great a part of the whole of nature we have discovered. If the order were distributive, that is, belonged to all things only by belonging to each thing, the difficulty would be that a character can only be known by comparing something which has with it something which has it not. *Being, quality, relation*, and other universals are not known except as characters of words or other signs, attributed by a figure of speech to things. Thus, in neither case could the order of things be known. But the order of things would not

help the validity of our reasoning—that is, would not help us to reason correctly—unless we knew what the order of things required the relation between the known reasoned *from* to the unknown reasoned *to*, to be.

But even if this order both existed and were known, the knowledge would be of no use except as a general principle, from which things could be deduced. It would not explain how knowledge could be increased, (in contradistinction to being rendered more distinct,) and so it would not explain how it could itself have been acquired.

Finally, if the validity of induction and hypothesis were dependent on a particular constitution of the universe, we could imagine a universe in which these modes of inference should not be valid, just as we can imagine a universe in which there would be no attraction, but things should merely drift about. Accordingly, J. S. Mill, who explains the validity of induction by the uniformity of nature,* maintains that he can imagine a universe without any regularity, so that no probable inference would be valid in it.† In the universe as it is, probable arguments sometimes fail, nor can any definite proportion of cases be stated in which they hold good; all that can be said is that in the long run they prove approximately correct. Can a universe be imagined in which this would not be the case? It must be a universe where probable argument can have some application, in order that it may fail half the time. It must, therefore, be a universe experienced. Of the finite number of propositions true of a finite amount of experience of such a universe, no one would be universal in form, unless the subject of it were an individual. For if there were a plural universal proposition, inferences by analogy from one particular to another would hold good invariably in reference to that subject. So that these arguments might be no better than guesses in reference to other parts of the universe, but they would invariably hold good in a finite proportion of it, and so would on the whole be somewhat better than guesses. There could, also, be no individuals in that universe, for there must be some general class—that is, there must be some things more or less alike—or probable argument would find no premises there; therefore, there must be two mutually exclusive classes, since every class has a residue outside of it; hence, if there were any individual, that individual would be wholly excluded from one or other of these classes. Hence, the universal plural proposition would be true, that no one of a certain class was that individual. Hence, no universal proposition would be true. Accordingly, every combination of characters would occur in such a universe. But this would not be disorder, but the simplest order; it would not be unintelligible, but, on the contrary, everything conceivable would be found in it with equal frequency. The notion, therefore, of a universe in which probable arguments should fail as often as hold true, is absurd. We can suppose it in general terms, but we cannot specify how it should be other than self-contradictory.*

Since we cannot conceive of probable inferences as not generally holding good, and since no special supposition will serve to explain their validity, many logicians have sought to base this validity on that of deduction, and that in a variety of ways. The only attempt of this sort, however, which deserves to be noticed is that which seeks to determine the probability of a future event by the theory of probabilities, from the fact that a certain number of similar events have been observed. Whether this can be done or not depends on the meaning assigned to the word probability. But if this word is to be taken in such a sense that a form of conclusion which is probable is valid; since the validity of an inference (or its correspondence with facts) consists solely in this, that when such premises are true, such a conclusion is generally true, then probability can mean nothing but the ratio of the frequency of occurrence of a specific event to a general one over it. In this sense of the term, it is plain that the probability of an inductive conclusion cannot be *deduced* from the premises; for from the inductive premises

$$S', S'', S''' \text{ are } M,$$
$$S', S'', S''', \text{ are } P,$$

nothing follows deductively, except that any M, which is S', or S'', or S''' is P; or, less explicitly, that some M is P.

Thus, we seem to be driven to this point. On the one hand, no determination of things, no *fact*, can result in the validity of probable argument; nor, on the other hand, is such argument reducible to that form which holds good, however the facts may be. This seems very much like a reduction to absurdity of the validity of such reasoning, a paradox of the greatest difficulty presented for solution.

There can be no doubt of the solution of this problem. According to the central question of philosophy, how synthetical judgments *a priori* are possible. But antecedently to this comes the question how synthetical judgments in general, and still more generally, how synthetical reasoning is possible at all. When the answer to the general problem has been obtained, the particular one will be comparatively simple. This is the lock upon the door of philosophy.

