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MADE UNDER THE DIRECTION OF THE LATE

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PHOTOMETRIC RESEARCHES.

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PHOTOMETRIC RESEARCHES.

BY

C. S. PEIRCE.

MADE IN THE YEARS 1872-1875.

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PREFACE.

The present investigation, as the title shows, forms part of the work of the Harvard College Observatory under the directorship of Professor Winlock. The immediate author of it is an Assistant in the United States Coast Survey; and, in making it, he availed himself, with the sanction of the Superintendent, of the services of his aids.

During the year 1871, in consequence of an arrangement between the two institutions, the undersigned was directed by the Superintendent of the Coast Survey to report to Professor Winlock, at Harvard College Observatory for duty as his assistant. After some months, it was decided to procure for the Observatory a Zöllner's astrophotometer, with the design of attaching it to the great equatorial and so comparing the scales of star-magnitudes of Struve and the Durchmusterung. When the instrument was received, the undersigned was requested by Professor Winlock to prepare a plan of work with it. The plan of observations described in this volume was accordingly reported and accepted. The writer was then directed to execute the work, but before much was done he was ordered to Washington to take temporary charge of the Coast Survey office. However, under a special agreement, the work was continued, the instrument being carried first to Washington and afterwards to several pendulum stations. According to this agreement, reports were made from time to time by the undersigned to Professor Winlock of the progress of the work. Upon the proposal of the undersigned some changes were made in the plan of the work, but Professor Winlock very considerately abstained from all interference, except in the case of a single suggestion which mechanical conditions rendered it impossible to carry out.

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It having become the duty of the writer to prepare a catalogue of latitude-stars and also a list of *errata* to Heis's catalogue for use in the Coast Survey*, it was found that by connecting these matters with the photometric researches some otherwise unoccupied time of some young aids might be used in both directions; and the plan of the work was consequently enlarged so as to include the historical researches herein contained. This part of the work was particularly facilitated by the intelligent assistance of Mr. Henry Farquhar of the Coast Survey service. Upon the completion of the whole work, it was submitted to and accepted by Professor Winlock as Director of the Observatory of Harvard College; but since his death the whole has been rewritten, all the computations revised, and many essential improvements made.

* Coast Survey Report for 1873. Appendices 14 and 15.

CHARLES S. PEIRCE.

PHOTOMETRIC RESEARCHES.

BY C. S. PEIRCE.

CHAPTER I.

THE SENSATION OF LIGHT.

When a point is emitting unpolarized and homogeneous undulations equally in all directions, its state may be defined by two numbers; as, for instance by the wave-length of the undulations and their amplitude at a certain distance from the point. If the light is not homogeneous, indefinitely more numbers will be required to define it. But when a point upon the retina is illuminated, just three numbers are in every case requisite to define the sensation produced. In other words, light is a triple sensation.

Since we have not yet succeeded in getting a clear general conception of any relation between different sensations, except that of more or less, it follows that when we have said that the sensation of light has three elements (arbitrarily taken as primary) we have gone as far toward describing that sensation as the present state of our ideas enables us to do.

When two lights which present precisely the same appearance separately, fall at once on the same point of the retina, without interference, the light is said to be doubled. From this convention we obviously deduce the principle that the brightness of two lights which in every other respect are alike are proportional to their energies. But this does not necessarily follow with regard to light of different wave-lengths.

Suppose that there are three sorts of measurement which we can apply to light, by the first of which we ascertain a quantity called its X, by the second a quantity called its Y, and by the third a quantity called its Z, so that if we know that

$$X = a \qquad Y = b \qquad Z = c,$$

we have completely determined the sensation. Then it will not be necessary to measure X, Y, and Z directly, but we may measure any three independent functions of these three quantities and we shall thus have three independent equations which will equally serve to define the sensation. There are therefore an infinite variety of sets of three quantities which will serve to define the sensation of light.

Light considered purely as something in the external world may be called *noumenal light*. Light considered as an appearance, and as a function of the sensation, such that it is measured by the convention just mentioned, may be termed *phenomenal light*. Photometry generally concerns phenomenal light; and in these researches, I shall nowhere touch upon the question of how the noumenal light of stars is constituted, (as to the difference of their spectra, for example), but shall confine myself to considering how it appears.

If the light A precisely matches the light A' in appearance, and the light B precisely matches the light B', then the mixture of A and B will precisely match the mixture A' and B'. This is by no means a self evident proposition, for as two lights may have precisely the same appearance and yet a totally different noumenal constitution, it might very well be that the effect of mixing should depend on something not represented in the phenomenal light.

This fact, that the appearance of a mixture of lights is determined entirely by the appearance of its constituents, renders it convenient to denote an appearance of light by an expression of the form

$$xX + yY + zZ$$

where x, y, and z are variable numbers, and X, Y, Z, are three different fixed lights arbitrarily chosen, subject to the condition that no one shall be a linear function of the other two. It is found possible to choose X, Y, and Z so that x, y, and z need never be negative, and so that nearly every positive system of values shall represent some sensation which may actually be experienced. When these conditions are satisfied, X, Y, and Z are a crimson red, an emerald green, and a blue or violet. We may denote these lights by R, G, and B, respectively.

Since phenomenal light is a triple quantity, the points of a solid are just adequate to represent it. Suppose a system of Cartesian coördinates, and let R, G, and B be measured along the three axes. Then we shall have a triangular pyramid having darkness for its apex and every point within it representing a light. Then to find the point which represents the light resulting from the mixture of two lights, draw lines from the points representing these lights to the apex, complete the parallelogram and the fourth angle will be the point sought. If the pyramid of light be cut by a plane, the points upon the triangular section so obtained will be adequate to represent the colour without the intensity of every kind of light. We may then imagine each point to be weighted proportionally to the brightness of the light; and in this case the centre of gravity of two points represents the colour of the mixture of the two lights represented by the two weighted points, while the brightness of the mixture is equal to the sum of the two weights. Since the inclinations of the three axes of our system of Cartesian

coördinates as well as the units of length are entirely arbitrary, we have precisely the degree of indeterminacy which is requisite to enable us to represent upon the plane section made as above, any three lights by a triangle of any size and shape. We may therefore begin by assuming any three points upon a plane to represent any three lights of different colours and then fill in the other colours according to the rule of centres of gravity. This is called Newton's Diagram.

If the colours of the spectrum be laid down on such a diagram, then, as Maxwell has shown*, that whole portion of the spectrum between C and a point considerably beyond F, appears as two straight lines. These lines meet near E; and there the corner is a little rounded off, but not more I suspect than may be accounted for by the impurity of the spectrum. Towards the violet the colours are all crowded together in an irregular manner, but this is to be accounted for, no doubt, by the known fluorescence of the retina. At the extreme red end there is also a departure from the straight line, which is such as might be produced by stray light which is difficult to keep out at this end of the spectrum. It seems, therefore, that light is composed of three elementary sensations of red, green, and blue. Upon examining Maxwell's experiments upon color I was led to conclude that there was a relation of a very singular nature between the wave-length and the apparent color of any part of the spectrum. Let λ_x, λ_y, λ_z, be the wave-lengths of three points of the spectrum which are all on the same side of the elementary green. If, then, C_x, C_y, C_z, are the apparent lights of these three parts and if

$$C_y = XC_x + ZC_z$$

I find from Maxwell's experiments that

$$\frac{X}{Z} = \frac{\lambda_y - \lambda_z}{\lambda_x - \lambda_y}.$$

I announced this fact to the American Academy of Arts and Sciences in April 1872, and afterwards to the Philosophical Society of Washington. The same thing was afterwards independently announced by W. von Bezold** and proved (in a less satisfactory manner, as it seems to me) by Helmholtz's experiments***. The following is the comparison of the law with Maxwell's Experiments. The quantities compared are the percentages of certain arbitrarily taken colors in the various colors of the spectrum.

* Philosophical Transactions CL, 57—84.

** Ueber das Gesetz der Farbenmischung. Poggend. Ann. Bd. 150, p. 71. 221.

*** Helmholtz. Archiv für Anat. und Physiol. 1852, p. 461.

OBSERVER K.

RED TO GREEN.

Point of Spectrum on Maxwell's Scale.	Per cent of blue. Obs.	Calc.	Per cent of green. Obs.	Calc.	Per cent of red. Obs.	Calc.
24	00	00	00	— 01	100	101
28	— 01	00	24	+ 25	77	75
32	— 00	— 01	51	51	49	50
36	— 02	— 01	73	72	29	29
40	— 01	— 01	91	92	10	09

GREEN TO BLUE.

48	11	12	97	96	— 08	— 08
52	38	37	69	70	— 07	— 07
56	63	62	43	44	— 06	— 06
60	84	84	21	20	— 05	— 04
64	100	100	04	04	— 04	— 04

OBSERVER J.

RED TO GREEN.

Point of Spectrum on Maxwell's Scale.	Per cent of blue. Obs.	Calc.	Per cent of green. Obs.	Calc.	Per cent of red. Obs.	Calc.
24	00	— 01	00	+ 01	100	100
28	— 03	— 02	26	27	77	75
32	— 02	— 02	56	55	43	47
36	— 01	— 01	78	77	23	24
40	— 02	— 02	95	97	07	05

GREEN TO BLUE.

48	29	31	75	74	— 04	— 05
52	60	59	44	45	— 04	— 04
56	93	84	12	20	— 05	— 04
60	101	107	02	— 03	— 03	— 04

The agreement is all that could be desired. Without regarding the wave-lengths but simply the fact that the loci of the two parts of the spectrum are straight lines on Newton's diagram, we may calculate the constitution of elementary green and we find it to be, in terms of the colors of (24) and (44) of Maxwell's Scale

Observer K. $-.09\,(24) + 1.09\,(44)$

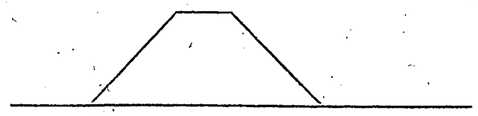
Observer J. $-.04\,(24) + 1.04\,(44)$

Having thus obtained the constituents of pure green we may by the law of wave-lengths get *two* values of the wave-length of it, one derived from the less refrangible and the other from the more refrangible part of the spectrum. Thus I find

Wave-length of pure green.

	From red end.	From blue end.
K	530	521
J	534	514

It appears from these calculations that the law of the variation of the green constituent of the sensation is such that if we represent the wave-length by the abscissas of rectangular coördinates, the ratio of the intensity of the green sensation to the whole light of any part of the spectrum being the ordinate, we have a curve like this:



Now we cannot suppose the real curve to consist of broken lines and we are therefore led to ask what curve resembles this. The curves which exhibits the intensity of resonance of a vibrating body for sounds which differ more or less from its natural pitch has a strong likeness to this; and the likeness suggests an explanation of the phænomena which falls in very well with current opinions about colour. In the present state of our knowledge of the action of the nerves, it would perhaps be useless to push our speculations on this subject any further.

So far we have adopted an arbitrary definition of the intensity of light which has no applicability except to lights differing in no respect except in intensity. We have now to consider another mode of measuring the intensity of all sensations which has much higher pretensions to real truth.

If a certain force x applied to irritate a nerve produces a certain sensation, there is perhaps no addition to it δx so slight that the sensation produced by $x + \delta x$ will not in some slight majority of trials be pronounced more intense than that produced by x. Nevertheless, a value can be assigned such that if δx is much larger than that, the sensation produced by $x + \delta x$ will be decidedly more intense than that produced by x, while if δx is much smaller than that, we shall feel uncertain which is the more intense sensation. Supposing δx to have that value, we find that $\frac{\delta x}{x}$ is a constant. Assuming that the addition to the sensation is the same and denoting this by δy, we have

$$\delta y = \frac{\delta x}{x} \quad \text{or} \quad y = A + B \log x.$$

The reasoning involved in this procedure would be open to criticism if the result were not fully confirmed in various ways. But it is so confirmed; and the (at least, approximate) truth of *Fechner's psychophysical law* is now fully admitted, that as the *vis viva* of the exciting force increases in geometrical ratio the sensation increases in arithmetical ratio.

Various circumstances interfere with the exactitude of this formula in the case of light; but still it is approximately true and this is why we do well to fix our scale

of magnitudes of stars so that equal increments of the numerical magnitude correspond to equal increments in the logarithm of the light.

Our sensibility to the three elementary colours is different. That is to say, the brighter a light is the more red and the less blue it appears. Consequently, observations on the colors of stars present very little agreement. I have given at the end of this paper a comparative catalogue showing the colors of stars according to Secchi at Rome, Schmidt at Athens, Sestini at Milan, and myself at different places. Secchi observed spectroscopically and he divides the stars into three classes, which I have indicated by I, II, III. I signifies that the star is a bluish one of the type of α Lyrae; II signifies that the star is a yellow one of the type of Capella; and III that it is a reddish one of the type of α Orionis. Schmidt has a numerical scale, the higher the number the warmer being the tone of the light. My own observations were partly made with the colorimeter of Zöllner's astrophotometer and these observations are italicized. The others are mere estimates. The former colors are all reduced to my day-time judgment of the colors. That is to say having matched the real star with the photometer star, I observed in the day-time what the color of the photometer star was. This is why I always make stars more blue and less red than other observers.

The different sensibility of the eye for the three primary colors also causes discrepancies in the observations of the relative brightness of different colored stars, made by different observers or under different atmospheric circumstances or with telescopes of different power. For if a red and a blue light which appear equally bright are both doubled in brightness according to the definition on page 1, they will no longer appear equally bright, but the red will appear the brighter.

CHAPTER II.

ON THE NUMBERS OF STARS OF DIFFERENT DEGREES OF BRIGHTNESS.

In Ptolemy's catalogue of stars, which is supposed to date from Hipparchus, we find the stars ranged in six orders of brightness called magnitudes. The early observers not only imitated this method of indicating the brightness from Ptolemy, but also, each of them derived immediately from the study of the Almagest and its comparison with the heavens the habit which determined the limit of brightness between stars which he would assign to different classes. This must, at least, have been the case with Sûfi and with Tycho Brahe. Ulugh Beg, was, no doubt, influenced by Sûfi as well as by Ptolemy directly; and Hevelius was in the same way influenced by Tycho. It appears that down to about 1840, Bayer's Uranometria enjoyed a high reputation. Argelander

showed however that its magnitudes were simply extracted from Tycho's catalogue* and he himself proceeded to make a Uranometria nova. It is to be presumed therefore that he endeavored to model his scale of magnitudes upon that of Tycho, although he may have sought to improve upon Tycho's scale by making the intervals between the limits of successive magnitudes such as would seem equal. All observers of stars visible to the naked eye since Argelander have sought to conform to his scale. It is, thus, easy to understand how all these observers have roughly speaking the same scale of magnitudes. On the other hand, the scale of Sir John Herschel, which was based on common English tradition from Flamsteed (who probably imitated Hevelius but was a careless observer of magnitudes), is very different.

It happened as a natural consequence of Fechner's psychophysical law that the ratio of light between successive magnitudes was approximately constant. Of course the observer would desire to have just as much difference between the 4th and 3rd magnitudes as between the 3rd and 2nd; and equal differences of sensation correspond nearly to equal ratios of light. So powerful is this natural influence that even Sir John Herschel's scale, which was conceived by its author to conform to a very different photometric law, really does conform to this and not to the one which he desired to follow. In reforming the scale of magnitudes by the aid of photometry, it should be so adjusted that equal numerical differences of magnitude shall precisely represent equal ratios of light. In this way, if Fechner's law were without error as applied to the eye, we should make equal numerical intervals correspond to equal differences of sensation and we should have a scale which would be independent of changes in the transparency of the atmosphere, of differences in the optical powers of our instruments, and of inequalities of visual sensibility. Since Fechner's law is unfortunately not in fact rigidly true, these important conditions cannot in any way be exactly fulfilled, but by making the ratio of light between successive magnitudes equal we at least approximate to their fulfillment.

I have desired to reduce all the scales of magnitudes of the different naked eye observers to one. I found I could not accomplish this very well by direct comparisons of single stars. The results obtained in this way were unsatisfactory and sometimes contradictory. The following table shows in the second column the result of endeavoring to find the mean value of each of Hevelius's classes of brightness by taking the mean of their magnitudes as given in the Durchmusterung, after the Durchmusterung magnitudes have themselves been corrected so as to make the scale uniform. The third column shows the value obtained for Hevelius's magnitudes by the process I am just about to describe.

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* And from the Almagest, in most cases, s. Argelander, De fide Uranometriae Bayeri, p. 15. (E.)

