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ART. XXVI.—*Note on the Sensation of Color;* by C. S. PEIRCE.

IT may, perhaps, be worth while to notice a few consequences of three theories concerning color which are usually regarded with some favor.

First hypothesis.—The appearance of every mixture of lights depends solely on the appearances of the constituents, without distinction of their physical constitution. This I believe is established.

Second hypothesis.—Every sensation of light is compounded of not more than three independent sensations, which do not influence one another. This is Young's theory. It follows that, if we denote the units of the three elementary sensations by i, j, and k, every sensation of light may be represented by an expression of the form,

$$Xi+Yj+Zk.$$

Third hypothesis.—The intensity of a sensation is proportional to the logarithm of the strength of the excitation, the barely perceptible excitation being taken of unit strength. Negative logarithms are to be taken as *zero*. This is Fechner's law. It is known to be approximately and only approximately true, for the sensation of light. From this it follows that, if x, y, z be the relative proportions of a mixture of three lights giving the elementary sensations i, j, k, the sensation produced by the mixture is

$$I \log x.i + J \log y.j + K \log z.k,$$

where I, J, K, are three constants.

From these principles, it follows that if a light giving any sensation such as that just written have its intensity increased in any ratio r, the resulting sensation will be,

$$\mathrm{I}\log rx.i+\mathrm{J}\log ry.j+\mathrm{K}\log rz.k=$$
$$\mathrm{I}\log x.i+\mathrm{J}\log y.j+\mathrm{K}\log z.k+\log r\,(\mathrm{I}i+\mathrm{J}j+\mathrm{K}k).$$

Thus, the result of increasing the brilliancy of any light must be to add to the sensation a variable amount of a constant sensation, $\mathrm{I}i+\mathrm{J}j+\mathrm{K}k$. And all very bright light will tend toward the same color, which may therefore be called the *color of brightness*. Moreover, if the three primary colors be mixed in the proportions which each by itself is just perceptible, the sensation produced will be

$$\log r\,(\mathrm{I}i+\mathrm{J}j+\mathrm{K}k),$$

and can only differ by more or less.

Now I find, in fact, that all colors are yellower when brighter. If two contiguous rectangular spaces, illuminated with the same homogeneous light, uniformly over each, but unequally in the two, they will appear of different colors.

If both are *red* the brighter will appear *scarlet;*
" " *green* " " *yellowish;*
" " *blue* " " *greenish;*
" " *violet* " " *blue.*

If we have the means of varying the wave-length of the light which illuminates the fainter rectangle, we can improve the match between the two, by bringing the fainter toward the yellow. Such motions will converge toward a certain point of the spectrum which they will never cross,—a point a little more refrangible than D and having a wave length of $582\cdot10^{-6}$ mm., according to Ångström's map. If both rectangles be illuminated with this light, the fainter appears white or even violet, but if it be varied in wave-length with a view of improving the match, it will be found to return to the same point with the utmost precision.

It appears, therefore, that, if our hypotheses are correct, the color $\log r\,(\mathrm{I}i+\mathrm{J}j+\mathrm{K}k)$ is like that of the spectrum at $\lambda=582$, only that it contains less blue or violet and is consequently of greater chromatic intensity.

It further follows from Fechner's law that, if any light be gradually reduced in brightness, one element of the sensation will disappear after another; and that when very faint it will exhibit only one primary color, which is the one which it contains in greatest proportion relatively to the proportion in the light which has the color of brightness. Now, although this does not seem to be exactly the case, yet we do get some approximation to it. It is true that any light whatever, when sufficiently faint, appears white, owing to the self-luminosity of the retina. We cannot, therefore, unfortunately, get sight of the primary colors by reducing the light of three parts of the spectrum. But we may, as has often been sug-

gested, make use of the principles of contrast. If any red spectral light be sufficiently reduced, it will perfectly match any less refrangible light. We may, therefore, say that a faint spectral red in contrast with a bright light of the same kind, excites with approximate purity one of the elementary sensations. The same thing is true of the violet; and therefore a rich violet may be taken as another primary color. In my book entitled Photometric Researches, the printing of which is nearly complete, I show reason to think that the pure green has a wave length intermediate between E and b. A faint green of this sort contrasted with a bright one appears as a very bluish green, and this may therefore be supposed to be the third primary color.

We have seen that it results from the theory that an increase in the brilliancy of any light adds to the sensation nothing of the peculiar color of that light, but only a certain amount of the color of brightness. If this be the fact, then the photometric sensibility of the eye should be the same for all colors. In order to ascertain whether this is so or not, I have made a series of determinations of my photometric probable error. Each determination was based on twenty-eight comparisons of two parts of the same colored disk. Since there were two unknown quantities, namely, the relative brightness of the two surfaces compared, and an instrumental constant, it follows that only twenty-six observations were effective for determining the probable error. Let R be my photometric probable error of a single comparison. Then the probable error of a single determination of R (which we may denote by r) would be $\frac{\cdot51}{\sqrt{26}}\times\mathrm{R}$, or say $\frac{1}{10}$R. Having made a considerable number of such determinations of R, with different colored disks, let us ascertain their probable error from their discrepancies, considering them as so many independent observations of the same unknown quantity, and denote this probable error by r'. If, then, R really is the same for all colors, we should have

$$r'=r,$$

or, at least, the difference should not exceed ρ, the probable error of r'; which may be calculated by the formula

$$\rho=\frac{\cdot51}{\sqrt{m}\,r'},$$

where m is the number of sets of experiments diminished by 1. If, on the other hand, R varies with the different colors, and not merely accidentally, r' should have a larger value. The following are the values I obtained for R, the sum of the brightness of the two surfaces compared being taken as unity.