All probable inference, whether induction or hypothesis, is inference from the parts to the whole. It is essentially the same, therefore, as statistical inference. Out of a bag of black and white beans I take a few handfulls, and from this sample I can judge approximately the proportions of black and white in the whole. This is identical with induction. Now we know upon what the validity of this inference depends. It depends upon the fact that in the long run, any one bean would be taken out as often as any other. For were this not so, the mean of a large number of results of such testings of the contents of the bag would not be precisely the ratio of the numbers of the two colors of beans in the bag. Now we may divide the question of the validity of induction into two parts: 1st, why of all inductions, premises for which occur, the generality should hold good, and 2d, why men are not fated always to light upon the small proportion of worthless inductions. Then, the first of these two questions is readily answered. For since all the members of any class are the same as all that are to be known; and since from any part of those which are to be known an induction is competent to the rest, in the long run any one member of a class will occur as the subject of a premise of a possible induction as often as any other, and, therefore, the validity of induction depends simply upon the fact that the parts make up and constitute the whole. This in its turn depends simply upon there being such a state of things that any general terms are possible. But it has been shown, p. 155, that being at all is being in general. And thus this part of the validity of induction depends merely on there being any reality.

From this it appears that we cannot say that the generality of inductions are true, but only that in the long run they approximate to truth. This is the truth of the [illegible] that the universality of an induction is only the analogue of [illegible]ality. Hence, also, it cannot be known an inductive conclusion [illegible]owever loosely we state it; we [illegible] that by accepting inductive conclusions, in the long run our errors balance one another. In fact, insurance companies proceed upon induction;—they do not know what will happen to this or that policyholder; they only know that they are secure in the long run.

The other question relative to the validity of induction, is why men are not fated always to light upon those inductions which are highly deceptive. The explanation of the former branch of the problem we have seen to be that there is something real. Now, since if there is anything real, then (on account of this reality consisting in the ultimate agreement of all men, and on account of the fact that reasoning from parts to whole, is the only kind of synthetic reasoning which men possess) it follows necessarily that a sufficiently long succession of inferences from parts to whole will lead men to a knowledge of it, so that in that case they cannot be fated on the whole to be thoroughly unlucky in their inductions. This second branch of the problem is in fact equivalent to asking why there is anything real, and thus its solution will carry the solution of the former branch one step further.

* Logic, Book 3, chap. 3, sec. 1.

† *Ibid.* Book 3, chap. 21, sec. 1. "I am convinced that any one accustomed to abstraction and analysis, who will fairly exert his faculties for the purpose, will, when his imagination has once learnt to entertain the notion, find no difficulty in conceiving that some one, for instance, of the many firmaments into which sidereal astronomy divides the universe, events may succeed one another at random, without any fixed law; nor can anything in our experience or mental nature constitute a sufficient, or indeed any, reason for believing that this is nowhere the case.

"Were we to suppose (what it is perfectly possible to imagine) that the present order of the universe were brought to an end, and that a chaos succeeded, in which there was no fixed succession of events, and the past gave no assurance of the future," &c.

* Boole (*Laws of Thought*, p. 370) has shown, in a very simple and elegant manner, that an *infinite* number of balls may have characters distributed in such a way, that from the characters of the balls already drawn, we could infer nothing in regard to that of the characters of the next one. The same is true of some arrangements of a finite number of balls, provided the inference takes place after a fixed number of drawings. But this does not invalidate the reasoning above, although it is an important fact without doubt.

dence with facts) consists solely in this, that when such premises are true, such a conclusion is generally true, then probability can mean nothing but the ratio of the frequency of occurrence of a specific event to a general one over it. In this sense of the term, it is plain that the probability of an inductive conclusion cannot be *deduced* from the premises; for from the inductive premises

$$S', S'', S''' \text{ are } M,$$
$$S', S'', S'''', \text{ are } P,$$

nothing follows deductively, except that any M, which is S', or S'', or S''' is P; or, less explicitly, that some M is P.

Thus, we seem to be driven to this point. On the one hand, no determination of things, no *fact*, can result in the validity of probable argument; nor, on the other hand, is such argument reducible to that form which holds good, however the facts may be. This seems very much like a reduction to absurdity of the validity of such reasoning; and a paradox of the greatest difficulty is presented for solution.

There can be no doubt of the importance of this problem. According to Kant, the central question of philosophy is "How are synthetical judgments *a priori* possible?" But antecedently to this comes the question how synthetical judgments in general, and still more generally, how synthetical reasoning is possible at all. When the answer to the general problem has been obtained, the particular one will be comparatively simple. This is the lock upon the door of philosophy.