H. Mag.	Reduced by comp. with *DM.*	By enumeration.
1	0.50	0.51
1½	2.34	1.44
2	2.07	2.29
2½	2.48	2.80
3	3.30	3.38
3½	3.94	3.79
4	4.24	4.22
4½	4.78	4.56
4½	4.61	4.57
5	5.09	4.89
5½	4.89	5.20
5½	5.38	5.20
6	5.36	5.40
6½	5.77	5.60
7	5.31	5.61

It thus appears that the method of taking the mean magnitudes according to one observer of all the stars of the stars assigned to the same class by another observer utterly breaks down, because it would lead to the absurd results that Hevelius's 7th magnitude is brighter than his 5½, etc. Probably everybody who ever undertook to reduce star-magnitudes began by attempting this method, but except where the numbers of stars at hand were very great, I do not remember to have seen any published reduction of a scale of magnitudes which was made in this way. No doubt therefore difficulties of the nature I have just indicated have always presented themselves. In this state of things computers have usually had recourse to the supposition that one scale was a linear function of the other. But this overlooks the great irregularities which exist in every scale and which are usually of more consequence than the mean discrepancy between two scales. The Durchmusterung is one of the best collections of magnitudes which we have; and here are some of the numbers of stars of different magnitudes according to that catalogue.

Mag.	*Number.*	*Mag.*	*Number.*
6.0	618	6.6	59
6.1	106	6.7	457
6.2	293	6.8	901
6.3	275	6.9	137
6.4	101	7.0	2141
6.5	1239		

It is impossible to believe that the stars are really distributed in the heavens in this singular way; so that there are really 1239 stars of the 6.5 magnitude and only 59 of the 6.6. My father has already introduced into another branch of astronomy the principle that the magnitude of the different parts into which an observer divides a scale in estimating tenths are respectively proportional to the numbers of

cases in which the figures denoting them are found to occur. Thus if we find in any observations of transits by eye and ear, twice as many transits the time of which ends with 0.5 as of those which end with 0.4, we assume that in that observer's mental subdivision of the second the fifth part was twice as long an interval as the fourth. We may extend the same idea to the comparison of scales of star-magnitudes. If one observer says there are 9 first magnitude stars in the northern heavens and another finds only 8, clearly the latter consigns some star to the 2nd magnitude which the other considers to be of the 1st, and therefore he makes the limit between the first and second magnitudes to be brighter than the other makes it. Suppose that neither of two observers made any errors in his estimations and that their discrepancies arose solely from the differences of their scales of magnitudes. Then if they observed the same stars, whichever had fewer stars brighter than the 4th magnitude, for example, would have made the limit between his third and fourth magnitudes the brighter.

In fact we might call these numbers a scale of magnitudes. If there were 175 stars brighter than the fourth magnitude we might say that the limit between the two was upon this new scale the 175th magnitude. This scale would not of course be near to an equable photometric scale but it would conform to two conditions, 1st the fainter stars would have the larger numerical magnitudes, and 2nd it would be entirely free from those irregularities which we find in existing scales, although it would doubtless have irregularities (much smaller ones) due to the irregularity in the distribution of the stars with reference to brightness. If we may be allowed to neglect the effect of this, then we may say that in this way we should succeed in reducing the scales of both our two observers to one, which (unlike those) would be capable of reduction to an equable photometric scale by means of a general formula.

This is in effect what I have done, except that instead of the numbers themselves I have used a certain function of them in order to make my "scale of equable distribution"*, as I call it, as near as possible to the ordinary one. I count all the stars which an observer finds of each magnitude in the northern heavens. Denote the sum of these, or the number as bright or brighter than each magnitude, by $\nu(m)$. Then for the scale of equable distribution the numerical magnitude being m we have

$$m = -\tfrac{1}{3} + 1.892958 \log \nu(m).$$

In this way we shall obtain the magnitudes upon the scale of equable distribution of the limits of each class in the scales of the different observers. But we want the mean magnitude upon the our scale of the stars of each of these classes. For this purpose we may use the formula

* My aid, Mr. Henry Tarquhar, has suggested this term.

$$\frac{\int_{\nu(m_1)}^{\nu(m_2)} m \,.\, d\nu(m)}{\nu(m_2) - \nu(m_1)} = \frac{m_2 \,.\, \nu(m_2) - m_1 \,.\, \nu(m_1)}{\nu(m_2) - \nu(m_1)} - 1.89\,..\log 6.$$

The numerical value of the last term is about $0^m.822$. For the first magnitude, $m_1 = -\infty$ and $\nu(m_1) = 0$. In this case, the rules of the differential calculus, give $m_1 \nu(m_1) = 0$ so that the formula becomes

$$m_2 - 0.82.$$

In other cases, as the above formula is rather complicated, we may use the expression

$$1.89\,\ldots \log \frac{\nu(m_1) + \nu(m_2)}{2} - \frac{1}{3}.$$

This expression gives practically the same values as the other, when $\frac{\nu(m_2)}{\nu(m_1)}$ is not very large. Thus if $\nu(m_1) = 10$ and $\nu(m_2) = 20$, we have $m_1 = 1.56$ and $m_2 = 2.13$ and the correct value of the mean magnitude is $1^m.878$ while the last formula gives $1^m.893$, which differs by $\frac{1}{18}$ of the interval between the limits.

Thus far we have supposed the observers to be absolutely accurate in their observations. In order to consider the effect of their errors, I must premise (what will be shown further on) that the scale of equable distribution is also nearly a scale of equal ratios of light between successive magnitudes; so that, by Fechner's law, there is an equal liability to positive and negative errors and also an equal liability to errors in different parts of the scale. But the number of stars within equal differences from any given magnitude m are much greater for positive than for negative differences, so that the errors increase the number of stars which appear brighter than m. This will have two opposite but unequal effects. For on the one hand there are observed to be more stars brighter than any given star than there really are, so that the value m comes out too great because $\nu(m)$ is too great. There is some liability to confusion in regard to the other effect owing to the inadequacy of the word probability and its congeners. It sounds paradoxical to admit that it is equally likely that a star will be observed too bright and too faint and yet to insist that it is much more likely that a star is fainter than it has been observed to be than that it is brighter. But what is meant is that though equal numbers of stars are observed too bright and too faint, yet of stars observed as having a given magnitude, a considerable majority are really fainter, because there are so many more fainter stars to be observed too bright, than there are brighter ones to be observed too faint.

To find the amounts of these two effects, denote the observed magnitude of a star (reduced to the scale of equable distribution) by m; and its real magnitude by $m + e$; then the probability of that error is by Fechner's law

$$\frac{h}{\sqrt{\pi}}\, 6^{-h^2 e^2}\, de$$

where h is nearly constant for all parts of the scale. The number of stars whose magnitudes lie between $m + e$ and $m + e + dm$ is

$$d\nu(m) = \frac{1}{\log 6 \,.\, 1.89\,\ldots}\,[1.89\,..]^e\,\nu(m)\,dm.$$

Multiply this number and this probability and integrate relatively to e from $-\infty$ to $+\infty$ and we get for the number of stars which are observed as if their magnitudes lay between m and $m + dm$,

$$dm \int_{-\infty}^{+\infty} \frac{1}{1.89\,..\log 6}\,[1.89\,\nu\,.]^e\,\nu(m)\,\frac{h}{\sqrt{\pi}}\,6^{-h^2e^2}\,de = \frac{1}{1.89 \log 6}\,6^{-(2\times 1.89\,..\log 6\,.\,h)^2}\,\nu(m)\,dm.$$

Integrating again, we get for the total number of stars observed to be brighter than magnitude m,

$$\int_{-\infty}^{m} \frac{1}{1.89\,..\log 6}\,6^{-(2\times 1.89\,..\log 6\,h)^2}\,\nu(m)\,dm = 6^{\frac{1}{2}(2\times 1.89\,..\log 6\,h)^2}\,\nu(m).$$

Accordingly since we necessarily use this quantity in place of $\nu(m)$ in reducing the magnitudes, all our magnitudes are by the errors of the observer increased by the constant quantity

$$\frac{1}{4 \times 1.89 \log 6 \,.\, h^2} = \frac{1}{2 \times 1.89 \log 6}\,\varepsilon^2$$

where ε is the so called *mean error* of the observer. This is the first effect. Now for the second.

We have seen that the number of stars observed as if their magnitudes lay between m and $m + dm$ but whose true magnitude is larger by an amount between e and $e + de$ is

$$\frac{h\,[1.89\,..]^e}{\sqrt{\pi}\,.\,\log 6\;1.89\,..}\,6^{-h^2e^2}\,\nu(m)\,.\,de\,.\,dm.$$

Then the mean value of the error e is the total integral of this times e divided by the total integral of this or

$$\frac{\int_{-\infty}^{+\infty} e\,[1.89\,..]^e\, 6^{-h^2\epsilon^2}\,de}{\int_{-\infty}^{+\infty} [1.89\,..]^e\, 6^{-h^2\epsilon^2}\,de} = \frac{1}{2 \times 1.89\,..\log 6\,.\,h^2} = \frac{1}{1.89\,..\log 6}\,\epsilon^2$$

and this is the amount by which the true mean magnitude of the stars observed as being as bright as m is increased by the second effect.

The difference of the two effects or

$$\frac{1}{2 \times 1.89 \log 6}\,\epsilon^2 = 0.608\,..\,\epsilon^2$$

is the amount by which an observers magnitudes when reduced to the scale of equable distribution by means of the count of his stars are in the mean too large numerically on account of the errors of his observations. The mean error of an ordinary good observer of magnitudes does not exceed $0^m.25$ and therefore the mean of his magnitudes is only increased by $\frac{1}{27}$ of a magnitude, which is insignificant. But in the case of very bad observers, such as Ptolemy and Hevelius, the effect becomes so large as even to afford a means of ascertaining the mean errors of their observations. The catalogue at the end of this memoir is uncorrected by this term.

The observer's error has also two effects in producing accidental errors in the reduced magnitudes*. Besides this, there is an irregularity in the distribution of the stars themselves, in consequence of which, when we attempt to reduce the scale of equable distribution by any simple formula to an equal photometric scale, there will be residual differences of an accidental character. The combined effect of these causes may be studied by comparing the numbers of stars in different parts of the heavens. The general relative concentration of fainter stars toward the milky way may be taken account of, as we shall see, in the formula for the reduction of the scale of equable distribution. But besides this there is a patchiness about the sky in consequence of which parts equally distant from the milky way are not equal in the distribution of their stars, which patchiness appears to be greater than it is owing to the observers errors, which results in accidental variations of the scale of equable distribution from what it ought to be in order to be reducible by a simple formula to an equal photometric scale.

* These terms constant and accidental are bad, as is the terminology of the theory of probabilities, generally. By the constant part of an observers error, I mean the mean of his errors (which in this particular case happens to be also constant with reference to the variable). By the accidental part I mean the remainder of his error, which affects the mean taken without regard to signs.

The following are the counts of stars on Heis's atlas in six equal regions equally distant from the milky way*

Mags.	1	2	3	4	5	6	6.7
Region *A*	1	3	6	6	18	58	74
„ *B*	1	3	4	11	22	69	90
„ *C*	0	0	11	9	34	62	112
„ *D*	0	7	8	4	40	87	103
„ *E*	0	3	11	17	34	83	92
„ *F*	0	3	7	9	26	71	72

As the mean discrepancies of these numbers must be proportional to their square roots, I will use the square roots in place of the numbers themselves.

Mags.	1	2	3	4	5	6	6.7
Region *A*	1	1.7	2.4	2.4	4.2	7.6	8.6
„ *B*	1	1.7	2.0	3.3	4.7	8.3	9.5
„ *C*	0	0.0	3.3	3.0	5.8	7.9	10.6
„ *D*	0	2.6	2 8	2.0	6.3	9.3	10.1
„ *E*	0	1.7	3.3	4.1	5.8	9.1	9.6
„ *F*	0	1.7	2.6	3.0	5.1	8.4	8.5
Mean	0.3	1.6	2.7	3 0	5.3	8.4	9.5

The differences from the means are

Mags.	1	2	3	4	5	6	6.7
Region *A*	+ 0.7	+ 0.1	− 0.3	− 0.6	− 1.1	− 0.8	− 0.9
„ *B*	+ 0.7	+ 0.1	− 0.7	+ 0.3	− 0.6	− 0.1	± 0.0
„ *C*	− 0.3	− 1.6	+ 0.6	± 0.0	+ 0.5	− 0.5	+ 1.1
„ *D*	− 0.3	+ 1.0	+ 0.1	− 1.0	+ 1.0	+ 0.9	+ 0.6
„ *E*	− 0.3	+ 0.1	+ 0.6	+ 1.1	+ 0.5	+ 0.7	+ 0.1
„ *F*	− 0.3	+ 0.1	− 0.1	± 0.0	− 0.2	± 0.0	− 1.0

The squares of the differences are

Mags.	1	2	3	4	5	6	6.7
Region *A*	0.5	0.0	0.1	0.4	1.2	0.6	0.8
„ *B*	0.5	0.0	0.5	0.1	0.4	0.0	0.0
„ *C*	0.1	2.6	0.4	0.0	0.2	0.3	1.2
„ *D*	0.1	1.0	0.0	1.0	1.0	0.8	0.4
„ *E*	0.1	0.0	0.3	1.2	0.3	0.5	0.0
„ *F*	0.1	0.0	0.0	0.0	0.0	0.0	1.0
Sums	1.4	3.6	1.3	2.7	3.1	2.2	3.4

The mean of these sums is 2.5 so that $\sqrt{\frac{2.5}{5}} = \sqrt{0.5}$ is the mean error of $\sqrt{\nu(m)}$. Then the mean error of $\nu(m)$ is $2\sqrt{\frac{\nu(m)}{2}}$, and that of m is

$$1.16\,\frac{1}{\sqrt{\nu(m)}}.$$

* They are what I shall call further on the six Beronicean apogalactic regions.

This I have no doubt is too large, on account of several circumstances. Taking it as it is, however, it gives for the probable irregularity and error of the scale, if determined by counting all the stars in the northern hemisphere,

Mag.	Prob. error.
0	$0^m.63$
1	0.34
2	0.19
3	0.10
4	0.05
5	0.03
6	0.02
7	0.00

Another estimate of the values of these probable errors based upon totally different considerations afforded me numbers about $\frac{1}{6}$ of these.

We have now to consider the ratios of light between successive magnitudes in different parts of the scale of equable distribution. The observations upon which as it appears to me we must most rely are those of Seidel, Rosén, and myself. Seidel's is distinctly the greatest piece of work which has yet been done in stellar photometry. It embraces all the stars visible in Munich down to the $3\frac{1}{2}$ magnitude. There is, therefore, about one or at most two magnitudes interval, and considering that besides the errors of eye estimations there is also a large uncertainty of the scale at that point, it will be seen that the accuracy of the photometric observations and the skill with which they have been planned is not of great avail in determining the ratio of light. In fact, since the probable error of the scale at $1\frac{1}{2}$ mag. is $0^m.27$ and at $3\frac{1}{2}$ is $0^m.07$, the probable error of the best possible determination of the interval must be $0^m.3$, so that the probable error of the logarithm of the ratio of light must be 0.15 of its whole amount. Having calculated a formula from Seidel's observations, I find for the logarithm of the ratio of light for the scale of equable distribution 0.44 which is nearly what he himself finds for Argelander's scale.

Dr. P. G. Rosén's observations at Pulkowa were especially designed to determine the quantity in question and are well adapted to ascertaining it with a high degree of precision. They relate to about 100 DM stars ranging from 5^m to 10^m. I have recomputed his observations according to the rigid method of least squares and find for the logarithm of the ratio of light

$$\log \rho = 0.369.$$

There is some appearance of a larger value of this quantity for the brighter stars.

My own observations were not made with a view of determining this quantity, and they are affected with certain ill-determined cap constants in consequence of which the brighter stars are likely to be affected by a constant error. The value which they

give for stars from the $1\frac{1}{2}$ to the $6\frac{1}{2}$ magnitude is $\log \rho = 0.443$. But when I throw out all the stars with which caps were used and confine myself, also, to direct comparisons of stars, I find for 280 stars whose residuals amount to 100 magnitudes,

$$\log \rho = 0.389.$$

This applies to stars of the $4\frac{1}{2}$ to $6\frac{1}{2}$ magnitudes, the mean magnitude being $5^m.6$.

We find then

from $1^m.5$ to $3^m.5$ in the mean 2.8 $\log \rho = 0.44$

„ 4.5 „ 6.5 „ „ „ 5.6 „ $= 0.389$. Much larger for brighter stars (?).