		R.	Diff. from mean.
Feb. 6.	White	·0041	+·0001
	Red, just before C	·0046	+·0006
	Chrome-yellow, A 2	·0032	−·0008
Feb. 7.	Red, just before C	·0040	±·0000
	Staat's emerald green	·0046	+·0006
	Carmine, B	·0044	+·0004
	Chrome-yellow, A 1	·0037	−·0003
	Purple, Hoffmann's violet RRR	·0033	−·0007
Feb. 13.	Red, just before C	·0048	+·0008
	Green, complementary to carmine	·0034	−·0006
	Blue violet, No. 2	·0048	+·0008
	Yellow, A 1, mixed with black	·0032	−·0008
	Mean	·0040	

After these experiments, the method of observing was changed, and I obtained the following:

Feb. 14.	White window-shade, ill. by sun	·0030	−·0002
	Brown	·0030	−·0002
	Greenish sky-blue	·0037	+·0005
	Very reddish purple	·0028	−·0004
	Yellow orange	·0032	±·0000
Feb. 15.	"Fundamental green of Müller"	·0030	−·0002
	Vermilion, half between C and D	·0034	+·0002
	Violet	·0032	±·0000
	Yellow	·0036	+·0002
	Mean	·0032	

We thus get from the

first twelve determinations, $r = \cdot 00040$, $r' = \cdot 00048$, $\frac{r'}{r} = 1\cdot 2$

last nine determinations, $r = \cdot 00032$, $r' = \cdot 00019$, $\frac{r'}{r} = 0\cdot 6$

and from the weighted mean, $\frac{r'}{r} = \cdot 96$, so that it appears from these experiments that the photometric susceptibility of the eye is the same for all colors. The result is, however, uncertain, because it may be that R is chiefly due to other sources of error than the limitation of sensibility; still, the experiments show as small a value of R as is usually obtained. I shall endeavor, by further observations, to obtain a conclusive result.

A further consequence of our hypotheses will be reached by differentiating the expression for a light-sensation. We have

$$d(\mathrm{I}\log x.i+\mathrm{J}\log y.j+\mathrm{K}\log z.k)=\frac{1}{x}dx.\mathrm{I}i+\frac{1}{y}dy.\mathrm{J}j+\frac{1}{z}dz.\mathrm{K}k.$$

Now, as x, y and z all exceed unity, the differential is greater the

nearer unity x, y and z are. Hence, since the variation of the proportions of the primary colors with a variation of postion in the normal spectrum is uniform,* it follows that the change of color of the normal spectrum should be most rapid about $\lambda=582$, as it of course is. It is also obvious that if the total quantities of the three colors are nearly the same in different parts of the spectrum (I here refer to these colors not as really objective, but as measured in the usual objective way) then the part about $\lambda=582$ must be the brightest, another familiar fact.

I may observe that there is a modification of our formula for a sensation of light, which probably better represents the relations of the sensations. Writing, in the first place,

$$\mathfrak{i}=\mathrm{I}i \qquad \mathfrak{j}=\mathrm{J}j \qquad \mathfrak{k}=\mathrm{K}k$$

the formula is

$$\log x.\mathfrak{i}+\log y.\mathfrak{j}+\log z.\mathfrak{k}.$$

This loses its validity when any of the logarithms become negative. If z is the smallest of the three quantities, we may substitute

$$\mathrm{X}=\frac{x}{z} \qquad \mathrm{Y}=\frac{y}{z}$$

and the formula becomes

$$\log \mathrm{X}.\mathfrak{i}+\log \mathrm{Y}.\mathfrak{j}+\log z(\mathfrak{i}+\mathfrak{j}+\mathfrak{k}).$$

When x or y is smallest there will be two other formulæ. Now, as the variation in the brilliancy of the light affects only the last term of the last formula, and not the first two depending on X and Y, it is more than probable that the eye is habituated to separating the element of sensation which this last term represents, and which is continually changing its values, from the rest which remains constant. It is, therefore, likely that the classification of light into three kinds, according as the *violet*, the *red*, or the *green*, is contained in the smallest proportion, is one which has a relation to the natural powers of discrimination.

My observations have been made with an instrument for which I am indebted to the liberality of the trustees of the Bache Fund. I shall describe it on the occasion of publishing some work of a more serious character. The colored disks made use of were very kindly lent me by Professor Rood.

* I will show this in a note in the next number of this Journal.