All probable inference, whether induction or hypothesis, is inference from the parts to the whole. It is essentially the same, therefore, as statistical inference. Out of a bag of black and white beans I take a few handfulls, and from this sample I can judge approximately the proportions of black and white in the whole. This is identical with induction. Now we know upon what the validity of this inference depends. It depends upon the fact that in the long run, any one bean would be taken out as often as any other. For were this not so, the mean of a large number of results of such testings of the contents of the bag would not be precisely the ratio of the numbers of the two colors of beans in the bag. Now we may divide the question of the validity of induction into two parts: 1st, why of all inductions, premises for which occur, the generality should hold good, and 2d, why men are not fated always to light upon the small proportion of worthless inductions. Then, the first of these two questions is readily answered. For since all the members of any class are the same as all that are to be known; and since from any part of those which are to be known an induction is competent to the rest, in the long run any one member of a class will occur as the subject of a premise of a possible induction as often as any other, and, therefore, the validity of induction depends simply upon the fact that the parts make up and constitute the whole. This in its turn depends simply upon there being such a state of things that any general terms are possible. But it has been shown, p. 155, that being at all is being in general. And thus this part of the validity of induction depends merely on there being any reality.

From this it appears that we cannot say that the generality of inductions are true, but only that in the long run they approximate to the truth. This is the truth of the statement, that the universality of an inference from induction is only the analogue of true universality. Hence, also, it cannot be said that we know an inductive conclusion to be true, however loosely we state it; we only know that by accepting inductive conclusions, in the long run our errors balance one another. In fact, insurance companies proceed upon induction;—they do not know what will happen to this or that policy-holder; they only know that they are secure in the long run.

The other question relative to the validity of induction, is why men are not fated always to light upon those inductions which are highly deceptive. The explanation of the former branch of the problem we have seen to be that there is something real. Now, since if there is anything real, then (on account of this reality consisting in the ultimate agreement of all men, and on account of the fact that reasoning from parts to whole, is the only kind of synthetic reasoning which men possess) it follows necessarily that a sufficiently long succession of inferences from parts to whole will lead men to a knowledge of it, so that in that case they cannot be fated on the whole to be thoroughly unlucky in their inductions. This second branch of the problem is in fact equivalent to asking why there is anything real, and thus its solution will carry the solution of the former branch one step further.

The answer to this question may be put into a general and abstract, or a special detailed form. If men were not to be able to learn from induction, it must be because as a general rule, when they had made an induction, the order of things (as they appear in experience), would then undergo a revolution. Just herein would the unreality of such a universe consist; namely, that the order of the universe should depend on how much men should know of it. But this general rule would be capable of being itself discovered by induction; and so it must be a law of such a universe, that when this was discovered it would cease to operate. But this second law would itself be capable of discovery. And so in such a universe there would be nothing which would not sooner or later be known; and it would have an order capable of discovery by a sufficiently long course of reasoning. But this is contrary to the hypothesis, and therefore that hypothesis is absurd. This is the particular answer. But we may also say, in general, that if nothing real exists, then, since every question supposes that something exists—for it maintains its own urgency—it supposes only illusions to exist. But the existence even of an illusion is a reality; for an illusion affects all men, or it does not. In the former case, it is a reality according to our theory of reality; in the latter case, it is independent of the state of mind of any individuals except those whom it happens to affect. So that the answer to the question, Why is anything real? is this: That question means, "supposing anything to exist, why is something real?" The answer is, that that very existence is reality by definition.

All that has here been said, particularly of induction, applies to all inference from parts to whole, and therefore to hypothesis, and so to all probable inference.

Thus, I claim to have shown, in the first place, that it is possible to hold a consistent theory of the validity of the laws of ordinary logic.

But now let us suppose the idealistic theory of reality, which I have in this paper taken for granted to be false. In that case, inductions would not be true unless the world were so constituted that every object should be presented in experience as often as any other; and further, unless we were so constituted that we had no more tendency to make bad inductions than good ones. These facts might be explained by the benevolence of the Creator; but, as has already been argued, they could not explain, but are absolutely refuted by the fact that no state of things can be conceived in which probable arguments should not lead to the truth. This affords a most important argument in favor of that theory of reality, and thus of those denials of certain faculties from which it was deduced, as well as of the general style of philosophizing by which those denials were reached.