„ 5 „ 10 „ „ „ $7\frac{1}{2}$ „ $= 0.369$. Larger for the brighter stars.

It appears then that there is a decrease of ρ as m increases. Such is my opinion. Still it must be admitted that there are certain other facts which weaken one's confidence in it.

The indication that Rosén's and my observations each show a larger value of $\log \rho$ for brighter stars is by no means a certain one in either case. Nothwithstanding the large number of stars observed, either determination might be wrong by 0.01, so that 0.38 might be the truth; and even Seidel's observations cannot be regarded as absolutely condicting that value, considering the probable error of the scale. At any rate, the main fact is that Seidel's observations give a large value of ρ as compared with Rosén's or even with mine. Now Rosén has shown that Zöllner's observations, which related to stars visible to the naked eye, give as small a value as his own. In fact, I have found that $\log \rho = 0.352$ (which happened to be a convenient value to use) satisfied Zöllner's observations well enough. Now Zöllner has compared 27 stars which had been determined by Seidel and these 27 stars give according to Rosén

from Seidel's measures $\log \rho = 0.444$

„ Zöllner's „ $\log \rho = 0.385$.

Furthermore there are 24 stars which have been photometrically measured by both Seidel and Herschel and (rejecting a red one among these) the others concur in giving a value of $\log \rho$ much less as derived from Herschel's than from Seidel's measures.

Seidel's measures were made with an instrument (Steinheil's photometer) of very peculiar construction. The object-glass is divided as in a heliometer and there are prisms to throw the light from the two stars into the two halves of the object-glass. Each half of the objective can be brought down towards the eye-piece or carried away from it the amount of the motion being shown on a graduated scale. Then both stars are put out of focus so that they appear as disks and then the brighter disk is enlarged by bringing the objective down or carrying it away until the two are equally bright; whenpon the positions of the two half objectives with reference to the focal distance show the relative amounts by which the lights have been reduced. Now this

instrument makes all red stars much too faint. Take λ and μ Ursae majoris for instance. μ is red, and every observer except Seidel makes it $0^m.4$ brighter than λ, and Seidel himself notes in his record that it appears so to his own eye, and yet his measures make λ $0^m.11$ brighter than μ. Considering, then, the two facts, that this instrument has such peculiarities, and that direct comparison with Zöllner and with Herschel indicate that it makes the difference of brightness of all stars too great, the question arises whether we ought not to think that it is this peculiarity of the instrument which causes the difference between the value of ρ, and not the greater brightness of the stars observed. Add that Pogson and Johnson, who observed on images of stars, both found $\log \rho = 0.38$; and that Steinheil, who observed with identically the same instrument as Seidel, found $\log \rho = 0.450$.

If there is such a difference between the two kinds of instruments, it is desirable to know which is right. I made a single experiment to ascertain whether the sum of two lights was equal to the light of the sum. I placed two kerosene lamps at a considerable distance and viewed them with Zöllner's sun and moon apparatus which reduced them, when placed side by side, to the appearance of a single star. I measured the light of each separately, and the light of their sum, and found

Lamp 1: Light	= 191
„ 2 „	= 85
Sum	276
Sum of 3 Errors	1.

A single experiment, however, cannot be considered as conclusive. On the whole I prefer to consider

$$\log \rho = 0.486 - .0162\,m$$

which gives

	$\log \rho$	
m	Obs.	Calc.
2.8	.44	.441
5.6	.389	.395
7.5	.369	.365

This will give for the logarithm of the light as compared with a star of the zero magnitude

m	$\log l$	m	$\log l$
0	0.000	4	— 1.685
1	— 0.470	5	— 2.025
2	— 0.907	6	— 2.333
3	— 1.312	7	— 2.608

and the following formula will give the nearest isophotometric scale to the one commonly in use:

$$M = -.813 - 2.87 \log l.$$

Denoting the magnitude on this scale by M and on our scale of equable distribution by m, we have

$$M = -.813 + 1.395\,m - .0465\,m^2.$$

Considering the uncertainty of this formula however I have thought it best to leave the magnitudes in the comparative catalogue at the end of this paper expressed in the scale of equable distribution, without attempting to reduce them to an isophotometric scale.

I now proceed to notice the various catalogues of stars whose magnitudes I have reduced.

The Durchmusterung or Bonner Sternverzeichniss.

I indicate this by the abbreviation *DM*, as Argelander desires. This great catalogue contains all the stars down to my 10^{th} magnitude, in the northern heavens, and extends two degrees beyond the equator. The observations were made with a telescope of $2\frac{1}{2}$ inches aperture. The estimates of magnitudes, if I understand the account rightly, were made to sixths of a magnitude. Every star was observed at least twice. The magnitudes are given to tenths. Mr. Proctor has plotted all the stars upon a single map. A glance at this shows that the scale of magnitudes for stars north of 81° of declination is different from that used south of this. Rosén's photometric measures show the same thing. He finds,

$\log \rho = 0.433 \pm 0.012$ for the Polar zones,
$\log \rho = 0.380 \pm 0.008$ for the others.

I have made no allowance for this important fact in my reduction as I ought perhaps to have done. The difference of scale cannot, however, be nearly so large as Rosén makes it, for in five magnitudes this would amount to more than half a magnitude. It would also cause the Polar zones to have only one fourth of the apparent density of stars in the rest of the heavens, where as they really have 83 per cent of the density of the rest; and as the absence of the milky way must make the circumpolar region really less dense the two scales cannot differ but very slightly.

K. von Littrow has published a count of the stars of each tenth of a magnitude in each zone of the Durchmusterung down to the equator. My aid Mr. Tarquhar has repeated the count for stars brighter than the 6.1 magnitudes. The discrepancies are insignificant. The accompanying table shows Mr. Tarquhar's count down to the sixth magnitude, and Littrow's beyond that point.

Zone	Var.	1.0	1.1	1.2	1.3	1.7	2.0	2.1	2.2	2.3	2.5	2.6	2.8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
89°																							
88							1																
87																							
86																							
85																							
84																							
83																							
82																							
81																							
80																							
79																							
78																							
77																							
76																			1				
75																							
74								1															
73																							
72													1									1	
71																							
70																	2						
69														1								1	
68																							
67																		1					
66																							
65														1				1				1	
64																							
63																			1				
62							1										1						
61											1			1			1				1		
60																							
59										1				2									
58	1								1										1				
57	1									1							1					1	
56							1												1				
55	1						1												1				
54											1												
53																							
52														2		1							
51									1														
50							1																
49							1																
48															1								
47																			2	1			
46																			1				
45		1																	2			1	
44						1	1							1									
43	2																		1				
42															1								
41							1											1			1		
40	1													1						1			
	Var.	1.0	1.1	1.2	1.3	1.7	2.0	2.1	2.2	2.3	2.5	2.6	2.8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9

4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	Sums	Z.
																					0	89°
										1											1	88
					1												1				1	87
										2										3	5	86
										1					1					2	4	85
										2		1								3	5	84
1										2					1					2	5	83
			1																	2	6	82
									1			1			2		1	1	1	1	2	81
												1			2					3	7	80
							1				1					2				3	7	79
							1	1		1	1					2	1	1		2	9	78
							1	1		1								1		4	8	77
										2		1	2		1					5	12	76
										1							1	3		2	7	75
						1						1			2			1		6	6	74
				1								1	1		1	1		1		5	11	73
								1		2		1	1		2					4	13	72
1							1					1	1		1			1		2	8	71
			1			1	1	1		1		2	1				1			8	19	70
							2			1		3			2				1	5	16	69
								2			1				2					5	10	68
											1	6	3		1			1		2	15	67
					1		1			3		1			3			1		1	11	66
					2			1		1			2		1		1	3		9	23	65
							1			3		1			1			2	1	5	14	64
					1									1	1			4	1	8	17	63
1					1					2		1	2				3		1	9	22	62
										2			1		2	1		2	1	7	20	61
					1					2			1	1	2		1	3	1	13	25	60
1			1		1					4							4	1		8	23	59
								1		5	1		1			2				6	19	58
	1				1					6			3	1			1	3		4	24	57
					1		1			2	1			2	1		2	2	1	11	26	56
										2	1	2	1		3		4	1	1	8	26	55
1									3		2						3	4		13	27	54
	1				1					4		1	1	2			1	4		11	26	53
1		1			2					3		1	2	2	7		1	1	1	5	30	52
1							1				1	1					1			11	17	51
									1		2	1	2			3	1	2	2	8	23	50
1	1	1	1		1				3	2	1	3		2		2		6	4	10	39	49
1		2			2	1				2	2				1	5			2	1	10	30	48
					1	1			2		1			2		5		1	4	5	25	47
3	1			1	1				1		2	1	2	1		4		1	3	2	16	40	46
									1		1		2	1	1	2	1	2	2	1	4	22	45
	1				1		1				1		1	3		2			1		4	18	44
1							1	1		3	1	4	1	2	3	2	2			8	32	43
1	2	1	1		1			1	1	2		1		1	1			1	1	10	26	42
			1		1			2				4	2		4			1	1	7	27	41
2							2	1		5	1	2			5		1	3	1	8	34	40
4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0		

Zone	Var.	1.0	1.1	1.2	1.3	1.7	2.0	2.1	2.2	2.3	2.5	2.6	2.8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9
39°													1	2		1							
38	1	1											1										
37														1									
36														1					1				
35														1									
															1								
34																							
33	1								1					1		1							
32	1					1						1		1				1					
31														1		1					1	1	
30														2					1			1	
																	1						
29																							
28	1				1		2							1		1							
27	1						1			1													2
26														1					1				
25														1		1					1		
24																							
23																1			1		1		
22							1							1		1							2
21										1	1			1		1			1				1
20	1						1				1				1								
19		1												1		1							
18														1		1						1	
17																			1		1		
16			1					1										1			1	1	
15							1									1			2			1	
14	1						1				1								2				
13																							1
12	2				1		1							1							1		
11																							
10														1		1							
														1					1		1	1	
9																							
8				1						1				1			1		1		1		
7	2													1								1	
6																							
5		1					1		1					1			1		1				
																		1					
4																							
3													2						1				
2											1												
1															1	1		1				1	
0	1																		1				
	18	4	1	1	2	2	17	2	4	5	6	1	3	31	4	12	12	6	30	2	10	14	6
	Var.	1.0	1.1	1.2	1.3	1.7	2.0	2.1	2.2	2.3	2.5	2.6	2.8	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9

SOUTHERN

−0°							1						1					1				1		1
−1							1						1					1						

1.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	Sums	Z.
1			1			1	2		3	2	2	1	1	4	2			2		7	33	39°
					2			1	1	2		1	1		3		3	1	1	9	28	38
4		2	1		3	1		3		3	1		5	1	3		5	1	1	10	46	37
					1		1	1		5				1	6			3	3	13	35	36
		1		1	1		1				1	1	1		1	1	2	3	1	4	20	35
1		3			1	1				2		1	1	4	1	1	1	2		9	31	34
	1							1		1	2		1	1	3		1	2	1	8	29	33
	1							2		1		1			4		2	5		11	33	32
1		1						1		3	1	2	3		3		1	4		8	32	31
			1		1		1			4		1	1		2	2		3		10	27	30
2		3		1			1			2		1	2		3		1	2		7	27	29
1					3				1	5	3	3		2	1		3	1	4	8	41	28
1		1					1	1		6	1	1	2		3		5	2		5	34	27
2		1			1		1		1	3			2	1	1		4	3	1	5	27	26
1							1	2		3		2	2		3		3	3		11	36	25
1	1	1	1							1			4	1	3		3	6		14	39	24
1					3	3	1	2		1	1	1	1		3			3	3	10	36	23
					3		1	1		5		2	2	1	2		1	1	1	11	36	22
1		1	1		2		1	1		2	2				3		1	6		19	43	21
	1	2	2		2		2			4	1	1	1		4	2	2	4	1	12	44	20
		1	1	1						5			1		4	2	2	3	1	10	35	19
2	1			1	3	1	1	2		4	1	1	1	1	2			3	1	7	34	18
		1	1		1		1	1				2		1	5		2	5	1	9	33	17
1	1	1							1	2	1	2	1		7	3	4	5		13	46	16
2									1	4	1		3		2		2	1		8	29	15
1		1			1	1		2		4		1		2	3		3	3	3	16	47	14
										1	1		3		2	2	4	4	2	4	25	13
2			1		1					2	1	1	1		1	1	1	1		12	32	12
	1							2		3	1	4	3	1	5		3	2		6	33	11
1	2									2		3	2		2	1		3		7	27	10
3		1	1			1	1			7		1	3	1	3		2	1		9	39	9
					2	1	2	1		3		4	1		1	1		2		5	26	8
			1		2			1		2			1		2		1	6	2	7	28	7
			1	1	2			1			1	3			1		1	2	1	3	23	6
2									1				2		3	1	1	1	1	4	22	5
2		1	1		2				1	4		1	2		3		2	1		7	30	4
							1	2	1	2			1		2		1			4	15	3
4		1	1		1		1	1	1	3			2		3	3	1	6		4	39	2
					1					5					2		2	3		5	18	1
	1				1	1				2			2	1	1		3	2	2	5	21	0
54	17	29	21	7	61	15	36	53	16	195	36	86	95	37	186	30	116	176	48	618	2125	
4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0		

ZONES.

1		2	1				1	2		2			1	2	2			2		5	26	−0°
		1			1		1			3		2	2		3	1	1	6	1	4	29	−1

Zone	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	Sums	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	Sums
+ 0°	5	..	6	3	..	12	1	2	10	3	42	23	1	13	12	3	27	11	16	24	18	148
1	5	3	2	3	..	12	..	3	12	1	41	19	3	12	8	6	25	7	29	37	8	151
2	4	4	3	2	1	13	2	2	11	4	46	14	9	12	12	4	33	3	27	41	10	165
3	5	..	3	4	..	10	1	5	6	3	37	23	4	13	17	5	34	5	20	27	11	159
4	7	2	..	1	1	6	..	6	4	..	27	26	4	7	10	3	37	4	22	31	3	147
5	1	3	3	4	1	9	..	6	10	3	40	16	5	9	18	3	31	3	14	32	12	143
6	3	1	2	2	2	16	1	6	4	3	40	28	4	13	16	7	18	10	19	31	20	169
7	7	1	2	3	..	9	1	1	6	1	31	18	3	11	13	3	43	8	17	32	8	156
8	5	1	6	1	2	7	..	4	7	1	34	25	3	8	14	5	30	4	23	32	10	154
9	9	..	4	1	..	15	1	3	15	3	51	30	5	6	10	7	30	11	20	30	13	162
10	7	3	4	2	2	9	2	4	9	3	45	34	9	12	18	5	32	10	24	24	13	181
11	6	1	2	4	3	11	2	6	10	7	52	22	8	13	19	7	46	10	25	32	7	189
12	11	2	4	1	2	12	..	4	12	4	52	27	5	12	18	5	32	8	20	32	10	169
13	4	1	6	4	3	21	3	7	11	1	61	35	2	18	10	3	42	2	18	30	10	170
14	16	2	2	1	1	31	1	6	13	4	77	25	4	8	18	5	35	12	31	40	18	196
15	8	1	5	6	2	24	1	7	18	6	78	36	6	11	26	7	43	8	23	33	15	208
16	13	6	5	1	2	23	5	5	6	5	71	37	3	15	24	2	40	10	23	41	10	205
17	9	2	6	6	..	24	..	7	15	2	71	27	6	17	20	2	32	7	21	29	8	169
18	7	3	2	2	2	18	2	5	13	2	56	39	6	11	17	7	44	9	23	32	23	211
19	10	3	9	5	2	27	3	6	19	1	85	33	7	20	14	8	38	12	22	33	9	196
20	12	1	7	5	1	18	2	5	12	5	68	24	3	15	17	9	47	6	26	39	9	195
21	19	..	2	1	..	14	2	5	13	4	60	27	2	14	20	7	50	6	29	37	13	205
22	11	1	2	3	..	19	2	10	14	4	66	36	5	14	16	9	51	7	29	34	8	209
23	11	1	7	5	1	29	1	4	15	3	77	32	4	9	27	11	45	9	16	39	22	214
24	14	2	4	4	1	21	1	9	16	2	74	31	7	4	18	11	35	6	18	36	14	180
25	14	3	5	7	1	13	5	3	10	2	63	32	5	15	24	11	55	12	26	35	14	229
26	5	2	5	6	..	20	1	5	15	2	61	29	8	14	20	7	37	7	28	27	12	189
27	5	1	3	1	1	20	3	3	10	1	48	28	7	15	14	3	38	13	24	31	18	191
28	8	..	3	5	..	17	..	6	14	1	54	34	6	10	26	7	32	7	20	23	18	183
29	7	3	3	1	1	15	1	3	16	1	51	31	1	20	9	8	54	15	33	44	15	230
30	10	2	2	2	1	17	2	4	16	8	64	29	5	12	14	8	44	8	17	43	20	200
31	8	..	7	4	3	19	3	12	12	3	71	35	9	23	20	7	42	14	27	43	13	233
32	12	..	1	5	..	21	3	10	15	5	72	32	3	20	17	14	50	13	20	37	17	223
33	8	2	5	6	1	15	3	7	15	4	66	34	7	19	21	9	49	7	29	43	9	227
34	10	1	5	3	3	20	10	5	9	3	69	35	4	20	27	9	41	8	25	44	26	239
35	4	1	3	1	2	22	2	9	14	1	59	39	7	14	21	7	47	10	22	38	21	229
36	13	3	2	6	2	28	4	6	15	7	86	36	2	18	28	9	51	8	28	40	22	242
37	10	1	8	4	3	24	1	7	14	8	80	38	6	19	19	6	51	13	24	52	18	216
38	9	2	4	4	2	20	4	9	26	4	84	30	5	16	24	11	47	13	28	41	17	232
39	7	..	7	4	1	19	3	9	25	5	80	44	7	16	27	14	40	16	37	56	16	273
40	8	3	8	6	..	18	3	5	23	7	81	39	9	23	37	13	52	22	29	43	21	285
41	7	1	6	4	1	19	3	9	12	8	70	49	5	11	34	6	60	12	19	36	12	244
42	10	2	3	5	3	26	2	12	16	6	85	23	4	12	22	13	65	11	29	35	22	236
43	8	1	5	6	..	17	1	13	17	9	77	45	4	15	29	10	42	9	20	44	14	232
44	4	1	3	5	3	18	..	10	17	5	66	27	2	14	25	4	49	9	23	51	9	213

7.5?

