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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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I have made use of a simple plate of glass having a breadth of 1 decimetre and a length of 1·5. Semi-silvered glass has great reflecting-power, and yet remains very transparent; it presents merely a slight brown shade.

It is known that M. Foucault advised the investing with this semi-silvering the objectives of telescopes for viewing the sun, in order to arrest nearly the whole of the rays of obscure heat*.

When the glass is silvered, washed, and dried, the silver (which might be removed by the slightest friction) is fixed by coating the glass with a transparent varnish. For this purpose it is heated to about 40°, and the following varnish is poured upon the silvered face:—alcohol, 100 cubic centims.; mastic tears, 10 grams. The thin film of resin which adheres is very transparent and has a very even surface. The reflecting-power of the glass is slightly diminished, but is still sufficient. The silvered surface could be covered with another glass plate; but this would give rise to multiple reflection, which is avoided by using the varnish.

The glass is then fixed, when the varnish is dry, by one of its edges, in a nipper fitted to a foot, permitting various inclinations to the horizontal to be given to the glass; if the object to be drawn is vertical, the angle of 45° should evidently be preferred. The paper on which the drawing is to be made is fixed beneath. It is indispensable that above the glass a sight-piece be placed, to give the eye a perfectly fixed position. If the object has a strong relief, the images of the various parts are formed at different distances behind the glass, and the perspective changes with the position of the eye; it is the same with the coincidence of the points on the paper and the different parts of the image to be drawn. The sight-piece consists simply of a small slip of blackened cardboard pierced with a small aperture; this can be supported by the apparatus which sustains the glass.

If the illumination of the object, placed at a suitable height and distance before the inclined glass, be in a certain correspondence with that of the drawing-paper, the image of the object, the pencil, and even the line of the drawing as it is being executed can all be seen at the same time without any fatigue. The conditions of the illumination can be easily realized by the aid of screens or shutters.

The advantage of this camera lucida over that generally used arises from the reflection taking place over a large surface, which gives more intensity, and especially from the circumstance that the simultaneous visibility of the pencil and the image is independent of the position of the eye of the observer, depending only on certain conditions of illumination which can be easily regulated before commencing the execution of the drawing. It would be easy, by taking two parallel glass plates, one semi-silvered and the other having received a thick coat of silver, to make a camera lucida that could be fitted to microscopes, and more convenient than those at present employed.

* The same arrangement would be very advantageously employed in photographic enlarging-apparatus, where the solar heat sometimes cracks the plates.

NOTE ON THE SENSATION OF COLOUR. BY C. S. PEIRCE.

It may, perhaps, be worth while to notice a few consequences of three theories concerning colour which are usually regarded with some favour.

First Hypothesis.—The appearance of every mixture of lights depends solely on the appearances of the constituents, without distinction of their physical constitution. This I believe is established.

Second Hypothesis.—Every sensation of light is compounded of not more than three independent sensations, which do not influence one another. This is Young's theory. It follows that, if we denote the units of the three elementary sensations by i, j, and k, every sensation of light may be represented by an expression of the form

$$\mathrm{X}i+\mathrm{Y}j+\mathrm{Z}k.$$

Third hypothesis.—The intensity of a sensation is proportional to the logarithm of the strength of the excitation, the barely perceptible excitation being taken as of unit strength. Negative logarithms are to be taken as *zero*. This is Fechner's law. It is known to be approximately (and only approximately) true for the sensation of light. From this it follows that, if x, y, z be the relative proportions of a mixture of three lights giving the elementary sensations i, j, k, the sensation produced by the mixture is

$$\mathrm{I}\log x\,.\,i+\mathrm{J}\log y\,.\,j+\mathrm{K}\log z\,.\,k,$$

where I, J, K, are three constants.

From these principles it follows that, if a light giving any sensation such as that just written have its intensity increased in any ratio r, the resulting sensation will be

$$\mathrm{I}\log rx\,.\,i+\mathrm{J}\log ry\,.\,j+\mathrm{K}\log rz\,.\,k$$
$$=\mathrm{I}\log x\,.\,i+\mathrm{J}\log y\,.\,j+\mathrm{K}\log z\,.\,k+\log r\,(\mathrm{I}\,i+\mathrm{J}j+\mathrm{K}k).$$

Thus the result of increasing the brilliancy of any light must be to add to the sensation a variable amount of a constant sensation, $\mathrm{I}\,i+\mathrm{J}\,j+\mathrm{K}\,k$; and all very bright light will tend toward the same colour, which may therefore be called the *colour of brightness*. Moreover, if the three primary colours be mixed in the proportions in which each by itself is just perceptible, the sensation produced will be

$$\log r\,(\mathrm{I}\,i+\mathrm{J}\,j+\mathrm{K}\,k),$$

and can only differ by more or less.

Now I find in fact that all colours are yellower when brighter. If two continuous rectangular spaces be illuminated with the same homogeneous light, uniformly over each, but unequally in the two, they will appear of different colours.