Upon our theory of reality and of logic, it can be shown that no inference of any individual can be thoroughly logical without certain determinations of his mind which do not concern any one inference immediately; for we have seen that that mode of inference which alone can teach us anything, or carry us at all beyond what was implied in our premises—in fact, does not give us to know any more than we knew before; only, we know that, by faithfully adhering to that mode of inference, we shall, on the whole, approximate to the truth. Each of us is an insurance company, in short. But, now, suppose that an insurance company, among its risks, should take one exceeding in amount the sum of all the others. Plainly, it would then have no security whatever. Now, has not every single man such a risk? What shall it profit a man if he shall gain the whole world and lose his own soul? If a man has a transcendent personal interest infinitely outweighing all others, then, upon the theory of validity of inference just developed, he is devoid of all security, and can make no valid inference whatever. What follows? That logic rigidly requires, before all else, that no determinate fact, nothing which can happen to a man's self, should be of more consequence to him than everything else. He who would not sacrifice his own soul to save the whole world, is illogical in all his inferences, collectively. So the social principle is rooted intrinsically in logic.

That being the case, it becomes interesting to inquire how it is with men as a matter of fact. There is a psychological theory that man cannot act without a view to his own pleasure. This theory is based on a falsely assumed subjectivism. Upon our principles of the objectivity of knowledge, it could not be based, and if they are correct it is reduced to an absurdity. It seems to me that the usual opinion of the selfishness of man is based in large measure upon this false theory. I do not think that the facts

bear out the usual opinion. The immense self-sacrifices which the most wilful men often make, show that wilfulness is a very different thing from selfishness. The care that men have for what is to happen after they are dead, cannot be selfish. And finally and chiefly, the constant use of the word "*we*"—as when we speak of our possessions on the Pacific—our destiny as a republic—in cases in which no personal interests at all are involved, show conclusively that men do not make their personal interests their only ones, and therefore may, at least, subordinate them to the interests of the community.

But just the revelation of the possibility of this complete self-sacrifice in man, and the belief in its saving power, will serve to redeem the logicality of all men. For he who recognizes the logical necessity of complete self-identification of one's own interests with those of the community, and its potential existence in man, even if he has it not himself, will perceive that only the inferences of that man who has it are logical, and so views his own inferences as being valid only so far as they would be accepted by that man. But so far as he has this belief, he becomes identified with that man. And that ideal perfection of knowledge by which we have seen that reality is constituted must thus belong to a community in which this identification is complete.

This would serve as a complete establishment of private logicality, were it not that the assumption that man or the community (which may be wider than man) shall ever arrive at a state of information greater than some definite finite information, is entirely unsupported by reasons. There cannot be a scintilla of evidence to show that at some time all living beings shall not be annihilated at once, and that forever after there shall be throughout the universe any intelligence whatever. Indeed, this very assumption involves itself a transcendent and supreme interest, and therefore from its very nature is unsusceptible of any support from reasons. This infinite hope which we all have (for even the atheist will constantly betray his calm expectation that what is Best will come about) is something so august and momentous, that all reasoning in reference to it is a trifling impertinence. We do not want to know what are the weights of reasons *pro* and *con*.—that is, how much *odds* we should wish to receive on such a venture in the long run—because there is no long run in the case; the question is single and supreme, and ALL is at stake upon it. We are in the condition of a man in a life and death struggle: if he have not sufficient strength, it is wholly indifferent to him how he acts, so that the only assumption upon which he can act rationally is the hope of success. So this sentiment is rigidly demanded by logic. If its object were any determinate fact, any private interest, it might conflict with the results of knowledge and so with itself; but when its object is of a nature as wide as the community can turn out to be, it is always a hypothesis uncontradicted by facts and justified by its indispensibleness for making any action rational.

THE LAÖKOÖN AS A WORK OF ART.

[Translated from the German of Goethe by E. S. MORGAN.]

[The editor takes pleasure in being able to offer in this number two of the most remarkable interpretations of Art-work that exist in all literature. Winckelmann and Goethe stand unrivalled among moderns for their appreciation of classic art. Goethe does more than recognize *classic* art—he esteems all styles of art each in its true spirit and time. This has been shown in the essay on Da Vinci's "Last Supper." The intensity of Winckelmann's admiration of the Classic art was well shown in the article on the "Torso" published in the last number of the *Journal*. But his appreciation extends only to outlines, and he is filled with disgust when he sees the paintings of the greatest Italians. Color does not distract his attention from the outlines. He who would see the beautiful in classic art must practise the same abstraction from color as well as from the action portrayed. Let him look at Correggio's "Night," for example; first, forgetting the outline in the magic of the coloring, and secondly, confining his attention merely to the forms, and he will see how beautiful and how ugly a picture may be, when viewed from two different standpoints.—EDITOR.]

A genuine work of art, like a work of nature, remains forever inexhaustible by the understanding. It is looked at, it impresses us, it produces an effect, but cannot be wholly comprehended, much less can its essence, its real value, be expressed in words.