Zone	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	Sums	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	Sums
+ 45°	4	1	6	4	3	17	4	4	19	5	67	44	5	13	20	7	37	12	20	39	21	218
46	16	2	2	2	2	28	3	3	12	1	71	36	7	25	21	7	27	10	29	41	14	220
47	5	2	6	5	3	19	5	11	16	4	76	31	6	20	28	10	55	12	24	36	24	246
48	10	..	7	1	2	19	4	7	10	5	65	36	5	16	24	12	48	15	18	45	13	232
49	10	..	5	6	1	16	4	8	6	3	59	31	5	16	25	10	38	8	28	36	20	217
50	9	..	8	2	1	17	4	8	7	5	61	33	6	20	29	10	42	14	20	42	14	230
51	11	1	5	7	2	16	2	6	10	1	61	27	7	10	21	4	39	11	16	32	23	190
52	5	2	6	6	1	15	2	8	11	3	59	23	6	15	17	9	41	6	18	31	25	191
53	11	..	2	4	2	17	3	1	9	..	49	27	11	11	19	9	31	8	17	22	10	165
54	13	2	2	2	3	13	3	6	9	5	58	26	4	8	22	5	44	4	15	22	15	165
55	8	..	1	7	..	17	1	8	8	3	53	25	5	15	6	11	34	4	19	38	7	164
56	11	2	4	4	2	12	7	11	10	2	65	24	4	11	15	10	37	7	14	25	11	158
57	4	4	3	9	2	20	2	5	8	3	60	20	6	14	11	2	31	7	11	28	12	142
58	6	1	3	7	1	10	3	4	4	2	41	19	6	3	15	7	30	5	8	32	8	133
59	8	..	6	4	2	12	3	10	10	4	59	26	3	15	18	10	26	10	20	31	12	171
60	12	2	3	3	..	16	5	7	13	3	64	26	3	9	16	5	31	3	17	27	10	147
61	7	2	3	4	1	17	..	6	11	2	50	19	2	9	19	2	30	6	14	29	15	145
62	9	2	2	2	1	15	3	8	7	1	50	19	2	11	13	6	33	6	11	18	7	126
63	8	1	..	5	2	10	2	3	7	2	40	32	..	10	12	4	26	2	17	38	5	146
64	5	..	2	1	..	13	1	1	10	4	37	21	1	9	8	6	30	3	10	15	9	112
65	9	1	1	5	1	5	2	6	10	4	46	24	4	7	12	2	22	6	7	16	7	107
66	1	..	3	..	1	7	1	7	10	..	30	14	1	8	14	6	18	5	13	21	6	106
67	2	1	1	2	..	8	1	1	11	3	30	16	..	6	9	5	24	4	7	22	11	104
68	5	..	3	..	2	10	..	2	6	..	28	12	..	7	8	5	13	3	7	11	5	71
69	5	2	3	2	..	11	1	4	8	1	37	18	3	7	7	2	17	4	15	15	7	95
70	8	1	1	4	..	5	..	2	6	..	27	14	3	10	11	2	12	1	9	22	7	91
71	2	..	..	2	..	5	1	5	10	1	26	6	1	7	4	2	19	2	10	13	2	66
72	4	..	2	5	1	6	1	4	6	..	29	18	3	5	8	..	21	1	11	9	5	81
73	5	..	3	3	1	9	..	2	9	2	34	10	1	5	8	2	16	1	2	14	1	60
74	..	1	..	1	3	8	1	3	3	..	20	6	2	8	5	4	15	1	6	12	4	63
75	2	..	3	..	1	6	1	5	8	..	26	12	2	5	5	3	14	4	6	7	3	61
76	5	..	..	1	1	5	..	3	2	1	18	7	1	3	7	..	14	2	2	10	1	47
77	4	1	2	1	..	5	..	1	4	..	18	12	1	5	4	3	13	1	9	13	2	63
78	2	..	2	..	..	7	..	1	7	2	21	9	1	2	6	1	12	..	6	6	2	45
79	3	..	..	3	2	6	1	2	2	..	19	10	..	3	7	1	14	..	7	10	..	52
80	..	2	1	2	..	3	..	5	3	..	16	7	2	2	3	6	7	..	1	9	5	42
81	1	..	4	3	..	1	..	2	5	..	16	3	..	2	3	4	6	3	3	5	6	35
82	2	..	..	..	..	2	..	2	..	..	6	5	..	..	..	..	6	3	2	..	..	16
83	2	..	2	..	..	2	..	..	1	..	7	4	2	2	2	2	4	..	3	2	..	21
84	3	..	..	..	..	3	..	..	..	..	6	3	..	4	1	..	5	..	3	1	1	18
85	2	..	..	..	..	2	..	..	..	..	4	3	..	1	..	..	7	..	1	1	..	13
86	3	..	..	..	..	1	..	..	1	..	5	5	..	2	..	..	2	..	..	1	..	10
87	..	..	..	..	..	1	..	..	..	..	1	1	..	..	..	..	1	..	1	..	2	5
88	..	..	..	..	..	2	..	..	..	..	2	..	..	..	..	..	2	..	..	..	..	2
89	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	1
Sums	618	106	293	275	101	1239	159	157	901	237	4386	2141	345	984	1356	516	2860	609	1537	2184	991	13823

Zone	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	Sums	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	Sums
+ 0°	5	..	6	3	..	12	1	2	10	3	42	23	1	13	12	3	27	11	16	24	18	148
1	5	3	2	3	..	12	..	3	12	1	41	19	3	12	8	6	25	7	29	37	8	151
2	4	4	3	2	1	13	2	2	11	4	46	14	9	12	12	4	33	3	27	41	10	165
3	5	..	3	4	..	10	1	5	6	3	37	23	4	13	17	5	34	5	20	27	11	159
4	7	2	..	1	1	6	..	6	4	..	27	26	4	7	10	3	37	4	22	31	3	147
5	1	3	3	4	1	9	..	6	10	3	40	16	5	9	18	3	31	3	14	32	12	143
6	3	1	2	2	2	16	1	6	4	3	40	28	4	13	16	7	18	10	19	31	20	169
7	7	1	2	3	..	9	1	1	6	1	31	18	3	11	13	3	43	8	17	32	8	156
8	5	1	6	1	2	7	..	4	7	1	34	25	3	8	14	5	30	4	23	32	10	154
9	9	..	4	1	..	15	1	3	15	3	51	30	5	6	10	7	30	11	20	30	13	162
10	7	3	4	2	2	9	2	4	9	3	45	34	9	12	18	5	32	10	24	24	13	181
11	6	1	2	4	3	11	2	6	10	7	52	22	8	13	19	7	46	10	25	32	7	189
12	11	2	4	1	2	12	..	4	12	4	52	27	5	12	18	5	32	8	20	32	10	169
13	4	1	6	4	3	21	3	7	11	1	61	35	2	18	10	3	42	2	18	30	10	170
14	16	2	2	1	1	31	1	6	13	4	77	25	4	8	18	5	35	12	31	40	18	196
15	8	1	5	6	2	24	1	7	18	6	78	36	6	11	26	7	43	8	23	33	15	208
16	13	6	5	1	2	23	5	5	6	5	71	37	3	15	24	2	40	10	23	41	10	205
17	9	2	6	6	..	24	..	7	15	2	71	27	6	17	20	2	32	7	21	29	8	169
18	7	3	2	2	2	18	2	5	13	2	56	39	6	11	17	7	44	9	23	32	23	211
19	10	3	9	5	2	27	3	6	19	1	85	33	7	20	14	8	38	12	22	33	9	196
20	12	1	7	5	1	18	2	5	12	5	68	24	3	15	17	9	47	6	26	39	9	195
21	19	..	2	1	..	14	2	5	13	4	60	27	2	14	20	7	50	6	29	37	13	205
22	11	1	2	3	..	19	2	10	14	4	66	36	5	14	16	9	51	7	29	34	8	209
23	11	1	7	5	1	29	1	4	15	3	77	32	4	9	27	11	45	9	16	39	22	214
24	14	2	4	4	1	21	1	9	16	2	74	31	7	4	18	11	35	6	18	36	14	180
25	14	3	5	7	1	13	5	3	10	2	63	32	5	15	24	11	55	12	26	35	14	229
26	5	2	5	6	..	20	1	5	15	2	61	29	8	14	20	7	37	7	28	27	12	189
27	5	1	3	1	1	20	3	3	10	1	48	28	7	15	14	3	38	13	24	31	18	191
28	8	..	3	5	..	17	..	6	14	1	54	34	6	10	26	7	32	7	20	23	18	183
29	7	3	3	1	1	15	1	3	16	1	51	31	1	20	9	8	54	15	33	44	15	230
30	10	2	2	2	1	17	2	4	16	8	64	29	5	12	14	8	44	8	17	43	20	200
31	8	..	7	4	3	19	3	12	12	3	71	35	9	23	20	7	42	14	27	43	13	233
32	12	..	1	5	..	21	3	10	15	5	72	32	3	20	17	14	50	13	20	37	17	223
33	8	2	5	6	1	15	3	7	15	4	66	34	7	19	21	9	49	7	29	43	9	227
34	10	1	5	3	3	20	10	5	9	3	69	35	4	20	27	9	41	8	25	44	26	239
35	4	1	3	1	2	22	2	9	14	1	59	39	7	14	21	7	47	10	22	38	21	229
36	13	3	2	6	2	28	4	6	15	7	86	36	2	18	28	9	51	8	28	40	22	242
37	10	1	8	4	3	24	1	7	14	8	80	38	6	19	19	6	51	13	24	52	18	216
38	9	2	4	4	2	20	4	9	26	4	84	30	5	16	24	11	47	13	28	41	17	232
39	7	..	7	4	1	19	3	9	25	5	80	44	7	16	27	14	40	16	37	56	16	273
40	8	3	8	6	..	18	3	5	23	7	81	39	9	23	37	13	52	22	29	43	21	285
41	7	1	6	4	1	19	3	9	12	8	70	49	5	11	34	6	60	12	19	36	12	244
42	10	2	3	5	3	26	2	12	16	6	85	23	4	12	22	13	65	11	29	35	22	236
43	8	1	5	6	..	17	1	13	17	9	77	45	4	15	29	10	42	9	20	44	14	232
44	4	1	3	5	3	18	..	10	17	5	66	27	2	14	25	4	49	9	23	51	9	213

7.5?

Zone	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	Sums	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	Sums
+ 45°	4	1	6	4	3	17	4	4	19	5	67	44	5	13	20	7	37	12	20	39	21	218
46	16	2	2	2	2	28	3	3	12	1	71	36	7	25	21	7	27	10	29	41	14	220
47	5	2	6	5	3	19	5	11	16	4	76	31	6	20	28	10	55	12	24	36	24	246
48	10	..	7	1	2	19	4	7	10	5	65	36	5	16	24	12	48	15	18	45	13	232
49	10	..	5	6	1	16	4	8	6	3	59	31	5	16	25	10	38	8	28	36	20	217
50	9	..	8	2	1	17	4	8	7	5	61	33	6	20	29	10	42	14	20	42	14	230
51	11	1	5	7	2	16	2	6	10	1	61	27	7	10	21	4	39	11	16	32	23	190
52	5	2	6	6	1	15	2	8	11	3	59	23	6	15	17	9	41	6	18	31	25	191
53	11	..	2	4	2	17	3	1	9	..	49	27	11	11	19	9	31	8	17	22	10	165
54	13	2	2	2	3	13	3	6	9	5	58	26	4	8	22	5	44	4	15	22	15	165
55	8	..	1	7	..	17	1	8	8	3	53	25	5	15	6	11	34	4	19	38	7	164
56	11	2	4	4	2	12	7	11	10	2	65	24	4	11	15	10	37	7	14	25	11	158
57	4	4	3	9	2	20	2	5	8	3	60	20	6	14	11	2	31	7	11	28	12	142
58	6	1	3	7	1	10	3	4	4	2	41	19	6	3	15	7	30	5	8	32	8	133
59	8	..	6	4	2	12	3	10	10	4	59	26	3	15	18	10	26	10	20	31	12	171
60	12	2	3	3	..	16	5	7	13	3	64	26	3	9	16	5	31	3	17	27	10	147
61	7	2	3	4	1	17	..	6	11	2	50	19	2	9	19	2	30	6	14	29	15	145
62	9	2	2	2	1	15	3	8	7	1	50	19	2	11	13	6	33	6	11	18	7	126
63	8	1	..	5	2	10	2	3	7	2	40	32	..	10	12	4	26	2	17	38	5	146
64	5	..	2	1	..	13	1	1	10	4	37	21	1	9	8	6	30	3	10	15	9	112
65	9	1	1	5	1	5	2	6	10	4	46	24	4	7	12	2	22	6	7	16	7	107
66	1	..	3	..	1	7	1	7	10	..	30	14	1	8	14	6	18	5	13	21	6	106
67	2	1	1	2	..	8	1	1	11	3	30	16	..	6	9	5	24	4	7	22	11	104
68	5	..	3	..	2	10	..	2	6	..	28	12	..	7	8	5	13	3	7	11	5	71
69	5	2	3	2	..	11	1	4	8	1	37	18	3	7	7	2	17	4	15	15	7	95
70	8	1	1	4	..	5	..	2	6	..	27	14	3	10	11	2	12	1	9	22	7	91
71	2	..	..	2	..	5	1	5	10	1	26	6	1	7	4	2	19	2	10	13	2	66
72	4	..	2	5	1	6	1	4	6	..	29	18	3	5	8	..	21	1	11	9	5	81
73	5	..	3	3	1	9	..	2	9	2	34	10	1	5	8	2	16	1	2	14	1	60
74	..	1	..	1	3	8	1	3	3	..	20	6	2	8	5	4	15	1	6	12	4	63
75	2	..	3	..	1	6	1	5	8	..	26	12	2	5	5	3	14	4	6	7	3	61
76	5	..	..	1	1	5	..	3	2	1	18	7	1	3	7	..	14	2	2	10	1	47
77	4	1	2	1	..	5	..	1	4	..	18	12	1	5	4	3	13	1	9	13	2	63
78	2	..	2	..	..	7	..	1	7	2	21	9	1	2	6	1	12	..	6	6	2	45
79	3	..	..	3	2	6	1	2	2	..	19	10	..	3	7	1	14	..	7	10	..	52
80	..	2	1	2	..	3	..	5	3	..	16	7	2	2	3	6	7	..	1	9	5	42
81	1	..	4	3	..	1	..	2	5	..	16	3	..	2	3	4	6	3	3	5	6	35
82	2	..	..	..	..	2	..	2	..	..	6	5	..	..	..	..	6	3	2	..	..	16
83	2	..	2	..	..	2	..	..	1	..	7	4	2	2	2	2	4	..	3	2	..	21
84	3	..	..	..	..	3	..	..	..	..	6	3	..	4	1	..	5	..	3	1	1	18
85	2	..	..	..	..	2	..	..	..	..	4	3	..	1	..	..	7	..	1	1	..	13
86	3	..	..	..	..	1	..	..	1	..	5	5	..	2	..	..	2	..	..	1	..	10
87	..	..	..	..	..	1	..	..	..	..	1	1	..	..	..	..	1	..	1	..	2	5
88	..	..	..	..	..	2	..	..	..	..	2	..	..	..	..	..	2	..	..	..	..	2
89	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	1
Sums	618	106	293	275	101	1239	159	157	901	237	4386	2141	345	984	1356	516	2860	609	1537	2184	991	13823