If both are *red*, the brighter will appear *scarlet*;
,, ,, *green* ,, ,, *yellowish*;
,, ,, *blue* ,, ,, *greenish*;
,, ,, *violet* ,, ,, *blue*.

If we have the means of varying the wave-length of the light which illuminates the fainter rectangle, we can improve the match between the two, by bringing the fainter toward the yellow. Such motions will converge toward a certain point of the spectrum which they will never cross—a point a little more refrangible than D and having a wave-length of 582.10^{-6} mm., according to Ångström's map. If both rectangles be illuminated with this light, the fainter appears white or even violet; but if it be varied in wave-length with a view of improving the match, it will be found to return to the same point with the utmost precision.

It appears, therefore, that, if our hypotheses are correct, the colour $\log r\,(\mathrm{I}\,i+\mathrm{J}\,j+\mathrm{K}\,k)$ is like that of the spectrum at $\lambda=582$, only that it contains less blue or violet, and is consequently of greater chromatic intensity.

It further follows from Fechner's law that, if any light be gradually reduced in brightness, one element of the sensation will disappear after another—and that when very faint it will exhibit only one primary colour, which is the one which it contains in greatest proportion relatively to the proportion in the light which has the colour of brightness. Now although this does not seem to be exactly the case, yet we do get some approximation to it. It is true that any light whatever, when sufficiently faint, appears white, owing to the self-luminosity of the retina. We cannot therefore, unfortunately, get sight of the primary colours by reducing the light of three parts of the spectrum. But we may, as has often been suggested, make use of the principle of contrast. If any red spectral light be sufficiently reduced, it will perfectly match any less-refrangible light. We may therefore say that a faint spectral red in contrast with a bright light of the same kind, excites with approximate purity one of the elementary sensations. The same thing is true of the violet; and therefore a rich violet may be taken as another primary colour. In my book entitled 'Photometric Researches,' the printing of which is nearly complete, I show reason to think that the pure green has a wave-length intermediate between E and *b*. A faint green of this sort contrasted with a bright one appears as a very bluish green; and this may therefore be supposed to be the third primary colour.

We have seen that it results from the theory that an increase in the brilliancy of any light adds to the sensation nothing of the peculiar colour of that light, but only a certain amount of the colour of brightness. If this be the fact, then the photometric sensibility of the eye should be the same for all colours. In order to ascertain whether this is so or not, I have made a series of determinations of my photometric probable error. Each determination was based on twenty-eight comparisons of two parts of the same-coloured disk. Since there were two unknown quantities

(namely, the relative brightness of the two surfaces compared, and an instrumental constant), it follows that only twenty-six observations were effective for determining the probable error. Let R be my photometric probable error of a single comparison. Then the probable error of a single determination of R (which we may denote by r) would be $\frac{\cdot 51}{\sqrt{26}}\times \mathrm{R}$, or say $\frac{1}{10}\mathrm{R}$. Having made a considerable number of such determinations of R with different coloured disks, let us ascertain their probable error from their discrepancies, considering them as so many independent observations of the same unknown quantity, and denote this probable error by r'. If, then, R really is the same for all colours, we should have

$$r'=r;$$

or at least the difference should not exceed ρ, the probable error of r', which may be calculated by the formula

$$\rho=\frac{\cdot 51}{\sqrt{mr'}},$$

where m is the number of sets of experiments diminished by 1. If, on the other hand, R varies with the different colours, and not merely accidentally, r' should have a larger value. The following are the values I obtained for R, the sum of the brightness of the two surfaces compared being taken as unity:—

		R.	Diff. from mean.
Feb. 6.	White	·0041	+·0001
	Red, just before C.............	·0046	+·0006
	Chrome-yellow, A 2	·0032	−·0008
Feb. 7.	Red, just before C............	·0040	±·0000
	Staat's emerald green	·0046	+·0006
	Carmine, B	·0044	+·0004
	Chrome-yellow, A 1	·0037	—·0003
	Purple, Hoffmann's violet RRR	·0033	—·0007
Feb. 13.	Red, just before C............	·0048	+·0008
	Green, complementary to carmine	·0034	—·0006
	Blue-violet, No. 2............	·0048	+·0008
	Yellow, A 1, mixed with black..	·0032	—·0008
	Mean	·0040	

After these experiments the method of observing was changed, and I obtained the following:—

		R.	Diff. from mean.
Feb. 14.	White window-shade, ill. by sun	·0030	—·0002
	Brown	·0030	—·0002
	Greenish sky-blue............	·0037	+·0005
	Very reddish purple	·0028	—·0004
	Yellow-orange	·0032	±·0000
Feb. 15.	"Fundamental green of Müller"	·0030	—·0002
	Vermilion, half between C and D	·0034	+·0002
	Violet......................	·2032	±·0000
	Yellow	·0036	+·0002
	Mean........	·0032	

We thus get from the

first twelve determinations, $r = \cdot 00040$, $r' = \cdot 00048$, $\frac{r'}{r} = 1\cdot 2$,

last nine determinations, $r = \cdot 00032$, $r' = \cdot 00019$, $\frac{r'}{r} = 0\cdot 6$,

and from the weighted mean, $\frac{r'}{r} = \cdot 96$; so that it appears from these experiments that the photometric susceptibility of the eye is the same for all colours. The result, however, is uncertain, because it may be that R is chiefly due to other sources of error than the limitation of sensibility; still the experiments show as small a value of R as is usually obtained. I shall endeavour, by further observations, to obtain a conclusive result.