Zone	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	Sums	9.0	9.1	9.2	9.3	9.4	9.5	Sums
+ 0°	58	21	59	77	27	166	49	117	217	94	885	417	172	331	663	383	2028	3994
1	69	16	61	68	20	139	35	112	221	62	803	466	142	294	619	346	1954	3821
2	86	13	50	71	31	155	42	107	221	89	868	458	147	326	579	329	1799	3638
3	75	23	56	52	24	172	51	122	179	119	873	510	114	309	590	363	1938	3854
4	75	21	69	73	37	186	46	140	187	79	913	537	138	313	578	350	2066	3982
5	79	27	48	86	45	184	59	157	196	99	980	510	146	331	594	438	2063	4082
6	87	31	58	66	28	160	47	139	186	109	911	418	189	338	619	457	2080	4101
7	60	26	54	71	32	181	52	113	187	93	869	437	181	326	539	477	2091	4051
8	72	19	64	60	40	144	80	131	208	109	927	410	206	375	643	444	1955	4033
9	66	28	72	71	33	140	80	130	162	116	898	405	221	339	603	511	2101	4180
10	70	31	59	64	37	166	59	120	154	125	885	336	224	310	567	496	1960	3893
11	84	30	52	69	36	128	83	140	184	118	924	334	197	337	522	540	1974	3904
12	75	26	50	68	33	173	67	128	175	108	903	362	230	341	529	536	1920	3918
13	63	24	45	70	40	136	54	127	165	96	820	351	222	331	516	502	2208	4130
14	59	22	64	68	45	138	54	104	151	112	817	336	242	353	531	519	1997	3978
15	76	25	55	55	31	151	63	108	144	101	809	377	199	300	538	480	1924	3818
16	79	22	40	72	42	135	48	102	161	101	805	316	215	336	543	514	2004	3928
17	73	16	52	64	37	142	55	102	155	111	807	355	225	318	522	516	2031	3967
18	90	31	61	86	26	120	58	81	165	121	842	350	275	333	516	540	2117	4131
19	84	24	60	75	34	136	50	94	169	87	813	378	244	396	496	536	2129	4089
20	74	22	69	69	36	152	73	110	183	81	869	409	284	320	525	609	2118	4265
21	87	32	66	71	29	140	67	104	161	102	859	348	255	310	488	561	1915	3877
22	85	27	55	61	47	131	73	102	181	132	894	298	249	283	520	562	1846	3758
23	92	26	51	69	40	161	59	105	152	113	868	330	280	304	497	596	1661	3668
24	79	25	62	80	36	139	76	112	160	144	913	322	273	318	513	618	1649	3693
25	91	34	57	78	65	164	64	110	162	127	952	337	251	316	473	661	1769	3807
26	66	30	57	61	55	140	83	90	146	144	872	309	262	334	465	612	1618	3600
27	77	28	72	65	41	132	71	98	133	101	818	317	262	315	493	586	1620	3593
28	76	33	67	70	46	123	59	115	157	116	862	314	250	292	530	627	1565	3578
29	85	34	60	87	51	129	74	97	181	132	930	350	274	369	461	642	1734	3830
30	76	39	61	77	64	131	81	109	162	143	943	330	280	351	500	653	1752	3866
31	87	38	60	76	46	140	75	90	188	134	934	349	326	351	482	639	1629	3776
32	90	29	48	72	63	127	66	91	153	132	871	316	285	340	480	647	1518	3586
33	72	33	64	89	56	129	61	97	173	121	898	332	286	329	477	606	1592	3622
34	78	36	65	84	63	113	62	89	144	142	906	352	264	322	480	655	1759	3832
35	84	33	71	79	60	156	88	110	174	150	1005	330	287	359	518	657	1712	3863
36	79	38	44	68	58	158	98	98	176	145	962	357	286	348	480	615	1756	3842
37	88	38	51	91	48	140	64	112	171	135	938	338	289	363	490	618	1538	3636
38	95	28	56	80	63	158	77	124	151	142	974	352	293	362	537	648	1617	3809
39	94	28	60	79	50	156	98	116	195	163	1039	338	295	356	470	653	1693	3805
40	87	28	66	95	67	162	87	118	207	149	1066	379	303	387	511	648	1547	3775
41	103	29	66	55	69	180	55	99	168	132	956	341	287	341	454	649	1577	3649
42	97	36	40	86	47	159	90	96	160	137	948	351	266	306	442	570	1618	3553
43	92	29	58	51	53	160	76	107	177	113	919	293	247	321	452	508	1565	3386
44	84	25	54	67	39	132	64	97	165	108	835	295	296	312	418	571	1536	3428

Zone	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	Sums	9.0	9.1	9.2	9.3	9.4	9.5	Sums
+ 45°	106	37	44	66	58	138	77	94	156	107	883	323	228	327	406	506	1442	3232
46	92	20	46	66	54	143	66	79	162	107	835	299	253	299	413	511	1338	3113
47	90	29	48	73	66	152	53	72	180	105	868	353	220	329	359	483	1427	3171
48	97	28	48	61	64	143	57	75	157	124	854	312	225	252	384	517	1388	3078
49	73	22	33	58	60	137	52	95	168	121	819	369	255	320	376	531	1362	3213
50	91	23	53	59	56	130	55	90	158	108	826	304	231	263	397	527	1385	3107
51	67	23	50	48	54	123	52	76	128	100	721	279	219	292	352	468	1107	2807
52	71	22	49	58	47	103	55	70	127	110	712	257	196	244	308	438	1171	2614
53	73	27	40	50	49	126	52	73	114	91	695	273	167	216	277	391	1043	2367
54	74	15	35	56	43	128	30	51	130	87	649	269	151	221	272	346	969	2228
55	71	16	43	44	36	104	43	78	107	86	628	254	198	222	256	343	953	2226
56	89	16	40	39	48	117	45	61	107	93	655	246	168	198	259	370	1010	2251
57	68	25	40	42	41	103	40	77	93	87	616	247	152	195	274	318	845	2031
58	77	18	34	30	39	102	35	62	102	69	568	212	149	165	271	330	821	1948
59	61	15	42	46	40	102	37	55	86	73	557	240	155	189	217	322	906	2029
60	54	18	28	48	29	97	42	59	80	64	519	202	143	186	209	306	878	1924
61	60	9	27	37	27	84	32	51	107	67	501	218	159	157	243	267	841	1885
62	62	14	43	30	24	83	36	45	94	54	485	158	123	156	224	258	775	1694
63	58	8	19	46	23	93	36	36	83	50	452	169	126	140	194	224	609	1462
64	36	12	27	38	17	79	19	34	82	68	412	135	98	124	178	207	588	1330
65	51	14	23	32	23	65	28	50	66	51	403	162	93	126	170	229	649	1429
66	12	16	13	39	18	59	26	42	66	40	361	127	85	101	144	205	513	1175
67	52	10	15	23	24	67	22	35	62	43	353	137	86	101	134	159	484	1101
68	30	12	14	23	21	60	16	38	51	31	296	112	85	88	131	169	444	1029
69	50	8	26	28	10	56	25	30	52	29	314	112	75	88	134	140	375	924
70	39	7	14	26	17	46	23	25	43	34	274	117	65	74	102	151	432	941
71	33	7	16	17	12	39	11	31	45	27	238	96	71	75	109	151	412	914
72	37	6	14	23	14	43	19	17	27	27	227	93	47	59	94	114	388	795
73	25	3	11	21	13	46	14	26	30	17	206	74	67	77	83	116	350	767
74	24	6	17	10	9	36	13	20	38	26	199	79	60	70	82	105	375	771
75	22	3	8	13	16	36	9	25	33	22	187	73	51	46	69	99	290	628
76	22	7	15	18	7	33	12	25	39	30	208	63	51	61	72	115	307	669
77	19	6	11	22	10	35	4	17	26	24	174	50	40	56	68	113	350	677
78	20	5	6	7	12	23	7	12	21	17	130	72	37	53	60	104	326	652
79	16	2	6	17	4	24	7	15	39	21	151	70	29	47	76	83	272	577
80	9	5	7	14	14	15	11	20	23	17	135	33	44	50	57	98	311	593
81	12	4	5	8	8	15	13	16	14	20	115	40	32	57	76	89	383	677
82	10	1	7	9	4	34	14	13	20	8	120	39	17	38	71	89	351	605
83	5	...	6	3	5	18	6	12	19	12	86	51	27	36	62	66	316	558
84	10	2	6	4	9	18	8	8	17	7	89	32	14	30	49	72	234	431
85	9	1	4	1	3	19	5	6	12	16	76	26	10	18	31	52	180	317
86	4	1	5	4	2	7	6	4	11	7	51	22	9	10	30	33	175	279
87	6	1	3	...	1	7	1	3	8	7	37	13	11	15	24	24	89	176
88	4	...	...	...	...	6	2	1	8	1	22	11	3	8	12	22	59	115
89	...	...	...	...	...	...	...	2	2	1	5	3	2	2	5	4	16	32
Sums	5622	1778	3650	4609	3101	9788	1189	6799	10963	7596	58095	23277	15615	20734	31278	34951	111276	237131

The following table shows the reduction of the Durchmusterung magnitudes to those of the scale of equable distribution.

Mag. DM.	Number as bright North of Equator.	Limiting mag.	Mean mag.	Mag. DM.	Number as bright North of Equator.	Limiting mag.	Mean mag.
1.0	4	0.81	−.02	5.7	1265	5.54	5.50
1.1	5	0.99	0.90	5.8	1441	5.65	5.59
1.2	6	1.14	1.07	5.9	1489	5.67	5.66
1.3	8	1.38	1.26	6.0	2107	5.96	5.82
1.7	10	1.56	1.47	6.1	2213	6.00	5.98
2.0	27	2.38	2.03	6.2	2506	6.10	6.05
2.1	29	2.44	2.41	6.3	2781	6.19	6.15
2.2	33	2.54	2.49	6.4	2882	6.22	6.21
2.3	38	2.66	2.60	6.5	4121	6.51	6.37
2.5	44	2.78	2.72	6.6	4280	6.54	6.53
2.6	45	2.80	2.79	6.7	4737	6.62	6.58
2.8	48	2.85	2.82	6.8	5638	6.77	6.70
3.0	79	3.26	3.07	6.9	5875	6.80	6.79
3.1	83	3.30	3.28	7.0	8016	7.06	6.94
3.2	95	3.41	3.36	7.1	8361	7.09	7.07
3.3	107	3.51	3.46	7.2	9345	7.18	7.14
3.4	113	3.55	3.53	7.3	10701	7.29	7.24
3.5	143	3.75	3.65	7.4	11217	7.33	7.31
3.6	145	3.76	3.75	7.5	14077	7.52	7.43
3.7	155	3.81	3.79	7.6	14686	7.56	7.54
3.8	169	3.88	3.85	7.7	16223	7.64	7.60
3.9	175	3.91	3.90	7.8	18707	7.75	7.70
4.0	229	4.13	4.03	7.9	19698	7.80	7.78
4.1	246	4.19	4.16	8.0	25320	8.00	7.90
4.2	275	4.28	4.24	8.1	27098	8.06	8.03
4.3	296	4.34	4.31	8.2	30748	8.16	8.11
4.4	303	4.36	4.35	8.3	35357	8.28	8.22
4.5	364	4.51	4.44	8.4	38458	8.35	8.31
4.6	379	4.55	4.53	8.5	48246	8.53	8.44
4.7	415	4.62	4.59	8.6	52435	8.60	8.57
4.8	468	4.72	4.67	8.7	59234	8.70	8.65
4.9	484	4.75	4.74	8.8	70197	8.84	8.77
5.0	679	5.03	4.90	8.9	77793	8.93	8.88
5.1	715	5.07	5.05	9.0	101069	9.14	9.04
5.2	801	5.16	5.12	9.1	116685	9.26	9.20
5.3	896	5.26	5.21	9.2	137422	9.39	9.33
5.4	933	5.29	5.27	9.3	168699	9.56	9.48
5.5	1119	5.44	5.37	9.4	203652	9.72	9.64
5.6	1149	5.46	5.45	9.5	314925	10.07	9.91

NOTE. Since writing the above, I have received the following kind explanation from Professor Schönfeld.

"Bei der Bonner Durchmusterung sind eigentlich verschiedene Theile zu unterscheiden, besonders zwei, die Zonenbeobachtungen an dem kleinen Fernrohr von 34 Linien Oeffnung, und die von Professor Argelander am Meridiankreise ausgeführten Revisionen. Als dritter Theil könnten die von Krüger und mir

beobachteten Revisionszonen, beobachtet an 5- und 14½-fussigen Fernröhren, betrachtet werden, doch stimmen diese bezüglich der Unterscheidung der Sterngrössen völlig mit unsern Beobachtungen am kleinern Fernrohr überein.

Bei diesen letztern war es ursprünglich die Absicht, in der Schätzung nichts genauer als die *halbe Grösse* anzugeben, also eine Scale 1^m, $1.2^m = 1^m5$, $2^m = 2^m0$, $2.3^m = 2^m5$, etc., in gleichen Intervallen von einer halben Grösse fortschreitend einzuführen. In den von Thormann beobachteten Zonen, sowie in den wenigen von Argelander kommen daher fernere Unterscheidungen nicht vor, und ebensowenig in der ersten von mir und von Krüger beobachteten. Erst ungefähr seit dem Ende des Jahres 1854 haben wir, mit wachsender Uebung, angefangen, *besonders auffällige* Unterschiede der Helligkeit von der nächstliegenden halben Grösse hervorzuheben; z. B. wurde ein Stern, der zu den schwächsten der Grösse 7 gezählt wurde, ohne jedoch [sonst] der Abtheilung $7.8^m = 7^m5$ zuzugehören, durch ein beigefügtes s (= schwach) unterschieden; ein Stern, der zu der hellsten der Abtheilung 7.8^m gezählt wurde, ebenso durch ein beigefügtes gt (= gut). Da nun die Abtheilung 7 die Sterne von 6^m75 bis 7^m25 umfasst, die Abtheilung 7.8^m die von 7^m25 bis 7^m75, etc., so wäre es zweckmässig gewesen, die Unterabtheilungen als die Grössen 7^m125, 7^m375 . . . zu definiren. Dies haben wir jedoch, wie eben bemerkt, nicht gethan, sondern nur die auffälligsten Unterschiede hervorgehoben, und deshalb die Bezeichnung "7^m s" nicht $= 7\frac{1}{8}$, sondern $= 7^m3$ genommen.

In dieser Zeit sind also die Grössen bei den Beobachtungen in 6 Unterabtheilungen getheilt worden; und es umfasst also z. B.

die Original-Bezeichnung 7^m	die Grössen	6^m9, 7^m0, 7^m1
" " 7^m s	" "	7^m2
" " 7.8^m gt	" "	7^m3
" " 7.8^m	" "	7^m4, 7^m5, 7^m6
" " 7.8^m s	" "	7^m7
" " 8^m gt	" "	7^m8
" " 8^m	" "	7^m9, 8^m0, 8^m1

etc.

Dass übrigens die Unterscheidung nicht alsbald noch weiter ausgedehnt wurde, hatte seinen Grund weniger darin, dass wir die Grössen 7^m4, 7^m5, 7^m6 z. B. nicht eben so gut von einander hätten unterscheiden können, wie die erstere von 7^m3 und die letztere von 7^m7; sondern vielmehr darin, dass die Beobachtungen sich meist so drängten, dass wir nicht Zeit fanden, die entsprechenden Noten während der Beobachtung zu *schreiben*. Es sind daher auch im Allgemeinen in den sternarmen Zonen mehr feinere Unterschiede notirt worden, als bei den Beobachtungen in der Milchstrasse.

Erst im Jahre 1857, als in vielen Gegenden des Himmels schon ziemlich grosse Declinationen, also durchschnittlich weniger reiche Zonen erreicht waren, gewöhnten wir uns daran, auch die bis dahin nicht besonders unterschiedenen Zehntel der Grössen nunmehr zu unterscheiden, sodass also von dieser Zeit an die Grössenklasse in 10 Abtheilungen getheilt wurde; doch ist dabei zu bemerken, dass auch dann, und bis zum Schluss der Arbeit, die Zehntel 1, 4, 6 und 9, besonders aber 1 und 6 viel seltener geschätzt wurden, als die übrigen.

Nach einem ungefähren, aber wahrscheinlich nicht sehr falschen Ueberschlage werden auf die erste dieser 3 Perioden etwa 20, auf die zweite 50, auf die dritte 30 Procent aller Beobachtungen fallen: die letzteren fast nur in Declinationen über 50°.

Argelander's Revisionen im Meridiane beginnen im Herbste 1853, und sind bei ihnen dieselben 3 Perioden zu unterscheiden, doch beginnen die zweite und dritte etwas früher, sodass auf die erste etwa 12 Procent, auf die zweite etwa 45 Procent, auf die dritte der Rest (etwas mehr als 50 Procent) zu rechnen sind. Die Zahl seiner Grössenschätzungen, soweit sie in das Bonner Sternverzeichniss eingegangen sind, mag etwa 35000 oder $\frac{1}{9}$ aller betragen.

Zur Beurtheilung der definitiven Grössen, wie sie gedruckt sind, erlaube ich mir noch eines hinzuzufügen. Sehr oft beruhen dieselben auf 2 Beobachtungen, welche um eine ungerade Anzahl von Zehntel-

grössen von einander abweichen. In diesem Falle ist im Allgemeinen das Zehntel so abgerundet, dass der Stern *schwächer*, als das wahre Mittel angiebt, angesetzt ist. Z. B. ein Stern, der in einer Zone 8.9^m, in der andern 9^m geschätzt ist, findet sich im Sternverzeichniss mit der Grösse 8^m8, nicht 8^m7. Eine Ausnahme ist nur dann gemacht, wenn Grund zu der Annahme vorhanden war, dass die Zonen im Allgemeinen zu schwache Schätzungen ergeben, oder wenn der Stern bei Bessel als heller angegeben war."

Argelander's Uranometria Nova. This catalogue, which I denote by A., is recognized as the standard for the magnitudes of the bright stars, as the Durchmusterung is for the telescopic stars. It contains all the stars easily visible to the naked eye in Bonn with their magnitudes to thirds of a unit. The observations were made with the naked eye. It is also generally accepted as the final authority in regard to the application of Bayer's Greek letters, although I have occasionally followed Baily in regard to the numbers attached to these letters. A list of errors of this catalogue has been published by Argelander; Astron. Nachrichten, vol. 26, col. 318. I have noted also the following

page 14 3rd Star from the end of Cephei. Probably $DM + 85^\circ$ 74 was seen, not $DM + 85^\circ$ 78 as given.
„ 15 *o* Persei is Flamsteed 38.
„ 20 P. VIII. 19 is 30 Lyncis.
„ 55 114 Tauri. For o (Greek type) read o (Roman type).
„ 61 *P* Leonis is 28 H.
„ 67 95 Leonis. For o (Greek type) read o (Roman type).
„ 68 P. X. 86 is 29 Sextantis.
„ 68 47 Leonis minoris is invisible. Argelander probably observed 46 Ursae majoris.
„ 96 for P. VII. 110 read P. VII. 100.
„ 103 78 Virginis. For o (Greek type) read o (Roman type).

The following table serves to convert the magnitudes of the Uranometria nova into those of our scale.

Mag. A.	Number as bright North of equator.	Limiting mag.	Mean mag.
1	6	1.14	0.32
	9	1.47	1.31
	11	1.64	1.68
	29	2.43	2.09
	43	2.76	2.62
	53	2.93	2.84
	89	3.36	3.16
4	138	3.72	3.55
4.3	175	3.91	3.82
4	271	4.27	4.10
4.5	354	4.49	4.39
5.4	444	4.68	4.59
5	800	5.16	4.94
5.6	903	5.26	5.22
6.5	1078	5.41	5.33
6	2340	6.04	5.77

Heis's Atlas Coelestis Novus. Catalogus Stellarum. I shall designate this catalogue by a German H. It is on the same plan as Argelander's Uranometria but is carried a third of a magnitude further. For stars brighter than the sixth, it is substantially a corrected copy of Argelander for though the author says that he made it a rule not to consult the Uranometria, the magnitudes of that work must have been so impressed upon his mind that he was greatly influenced by them; for as will be seen in another chapter, the mean disagreement of the two catalogues is far less than the square root of the sum of the squares of their mean errors. This is simply incontrovertible; and were their errors as small as their mean disagreement would indicate, both observers would have been to blame for not giving their magnitudes to hundredths.

Some copies of this catalogue contain two pages of corrigenda. There is a list of others by Schönfeld in the Vierteljahrsschrift der Astronomischen Gesellschaft. Vol 8, page 67. I have noted a considerable number of other errors. The following list embraces all that are marked in my copy.

Page VIII note. For $\delta = 30^\circ\ 27'$ read $\delta = 31^\circ\ 37'$.
„ XII Line 2. For sexta read septima.
„ „ „ 5. Sentence inserted in Heis's corrigenda.
Corrigenda 1st page 5th line from the bottom. For "b loco b^1", read "dele b^1".
Same page. 4th line from the bottom. For "dele b^2" read "b loco b^2".
Second page of corrigenda. 4th line from the bottom. For "dele g^1", read "g loco g^1".
Second page of corrigenda. 3rd line from the bottom. For "g loco g^2", read "dele g^2".
Page 1, Number 14. For B.A.C. read R.
„ 1 „ 16 Strike out the asterisk.
„ 1 „ 20 4966 B.A.C. = 8 Ursae minoris.
„ 3 „ 4 Add an asterisk.
„ 3 „ 8 This is 2 Draconis.
„ 3 „ 13 Dele second line.
„ 3 „ 16 For 4122 B.A.C. read 4112 B.A.C.
„ 3 „ 29 For 2983 R. read 2985 R. and change the place to R.A. 197° 7′ Dec. + 68° 3′.
„ 4 „ 37 For 26209 LL. RA. 212° 50′, Dec. + 57° 22′ read {+ 57° 1498 B. RA. 212° 49′, Dec. + 57° 22′ / + 57° 1499 B. RA. 212° 52′, Dec. + 57° 22′}
„ 4 „ 46 This is 294 of Struve's 2nd Catalogue.
„ 7 „ 112 Dele the asterisk.
„ 7 „ 161 Dele 18.
„ 9 „ 216 = 7178 B.A.C.
„ 9 „ 220 = 7299 B.A.C.
„ 10 „ 33 = 440².
„ 11 „ 40 The places should be

AR.	Decl.
322° 52′	+ 66° 5′
322 54	+ 66 8

Page 11, Number 51. = Σ 2836.
„ 11 „ 52 Strike out this star which is Heis 196 Cygni.
„ 11 Note For "min 5^m" read "min 6^m5".
„ 12 Number 76 = 7799 B.A.C.
„ 12 „ 78 = Σ 2903.
„ 12 „ 88 For 7799 B.A.C. read 7871 B.A.C.
„ 13 „ 99 Decl. + 67° 48′. This is Σ 2947.
„ 13 „ 101 = Σ 2950.
„ 13 „ 131 For B.A.C. read R.
„ 15 „ 19 = Σ 3049.
„ 16 „ 35 Transpose the designations of the two stars.
„ 16 „ 40 = 16 Cassiopeae.
„ 16 „ 46 Should be 175 B.A.C. RA. 8° 22′, Dec. + 65° 20′.
„ 17 „ 80 = 35 Cassiopeae.
„ 18 „ 99 Dele the asterisk.
„ 18 „ 120 = Σ 302.
„ 18 Insert as number 127* DM. 71° 201′ RA. 48° 50′ D. 71° 21′. Mag. 6.7. This star is on map II.
„ 19 „ 9 = *g*.
„ 19 „ 20 = Σ 268.
„ 19 „ 21 = Σ 279.
„ 20 „ 61 For B.A.C. read R.
„ 21 „ 80 Should be DM. + 46° 762, AR. 49° 53′, Dec. + 46° 27′.
„ 21 „ 94 = 38 Persei.
„ 22 „ 108 Should read 1219 B.A.C.
„ 22 „ 109 „ „ 1228 B.A.C.
„ 22 „ 115 „ „ 73^2.
„ 23 „ 9 For 1415 B.A.C. read 3936 A.Oe.
„ 23 „ 19 Should read DM. 68° 286, AR. 54° 22′, Dec. + 68° 3′.
„ 24 „ 29 „ „ 68° 310 B. AR. 59° 35′, + 68° 8′.
„ 25 „ 59 = 8 Camelopardali.
„ 25 „ 77 = 18 Camelopardali.
„ 25 „ 79 = 19 Camelopardali.
„ 26 „ 114 For 2659 B.A.C. read 2059 R.
„ 26 „ 115 Should be 2650 B.A.C.
„ 26 „ 117 For 2092 R. read 2722 B.A.C.
„ 28 „ 28 Dele 475^2.
„ 28 „ 32 Should be 44484 L.L. AR. 339° 12′, Dec. + 46° 24′.
„ 30 „ 51 = 2732 B.A.C.
„ 30 „ 56 = 30 Lyncis.
„ 31 „ 65 Add an asterisk and for 33 read 32.
„ 31 „ 67 Dele asterisk. This is 33 Lyncis.
„ 31 Insert as number $38\frac{1}{2}$* DM. 47° 1660, RA. 137° 38′, Dec. + 47° 34′, Mag. 6.7. Heis has this star on his maps III and IV.
„ 34 „ 66 = 521^2.
„ 34 „ 80 Should be 2469 R. AR. 152° 9′, Dec. 42° 10′.
„ 35 „ 115 Dele asterisk, and for 40 read 46.
„ 35 „ 128 For B.A.C. read R. both lines.
„ 36 „ 148 Should be 3904 B.A.C.

Page 37, Number 177. Dele 241^2.
„ 38 „ 200 = 4300 B.A.C.
„ 38 „ 208 = 4392 B.A.C.
„ 40 „ 43 15 Canum venat. = 4408 B.A.C.
17 „ „ = 4415 B.A.C.
„ 40 „ 45 = 261^2.
„ 40 „ 57 Dele 269^2.
„ 40 „ 61 = 269^2.
„ 41 „ 81 and 82. Brace the star identified with 82 as a part of 81. And insert for 82 4632 B.A.C. AR. 206° 21′, Dec. + 35° 10′.
„ 42 „ 25 = 500^2.
„ 43 „ 42 = 8345 B.A.C.
„ 43 „ 61 = 26 Andromedae = 5^2.
„ 44 „ 86 = 178 B.A.C.
„ 44 „ 87 = 184 B.A.C.
„ 44 „ 101 = 515^2.
„ 45 „ 108 = Σ 108.
„ 46 All the stars of Equuleus except 10 are wrongly identified. They should be as follows
1 = 7255 B.A.C.
2 = 7276 B.A.C.
3 = 7302 B.A.C.
4 = 40806 L.L.
5 = 7318 B.A.C.
6 = 7324 B.A.C.
7 = 20^h 484 P.
8 = 7350 B.A.C.
9 = 41136 L.L. / 41147 L.L.
11 = 7372 B.A.C.
12 = 7380 B.A.C.
13 = 7405 B.A.C.
14 = 41533 L.L.
15 = 7421 B.A.C.
16 = 41615 L.L.
„ 47 „ 7 For W read W^2.
„ 47 „ 11 „ „ „ „
„ 47 „ 17 = 7528 B.A.C.
„ 47 „ 22 For W read W^2.
„ 47 „ 26 „ „ „ „
„ 47 „ 28 „ „ „ „
„ 48 „ 38 „ „ „ „, both lines.
„ 48 „ 54 „ „ „ „. Dele π, and 29.
„ 48 „ 55 Add π. Dele 28, and add 29 in its place.
„ 48 „ 56 Add 28 in col. headed Flamst.
„ 48 „ 62 For W read W^2.
„ 49 „ 64 „ „ „ „
„ 49 „ 65 „ „ „ „
„ 49 „ 83 „ „ „ „

Page 49, Number 84. For W read W².
„ 49 „ 85 „ „ „ „
„ 49 „ 89 „ „ „ „
„ 49 „ 96 „ „ „ „
„ 50 „ 110 „ „ „ „
„ 54 „ 71 = 299 B.A.C.
„ 54 „ 77 = 321 B.A.C.
„ 56 „ 12 Add 5 in Flamst. col.
„ 58 „ 42 = 834 B.A.C.
„ 59 „ 54 Insert before 54
54ᵃ′ = 45 Arietis = 901 B.A.C. AR. 41ᵐ 55′, Decl. + 17° 44′.
Also, change 54 to 54ᵇ and strike out the first line.
„ 60 „ 5 Above, in cols. 4, 5 and 6, place 45° 992 B., RA. 70° 16′, Dec. + 45° 36′, and connect by brackets.
„ 61 „ 48 Above 5ʰ 702 W in col. aliorumque, place 5ʰ 691 W and connect by bracket: above 81° 4′ in col. AR. place 81° 0′: above + 32° 38′ in col. Decl., place + 32° 42′ and connect by bracket.
„ 61 „ 49 In col. Decl. for + 34 18, read 54 18.
„ 62 „ 56 Over 10569 L.L. write 10533 L.L. and 10560 L.L. and connect the three lines by bracket; over 82 30 write 82 17 and 82 25; over + 37 53 write + 37 52, and + 37 54, and connect by bracket.
„ 62 „ 72 For 1857 B.A.C. read 1850 B.A.C.
„ 62 „ 75 = 1875 B.A.C.
„ 62 „ 83 For + 48 51 read + 48 57.
„ 65 „ 35 For 1182 B.A.C. read 1177 B.A.C., and for 55 7 read 55 9.
„ 66 „ 59 For A^1 read A.
„ 66 „ 61 Dele A^2.
„ 67 „ 89 For 4ʰ 311 W read 1342 B.A.C.
„ 67 „ 91 For δ^1 read δ.
„ 67 „ 92 For 1342 B.A.C. read 4ʰ 311 W.
„ 67 „ 94 Dele δ^2.
„ 67 „ 99 Dele δ^3.
„ 67 „ 100 Dele υ^1.
„ 68 „ 101 For υ^2 read υ.
„ 68 „ 130 For c^1 read c.
„ 69 „ 134 Dele c^2.
„ 70 „ 165 For 5ʰ 606 W read 1734 B.A.C.; for 80° 13′, read 81° 16′; and for + 18° 15′ read + 18° 26′.
„ 70 „ 175 = 1793 B.A.C.
„ 71 „ 33 Under 39 write 40; under 2275 B.A.C., write 2278 B.A.C.; for 102 38 read 102 28, and under it write 102 38, for + 26 6 read + 26 16, and under it write + 26 6 and inclose the whole in brackets.
„ 71 „ 38 For ω^1 read ω.
„ 72 „ 45 Dele ω^2.
„ 72 „ 70 Dele b^1.
„ 74 „ 16 Dele δ^3.
„ 75 „ 28 For 2647 read 2636.