A further consequence of our hypotheses will be reached by differentiating the expression for a light-sensation. We have

$$d(\mathrm{I}\log x \,.\, i + \mathrm{J}\log y \,.\, j + \mathrm{K}\log z \,.\, k) = \frac{1}{x}dx \,.\, \mathrm{I}i + \frac{1}{y}dy \,.\, \mathrm{J}j + \frac{1}{z}dz \,.\, \mathrm{K}k.$$

Now, as x, y, and z all exceed unity, the differential is greater the nearer unity x, y, and z are. Hence, since the variation of the proportions of the primary colours with a variation of position in the normal spectrum is uniform, it follows that the change of colour of the normal spectrum should be most rapid about $\lambda = 582$, as it of course is. It is also obvious that, if the total quantities of the three colours are nearly the same in different parts of the spectrum (I here refer to these colours not as really objective, but as measured in the usual objective way), then the part about $\lambda = 582$ must be the brightest—another familiar fact.

I may observe that there is a modification of our formula for a sensation of light, which probably better represents the relations of the sensations. Writing, in the first place,

$$\mathfrak{i} = \mathrm{I}i, \quad \mathfrak{j} = \mathrm{J}j, \quad \mathfrak{k} = \mathrm{K}k,$$

the formula is

$$\log x \,.\, \mathfrak{i} + \log y \,.\, \mathfrak{j} + \log z \,.\, \mathfrak{k}.$$

This loses its validity when any of the logarithms become negative. If z is the smallest of the three quantities, we may substitute

$$\mathrm{X} = \frac{x}{z}, \quad \mathrm{Y} = \frac{y}{z};$$

and the formula becomes

$$\log \mathrm{X} \,.\, \mathfrak{i} + \log \mathrm{Y} \,.\, \mathfrak{j} + \log z\,(\mathfrak{i} + \mathfrak{j} + \mathfrak{k}).$$

When x or y is smallest there will be two other formulæ. Now, as the variation in the brilliancy of the light affects only the last term of the last formula, and not the first two depending on X and Y, it is more than probable that the eye is habituated to separating the element of sensation which this last term represents, and which is continually changing its values, from the rest which remains constant. It is therefore likely that the classification of light into three kinds, according as the *violet*, the *red*, or the *green* is contained in the smallest proportion, is one which has a relation to the natural powers of discrimination.—Silliman's *American Journal*, April 1877.

ON ACCIDENTAL DOUBLE REFRACTION. BY J. MACÉ.

The phenomena of accidental double refraction produced by compression have, since their discovery by Brewster, given rise to numerous researches, while those produced by tempering have been but little studied. Since the discovery of the latter by Seebeck, we find but little concerning them except the papers of M. de Luynes, in which the author studies chiefly the mechanical properties acquired through an energetic tempering, and those of M. Mascart on the tempering of cast plates of glass (*Journal de Physique*, 1876), in which the author applies himself especially to the practical side of the question.

The aim of the investigation which I commenced some months since was, on the contrary, to discover what are the laws which govern this phenomenon, and to search out the analogies or the differences which might exist between that which results from tempering and the other cases of accidental double refraction. I therefore proposed to myself to study plates of simple geometrical forms, especially rectangular and square, differing in dimensions, thickness, and constitution. If it be remarked that Wertheim, by the study of the regular compression of glass, has been able to show that the differences in the course are proportional to the pressures exerted, it will be seen that this led me to investigate the distribution of the differences of the course in the various plates submitted to experiment.

The arrangement employed was very simple:—A car carrying the plate permitted it to be displaced parallel to a micrometer formed, as in M. Jamin's apparatus, of two parallel fine threads and, besides, of a third, horizontal thread, rendered necessary by the curved form often (and particularly in square plates) affected by the fringes to be studied. As always, the polarizer was placed at 45°; the analyzer was alternately transverse and parallel, so as to exhibit fringes corresponding to differences of course varying by half wavelengths. The compensator was suppressed, it having the inconvenience of deforming the fringes and often rendering them difficult to observe; but it was made use of for the rectangular plates, in order to measure the central difference of course. For the illumination the burner of Laurent's saccharimeter was used. The position of each fringe was measured to within 0·1 of a millimetre at the least.

In studying the distribution of the course-differences along a line parallel to and equally distant from two of the sides of the plate, the phenomenon could in all cases be represented by a formula