Page 75. Insert as number 29½* 2647 B.A.C., RA. 117° 21′, Dec. + 9° 2′, Mag. 6.7.
„ 75, Number 33. In col. Dupl. Str. write 1168.
„ 75 „ 36 = 2673 B.A.C.
„ 75 „ 12 Dele μ^1.
„ 75 „ 13 For μ^2 read μ.
„ 76 „ 47 For 127 59 read 128 2.
„ 77 „ 57 = 2991 B.A.C.
„ 77 „ 66 Dele α^1.
„ 77 „ 74 For α^2 read α.
„ 78 „ 8 For + 23 34 read + 23 36.
„ 79 „ 27 For 9ʰ 809 W read 9ʰ 780 W, for 144 27 read 144 6; for + 19 21 read + 19 32.
„ 79 „ 32 In col. Flamst. place 20.
„ 80 „ 66 In col. Flamst. place 42.
„ 83 „ 161 For 177 32 read 177 34.
„ 84 „ 1 Dele 1596.
„ 84 „ 2 In col. Dupl. Str. place 1596.
„ 85 „ 13 In col. Flamst. place 9 opposite 4142 B.A.C., and 10 opposite 4147 B.A.C.
„ 85 „ 14 = 4152 B.A.C.
„ 85 „ 37 For 4232 B.A.C. read 12 757 *W*.
„ 87 „ 1 For + 13° 10′ read + 13° 16′.
„ 88 „ 28 For 14° 271 B. read 14° 2718 B.
„ 90 „ 102 In col. Dupl. Str. place 288².
„ 90 „ 103 „ „ „ „ „ 289².
„ 91 „ 135 = 5084 B.A.C.
„ 93 „ 9 Dele 10.
„ 93 „ 15 In col. Flamst. place 14.
„ 93 „ 27 For o (Greek type) read o (Roman type).
„ 93 „ 39 In col. Flamst. place 34.
„ 94 „ 48 For 16ʰ 147 P. read 5581 B.A.C., and for 16ʰ 149 P. read 5582 B.A.C.
„ 94 „ 63 = 5647 B.A.C.
„ 94 „ 69 Dele 49.
„ 96 „ 108 For 228² read 328².
„ 96 „ 110 For 31545 read 31544.
„ 96 „ 117 For ω read *w*.
„ 98 „ 192 For 6154 read 6151, and write under it 6152 B.A.C., and connect by brackets.
„ 98 „ 197 In col. Dupl. Str. place 344².
„ 101 „ 33 = 6456 B.A.C.
„ 101 „ 35 In col. Dupl. Str. place 525².
„ 103 „ 41 „ „ „ „ „ 390².
„ 104 „ 70 In col. Flamst. place 30 opp. 6962, and 31 opp. 6965, and dele bracket. For o^1 (Roman type) read o^1 (Greek type).
„ 104 „ 75 For o^2 (Roman type) read o^2 (Greek type).
„ 105 „ 92 In col. Flamst. place 43.
„ 105 „ 98 For 7091 read 7085.

Page 105, Number 100. For 7085 read 7091.
„ 107 „ 159 In col. Litt. Bay. al. place *A*.
„ 107 „ 161 Dele *A*.
„ 108 „ 179 = 7489 B.A.C.
„ 112 „ 1 In col. Flamst. place 2.
„ 113 „ 15 In col. Litt. Bay. al. place ι.
„ 113 „ 24 In col. Flamst. place 14.
„ 113 „ 29 In col. Litt. Bay. al. place φ^1.
„ 115 „ 98 Dec. + 8° 10′.
„ 117 „ 156 In col. Flamst. place 93.
„ 118 „ 25 For 997 B.A.C. read 5925 L.L.
„ 118 „ 27 For 1013 read 997.
„ 118 „ 29 = 1013 B.A.C.
„ 125 „ 20 In col. Flamst. place 68.
„ 127 „ 18 = 2070 B.A.C.
„ 127 „ 19 For 2070 B.A.C. read 12278 L.L.
„ 127 At bottom of page read 1868 for 1856.
„ 128 „ 76 In col. Flamst. place 24; in col. Dupl. Str. place 169^2.
„ 128 „ 77 Dele 24.
„ 129 „ 105 For 16559 L.L. read 2825 B.A.C.
„ 132 „ 22 In col. Flamst. place 6.
„ 133 „ 35 „ „ „ „ 12.
„ 134 „ 18 „ „ „ „ 1.
„ 139 „ 90 Dele λ^1.
„ 139 „ 91 For λ^2 read λ.
„ 139 „ 113 Dele b^2.
„ 139 „ 115 For b^3 read b^2.
„ 140 „ 117 For χ^1 read χ.
„ 140 „ 118 Dele χ^2.
„ 140 „ 143 In col. Flamst. place 48.
„ 141 „ 1 = 3286 B.A.C.
„ 141 „ 18 For 140 9 read 149 9.
„ 141 „ 19 = 3449 B.A.C. In col. Flamst. place 14.
„ 142 „ 35 For 5^h read 10^h.
„ 143 „ 10 For ^{2} read A^2.
„ 143 „ 21 For 503 M. read 12^h 46 W^2, and for 180 45 read 180 59.
„ 144 „ 38 = 4230 B.A.C.
„ 146 „ 113 For o (Greek type) read o (Roman type).
„ 147 „ 136 = 4665 B.A.C.
„ 147 „ 157 For o^1 read o.
„ 147 „ 159 Dele o^2.
„ 149 „ 27 = 3975 B.A.C.
„ 149 „ 33 For B^2 read W^2.
„ 152 „ 41 For B^2 read B.
„ 154 „ 31 = 5760 B.A.C.
„ 154 „ 44 For *c* read *e*.
„ 155 „ 66 For + 13 59 read + 13 55.
„ 155 „ 71 = 5985 B.A.C.
„ 156 „ 93 For 268 16 read 268 21.

Page 158, Number 30. For ω^1 read ω.
„ 158 „ 35 Dele ω^2.
„ 159 „ 50 = 6679 B.A.C.
„ 159 „ 54 Dele *k*.
„ 161 „ 123 = 7130 B.A.C.
„ 162 „ 14 Dele ξ^1.
„ 162 „ 15 For ξ^2 read ξ.
„ 162 „ 35 Dele ζ^1.
„ 162 „ 36 Dele ζ^3.
„ 162 „ 38 For ζ^2 read ζ.
„ 163 „ 2 For ψ read γ.
„ 163 „ 4 For ξ read μ.
„ 164 „ 28 For *d* read c^2.
„ 165 „ 12 = 6161 B.A.C.
„ 166 „ 28 = 6343 B.A.C. For — 23 38 read — 23 37.
„ 166 „ 43 Dele ξ^1.
„ 166 „ 44 For ξ^2 read ξ.
„ 167 „ 70 Dele h^1.
„ 167 „ 71 For h^2 read *h*.
„ 167 „ 76 Dele e^1.
„ 167 „ 77 For e^2 read *e*.
„ 167 „ 89 In col. Flamst. place 65.
„ 168 „ 24 = 7221 B.A.C.; for — 13 4 read — 13 5.
„ 169 „ 37 For 314 49 read 314 54; for — 18 1 read — 18 2.
„ 170 „ 7 = 7242 B.A.C.
„ 171 „ 42 = 7672 B.A.C.
„ 171 „ 47 In col. Flamst. place 36.
„ 172 „ 66 For 7793 read 7804; for 334 11 read 333 59, and for — 7 58 read — 7 56.
„ 172 „ 84 For g^1 read *g*.
„ 172 „ 85 Dele g^2.
„ 173 „ 96 For — 26 56 read — 25 56.
„ 174 „ 146 Dele *h*. For 6.7^m read 6^m.
„ 175 „ 15 For 7986 read 7987.

Page 176—177, Number 3. Cepheus: In col. $\overset{m.}{6}$ for 39 read 38.
„ 176—177 „ 9 Urs. Maj: „ „ $\overset{m.}{6.7}$ for 100 read 101.
„ 176—177 „ 16 Aries: „ „ $\overset{m.}{2}$ place 1; in col. $\overset{m.}{2.3}$ dele 1; in col. $\overset{m.}{6}$ for 22 read 23; in col. Summa for 80 read 81.
„ 176—177, Mediae . . . . . 22. In col. $\overset{m.}{2}$ for 7 read 8; in col. $\overset{m.}{6}$ for 584 read 585; in col. Summa for 2184 read 2185.
„ 176—177, Omnes . . . . . 57. In col. $\overset{m.}{2}$ for 27 read 28; in col. $\overset{m.}{6}$ for 1533 read 1534; in col. Summa for 5421 read 5422.
„ 176—177, Cassiopeia. In col. $\overset{m.}{5.4}$ for 5 read 4, and in col. $\overset{m.}{6}$ for 24 read 25.
„ 176—177, Pegasus. „ „ $\overset{m.}{4}$ for 3 read 5, and in col. $\overset{m.}{4.5}$ for 5 read 3.

The following table serves to convert Heis's magnitudes into those of our scale of equable distribution.

Mag. Φ.	ψl.	C.S.P. Limiting mag.	C.S.P. Mean mag.
1	5	0.99	0.17
1.2	8	1.38	1.20
2.1	10	1.56	1.47
2	29	2.43	2.07
2.3	41	2.72	2.59
3.2	52	2.91	2.82
3	83	3.22	3.12
3.4	134	3.69	3.51
4.3	165	3.86	3.78
4	256	4.23	4.06
4.5	340	4.46	4.35
5.4	446	4.68	4.57
5	720	5.08	4.89
5.6	921	5.28	5.18
6.5	1288	5.55	5.42
6	2362	6.05	5.83
6.7	3904	6.47	6.28

I have now to notice six catalogues, those of Ptolemy, Sûfi, Ulugh Beg, Tycho Brahe, and Hevelius, from which a great many stars bright enough to be observed were omitted. It was plainly necessary to increase the numbers of stars in these catalogues before deducing the magnitudes. This I did by counting the numbers of stars of different Durchmusterung magnitudes omitted by each of these observers and so obtaining an equation between the true ψl and the number of stars down to brightness l omitted by him. The mean magnitudes had to be determined by integration by quadratures.

Ptolemy. The catalogue of stars which is contained in the Almagest, (and which I designate by Π,) is the oldest which has come down to us. Delambre attributes it to Hipparchus, in which case it dates from B.C. 127. The best printed edition is that of M. Halma, which rests principally on a MS. (No. 2389), written in the uncial characters of the IX century, which is contained in the Bibliothèque Nationale in Paris. This manuscript is a most precious authority, not only on account of its great age, but also because of the extreme care which was taken in making it; and its readings are seldom in fault. It contains, in the catalogue of stars, a considerable number of various readings, in the original hand and ink, for which room has frequently been made by writing the reading first written, a little below the line. M. Halma has not observed this peculiarity of the manuscript; and, unfortunately, the copy of it, taken by him, abounds in errors. For example, the characters for $\frac{1}{6}$ and $\frac{1}{2}$, which do not in the least

resemble one another in the original, are perpetually confused by Halma, who has transcribed them by ς″ and ς′, respectively. It is curious that Halma frequently adopts a reading really taken from this MS., when, according to his table of various readings, no original authority supports him. As the writing is as distinct as possible, these errors must have arisen from some confusion in Halma's papers. But the worst fault in M. Halma's edition of the catalogue, as well as in that of Mr. Baily, is that the words μείζων and ἐλάσσων, which, in the original, are affixed to the magnitudes of many stars to show that they are brighter or fainter, respectively, than the middling stars of the same magnitudes, are entirely omitted by both editors. The result is, that, while the magnitudes of Ptolemy's catalogue are really expressed to thirds, they appear in these editions to be given only to whole magnitudes. Halma gives no reason at all for this strange omission, but simply says, "J'ai omis les μείζων et les ἐλάσσων". Baily admits the importance of the omission, but says that in the old editions these words are so often printed on the wrong line, that he has thought it best not to attempt to give them. M. Schjellerup, in the recent important publication in which has brought to light the work of Sûfi, has undertaken to supply Ptolemy's thirds of a magnitude upon Sûfi's authority. In doing this, however, he seems to have assumed that Sûfi is to be considered as saying that Ptolemy's magnitude agrees with his own, whenever he does not expressly say that it differs. He has, also, adopted much too often, indeed almost invariably, Sûfi's identifications of Ptolemy's stars. For these reasons, and also because Sûfi sometimes misquotes Ptolemy, Ptolemy's thirds of a magnitude as given in the catalogue at the end of this memoir, and as given by Schjellerup, will be found to differ very often. In order, therefore, to place my indications beyond question, I give, in the following table, the readings of eight independent authorities in regard to the μείζων's and ἐλάσσων's, for all stars to the magnitudes of which any of these authorities attach either of these qualifications.

The first column of the table, indicates the star by its constellation and number in the constellation, in Ptolemy's catalogue. The second column gives the modern designation. The third, shows the reading adopted, where μ stands for μείζων and ἐ for ἐλάσσων. The next eight columns, headed P., G., V., F., K., T., S., Sj., shows the readings of eight authorities.

P. is the MS. No. 2389 of the Bibliothèque Nationale, referred to above. This MS. is never wrong in regard to the number of the magnitude, and only four times in regard to the qualification μείζων or ἐλάσσων. Of these four errors, three relate to the three stars without the figure of Aquarius, where P. omits the μείζων's which all the other authorities give. The other case is one in which P. erroneously puts an ἐλάσσων on the line below the one on which it rightly belongs.

G. is the first printed edition of the Almagest, in Greek; that of Grynaeus, Basle, 1538. The allineation of the words μείζων and ἐλάσσων is very bad, so that they are frequently placed between two stars, or even quite on the wrong line. But, except for this negligence of printing, G. is a very accurate authority. There are only two stars for which it makes any error in regard to the assignment of the qualifications μείζων or ἐλάσσων, other than of the sort just mentioned. One of these two stars is the first of Eridanus, where it omits the μείζων which is found in P., K., and T. F. also has a μείζων which was probably intended for this star, although it rather looks as if it were meant for the next one. The μείζων in P. is in this case written much further from the number indicating the magnitude than is usual in that MS., so that it might easily be overlooked. G. was certainly not copied from P., but it is possible that in the original MS. of Ptolemy, this μείζων was written so far from the number that some copyists overlooked it and others put it on the wrong line. The only other star for which G. has an error in the qualification of the magnitudes is the 32nd of Argo, where it omits an ἐλάσσων given by all the other authorities.

V. is MS. No. 2394 of the Bibliothèque Nationale. Halma made some use of it. It is of the XVI century and the μείζων's and ἐλάσσων's are very incorrectly copied. Nevertheless, it seems to have been copied from a good MS., and its text resembles that of G.

F. is MS. No. 2390 of the Bibliothèque Nationale. It was used by Halma, and belongs to the XIII century. It is a good manuscript but frequently omits the qualification of the magnitude.

K. is an Arabic MS. which is contained in the British Museum. It is a very beautiful MS., pretty accurate, and interesting as showing the concordance of Greek and Arabic sources. While it supports few of Schjellerup's variations, it differs more from P., G., V., and F., than these authorities do from each other.

T. is the printed Latin edition of George of Trebizonde. It exhibits a text which rather resembles that of K., than those of G. or F.

S. is a very faulty Latin MS. of the XIII century, belonging to the British Museum. Its text resembles that of T. but seems to be almost verbally the same as the translation of Liechtenstein, which I know only from the few citations made by Baily.

Sj. is Schjellerup. I have enclosed his reading in parenthesis when it agrees with Sûfi. In these cases, I do not think that the least weight should be accorded to the reading. In other cases we really have the authority of Sûfi, a rather early but not apparently remarkably accurate reporter.

These MSS. fall so distinctly into two classes that whenever any one of P., G., V., or F., is supported by any one of K., T., or S. the reading is almost sure to be right.

In the body of the table I have used the following signs to signify the readings of the different authorities.

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					P.	G.	V.	F.	K.	T.	S.	Sj.
7	Ἄρκτου μεγάλης	τ	Ursae maj.			—	—	—	—	—		(ε)
10	..	63 Ḧ	..	ε		—	—	′	0	—	—	0 (—)
12	..	ι	..			—	—	—	—	—	+	(ε)
13	..	κ	..			—	—	—	—	—	—	(ε)
20	..	λ	..			—	—	—	—	—	—	(ε)
21	..	μ	..			—	—	—	—	—	—	(ε)
22	..	ψ	..		μ	—	—	—	—	—	0	0 0
23	..	ν	..			—	—	—	—	—	—	(ε)
24	..	ξ	..			—	—	—	—	—	—	(ε)
2	Δράκοντος	ν	Draconis		μ	—	—	′	—	—	0	0 (0)
3	..	β	..			—	—	—	—	—	—	(ε)
12	..	ε	..			—	—	—	—	—	—	(μ)
14	..	σ	..			—	—	—	—	—	—	(μ)
15	..	υ	..			—	—	—	—	—	—	(μ)
16	..	τ	..			—	—	—	—	—	—	(μ)
19	..	φ	..			—	—	—	—	—	—	(μ)
26	..	θ	..		μ	—	—	—	, —	— ,	0	(0)
27	..	ι	..			—	—	—	—	—	—	(ε)
29	..	α	..			—	—	—	—	—	—	(ε)
30	..	χ	..			—	—	—	—	—	—	(ε)
31	..	λ	..			—	—	—	—	—	—	(ε)
8	Κηφέως	ι	Cephei		μ	—	—	—	—	—	0	(—)
2	Ἀμ ..	δ	..			—	—	—	—	—	—	(μ)
6	Βοώτου	β	Bootae		μ	—	—	—	—	—	0	(—)
7	..	δ	..		μ	—	—	—	—	—	0	(—)
10	..	η	Coronae Borealis			—	—	—	—	—	—	μ
18	..	ρ	Bootae		μ	—	′?	″	—	—	§	′ (—)
1	Στεφάνου βορείου	α	Coronae Borealis		μ	—	—	—	—	—	—	— (0)

G. is the first printed edition of the Almagest, in Greek; that of Grynaeus, Basle, 1538. The allineation of the words μείζων and ἐλάσσων is very bad, so that they are frequently placed between two stars, or even quite on the wrong line. But, except for this negligence of printing, G. is a very accurate authority. There are only two stars for which it makes any error in regard to the assignment of the qualifications μείζων or ἐλάσσων, other than of the sort just mentioned. One of these two stars is the first of Eridanus, where it omits the μείζων which is found in P., K., and T. F. also has a μείζων which was probably intended for this star, although it rather looks as if it were meant for the next one. The μείζων in P. is in this case written much further from the number indicating the magnitude than is usual in that MS., so that it might easily be overlooked. G. was certainly not copied from P., but it is possible that in the original MS. of Ptolemy, this μείζων was written so far from the number that some copyists overlooked it and others put it on the wrong line. The only other star for which G. has an error in the qualification of the magnitudes is the 32nd of Argo, where it omits an ἐλάσσων given by all the other authorities.

V. is MS. No. 2394 of the Bibliothèque Nationale. Halma made some use of it. It is of the XVI century and the μείζων's and ἐλάσσων's are very incorrectly copied. Nevertheless, it seems to have been copied from a good MS., and its text resembles that of G.

F. is MS. No. 2390 of the Bibliothèque Nationale. It was used by Halma, and belongs to the XIII century. It is a good manuscript but frequently omits the qualification of the magnitude.

K. is an Arabic MS. which is contained in the British Museum. It is a very beautiful MS., pretty accurate, and interesting as showing the concordance of Greek and Arabic sources. While it supports few of Schjellerup's variations, it differs more from P., G., V., and F., than these authorities do from each other.

T. is the printed Latin edition of George of Trebizonde. It exhibits a text which rather resembles that of K., than those of G. or F.

S. is a very faulty Latin MS. of the XIII century, belonging to the British Museum. Its text resembles that of T. but seems to be almost verbally the same as the translation of Liechtenstein, which I know only from the few citations made by Baily.

Sj. is Schjellerup. I have enclosed his reading in parenthesis when it agrees with Sûfi. In these cases, I do not think that the least weight should be accorded to the reading. In other cases we really have the authority of Sûfi, a rather early but not apparently remarkably accurate reporter.

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					P.	G.	V.	F.	K.	T.	S.	Sj.
7	Ἄρκτου μεγάλης	τ	Ursae maj.			—	—	—	—	—		(ε)
10	..	63 Ḧ	..	ε		—	—	′	0	—	—	0 (—)
12	..	ι	..			—	—	—	—	—	+	(ε)
13	..	κ	..			—	—	—	—	—	—	(ε)
20	..	λ	..			—	—	—	—	—	—	(ε)
21	..	μ	..			—	—	—	—	—	—	(ε)
22	..	ψ	..		μ	—	—	—	—	—	0	0 0
23	..	ν	..			—	—	—	—	—	—	(ε)
24	..	ξ	..			—	—	—	—	—	—	(ε)
2	Δράκοντος	ν	Draconis		μ	—	—	′	—	—	0	0 (0)
3	..	β	..			—	—	—	—	—	—	(ε)
12	..	ε	..			—	—	—	—	—	—	(μ)
14	..	σ	..			—	—	—	—	—	—	(μ)
15	..	υ	..			—	—	—	—	—	—	(μ)
16	..	τ	..			—	—	—	—	—	—	(μ)
19	..	φ	..			—	—	—	—	—	—	(μ)
26	..	θ	..		μ	—	—	—	, —	— ,	0	(0)
27	..	ι	..			—	—	—	—	—	—	(ε)
29	..	α	..			—	—	—	—	—	—	(ε)
30	..	χ	..			—	—	—	—	—	—	(ε)
31	..	λ	..			—	—	—	—	—	—	(ε)
8	Κηφέως	ι	Cephei		μ	—	—	—	—	—	0	(—)
2	Ἀμ ..	δ	..			—	—	—	—	—	—	(μ)
6	Βοώτου	β	Bootae		μ	—	—	—	—	—	0	(—)
7	..	δ	..		μ	—	—	—	—	—	0	(—)
10	..	η	Coronae Borealis			—	—	—	—	—	—	μ
18	..	ρ	Bootae		μ	—	′?	″	—	—	§	′ (—)
1	Στεφάνου βορείου	α	Coronae Borealis		μ	—	—	—	—	—	—	— (0)

					P.	G.	V.	F.	K.	T.	S.	Sj.
2	Στεφάνου βορείου	β	Coronae Borealis	μ	—	—	—	—	—	—	—	(0)
4	Τοῦ ἐν γόνασιν	x	Herculis		—	—	—	—	—	—	μ	—
6	,,	λ	,,	μ	—	—	—	—	—	÷	÷	—
7	,,	μ	,,	μ	—	—	—	—	—	—	—	—
8	,,	ο	,,	μ	—	—	—	—	—	—	—	—
9	,,	ν	,,	μ	—	—	—	—	÷	—	÷	÷
12	,,	ε	,,		—	—	—	—	μ	—	μ	(—)
17	,,	ρ	,,	μ	—	?′	′	—	—	§	′	—
23	,,	η	,,	μ	—	—	—	—	÷	—	—	—
25	,,	τ	,,	μ	—	—	′	—	÷	—	0	0
2	Λυράς	ε	Lyrae	μ	—	—	—	—	—	—	0	(—)
3	,,	ζ	,,	μ	—	—	—	—	—	—	0	(÷)
6	,,	ϑ	,,		—	—	—	—	ε	—		—
8	,,	8	,,	ε	—	—	,	—	—	—	0	(—)
10	,,	λ	,,	ε	—	—	—	—	—	—	0	(—)
3	Ὄρνιθος	η	Cygni	μ	—	—	—	—	—	—	0	—
8	,,	ι	,,	μ	—	?,	,	—	—	—	0	—
9	,,	x	,,	μ	—	?,	,	—	—	—	0	—
11	,,	λ	,,	μ	—	?,	,	—	—	—	0	—
13	,,	ν	,,	μ	—	?,	—	—	—	0	0	—
14	,,	ξ	,,	μ	—	?,	—	—	—	—	0	—
15	,,	o^1	,,		—	—	—	—	—	—		(μ)
16	,,	32	,,		—	—	—	μ	—	—		(—)
1	Ἀμ ,,	τ	,,	μ	—	—	—	—	—	—	0	(—)
2	Ἀμ ,,	σ	,,	μ	—	—	—	—	—	—	0	*
1	Καρτεπείας	ζ	Cassiepeae	μ	—	—	—	—	—	—	0	—
4	,,	γ	,,	μ	—	—	÷	—	—	—	0	(—)
11	,,	x	,,	ε	—	″	—	—	—	—	0	(—)
3	Περσέως	γ	Persei	ε	—	—	—	—	—	—	0	(—)
6	,,	ι	,,		—	—	μ	—	—	—		(—)
22	,,	ν	,,	μ	—	—	—	—	÷	—	—	(0)
25	,,	ο	,,	ε	—	—	—	—	—	—	′	(—)
26	,,	ζ	,,	μ	—	—	—	—	—	—	′	(§)
6	Ἡνίοχου	ϑ	Aurigae	μ	—	—	—	—	—	—	′	—
7	,,	ε	,,	μ	—	-	—	—	—	—	′	(0)
8	,,	η	,,	μ	—	—	—	—	—	—	—	(0)
10	,,	ι	,,	ε	—	′	—	0	—	—	÷	(—)
11	,,	β	Tauri	μ	—	—	—	0	§	—	—	—
1	Ὀφιούχου	α	Ophiuchi	μ	—	—	0	—	—	—	0	(0)
2	,,	β	,,	μ	—	—	0	—	—	—	—	0
7	,,	δ	,,		—	—	—	—	ε	—		(—)
10	,,	ν	,,	ε	—	,	§	—	—	—	0	—
13	,,	40	,,	μ	—	—	—	—	—	—	0	—
15	,,	ϑ	,,	μ_0	—	—	—	—	—	—	0	(—)
20	,,	φ	,,	μ	—	—	—	—	—	,	0	(0)
22	,,	ψ	,,	μ	—	—	÷	—	—	—	0	(0)

* Mag. altogether omitted, in Sj.

					P.	G.	V.	F.	K.	T.	S.	Sj.
14	Ὀφέως ὀφιούχου	ξ	Serpentis	μ	—	?,	—	—	—	—	0	(—)
17	,,	η	,,	μ	—	?,	—	—	—	—	0	(—)
3	Ἀετοῦ	α	Aquilae	μ	—	?′	—	0	—	—	0	(—)
4	,,	ο	,,	ε	—	?′	—	—	§	—	0	—
8	,,	σ	,,	μ	—	?′	—	—	—	—	0	0
3	Ἀμ ,,	δ	,,	μ	—	—	—	—	—	—	—	—
1	Δελφῖνος	ε	Delphini	ε	—	—	—	—	—	—	—	—
2	,,	ι	,,	ε	—	—	—	—	0	—	0	(0)
4	,,	β	,,	ε	—	′	′	—	—	—	0	(—)
5	,,	α	,,	ε	—	′	′	—	—	—	0	(—)
6	,,	δ	,,	ε	—	′	′	—	—	—	0	(—)
7	,,	γ	,,	ε	—	′	′	—	—	—	0	(—)
1	Ἵππου	α	Andromedae	ε	—	—	—	—	—	—		(—)
2	,,	γ	Pegasi	ε	—	—	—	—	—	—		(—)
3	,,	β	,,	ε	—	—	—	—	—	—		(—)
4	,,	α	,,	ε	—	—	—	—	—	—		(—)
17	,,	ε	,,	μ	—	,	—	—	—	—		(0)
18	,,	π^2	,,	μ	—	,	—	—	—	—		—
19	,,	ι	,,	μ	—	,	—	—	—	—		—
20	,,	x	,,	μ	—	,	—	—	—	—		—
16	Ἀνδρομέδας	φ	Persei	ε	—	?,	—	—	§	—		—
17	,,	υ	,,	μ	—	?,	—	—	0	—		0
18	,,	υ	Andromedae		—	—	—	—	—	μ	—	(μ)
1	Κριοῦ	γ	Arietis	ε	—	?,	—	—	—	—		0
13	,,	μ	Ceti	μ	—	?,	,	—	—	—		(0)
1	Ἀμ ,,	α	Arietis	μ	—	?,	,	—	—	—		(—)
11	Ταύρου	γ	Tauri	ε	—	?,	—	—	—	—		(—)
12	,,	δ^1	,,	ε	—	?,	—	—	—	0		(—)
13	,,	ϑ^1	,,	ε	—	?,	—	—	—	0		(—)
15	,,	ε	,,	ε	—	?,	—	—	—	—		(—)
14	Διδύμων	η	Geminorum	μ	—	?,	—	—	—	—		(—)
15	,,	μ	,,	μ	—	?,	—	—	—	—		(—)
16	,,	ν	,,	μ	—	?,	—	—	—	—		—
2	Ἀμ ,,	x	Aurigae	μ	—	?,	0	—	0	—		—
3	Ἀμ ,,	36 = d	Geminorum		—	—	—	—	—	—		(ε)
2	Καρκίνου	η	Cancri	ε	—	—	—	0	—	—		(—)
3	,,	ϑ	,,	ε	—	—	—	0	—	—		(—)
4	,,	γ	,,	μ	—	—	—	—	—	—		—
5	,,	δ	,,	μ	—	—	0	—	—	—		—
9	,,	β	,,	μ	—	—	—	—	—	—		(0)
1	Ἀμ ,,	π^1	,,	ε	—	—	—	—	—	—		0
2	Ἀμ ,,	x	,,	ε	—	—	—	—	—	—		0
4	Λέοντος	ε	Leonis	μ	—	′	′	—	—	—		(—)
20	,,	δ	,,	ε	—	?,	—	—	—	—		(0)
27	,,	β	,,	ε	—	—	0	—	—	—		(0)
3	Ἀμ ,,	χ	,,	ε	—	?,	—	—	—	§		0
13	Παρθένου	ε	Virginis	μ	—	—	—	—	—	0		(§)
14	,,	α	,,		—	—	—	—	—	—		(ε)

				P.	G.	V.	F.	K.	T.	S.	Sj.
18	Παρθένου	82=*m* Virginis	ἐ	—	—	—	—	—	§		—
24	„	φ „		—	—	—	—	—	—		(ἐ)
26	„	μ „		—	—	—	—	—	—		(μ)
8	Χηλῶν	θ Librae	ἐ	—	—	—	—	—	—		(0)
2	Ἀμ „	48 „	ἐ	—	—	—	—	—	—		(—)
3	Ἀμ „	ξ Scorpii	ἐ	—	—	—	—	—	—		(—)
1	Ἀμ Σκορπίου	65 Ⅎ „		—	—	—	—	—	—		(—)
2	Ἀμ „	45=*d* Ophiuchi	μ	—	—	—	—	—	—		(ἐ)
7	Τοξότου	φ Sagittarii		—	—	—	—	μ	—		(μ)
21	„	τ „	μ	—	—	—	—	—	0		(—)
24	„	α „	ἐ	—	—	—	—	—	—		(—)
1	Ἀμ Ὑδροχόου	2 Ceti	μ	0	—	—	—	—	—		(—)
2	Ἀμ „	6 „	μ	0	—	—	—	—	—		(—)
3	Ἀμ „	7 „	μ	0	—	—	—	—	—		(—)
1	Ἰχθύων	β Piscium	μ	—	?,	—	0	—	—		—
19	Κήτους	Ceti	μ	—	—	—	0	—	—		—
20	„	φ¹ „	μ	—	—	—	0	—	—		—
21	„	ι „	ἐ	—	—	§	0	—	—		(—)
22	„	β „		—	—	ἐ	—	ἐ	—		(μ)
2	Ὠρίονος	α Orionis	ἐ	—	′	′	—	—	—		(—)
3	„	γ „		—	—	—	—	μ	—		μ
		32=*A* „	ἐ	—	?,	—	—	—	—		(—)
		θ „	ἐ	′	′	—	—	—	—		(—)
36	„	τ „	μ	—	?′	—	—	—	—		(—)
38	„	κ „	μ	—	?′	—	—	—	—		(§)
1	Ποταμοῦ	λ Eridani	μ	—	0	0	?,	—	—		(0)
5	Λαγωοῦ	μ Leporis	μ	—	—	—	—	—	—		(—)
6	„	ε „	μ	—	—	—	—	—	—		(—)
9	„	δ „	μ	—	—	—	—	—	—		(—)
10	„	γ „	μ	—	—	—	—	—	—		(—)
11	„	ζ „	μ	—	—	—	—	—	—		(—)
12	„	η „	μ	—	—	—	—	—	—		(—)
14	Κυνός	δ Canis majoris	ἐ	—	?,	—	—	—	—		(0)
18	„	η „	ἐ	—	?′	—	—	—	—		0
15	Ἀργοῦς	φ² Argûs (*c* Puppis)		—	—	—	μ	—	—		(—)
22	„	*p*¹ „	μ	—	—	—	—	—	—		(0)
23	„	*p*² „ (*d* Velorum)	μ	—	—	—	—	—	—		(0)
24	„	*p*³ „ (*e* „)	μ	—	—	—	—	—	—		(0)
25	„	(*a* „)	μ	—	—	—	—	—	—		(—)
26	„	(*b* „)	μ	—	—	÷	—	—	—		(—)
2[illegible]	„	o² „ (*a* Mali)		—	—	μ	—	—	—		—
		(ψ)	ἐ	—	0	—	—	—	—		—
40	„	*b* „ (x)		—	—	ἐ	—	—	—		—
42	„		μ	—	—	—	—	—	—		—
43	„	*g* „ (ν)	μ	—	—	—	—	—	—		0
45	„	*h* „ (τ)	μ	—	?′	—	—	—	—		0
16	Ὑδρου	μ Hydrae		—	—	—	—	—	—		(ἐ)
19	„	β Crateris	μ	—	—	—	—	—	0		—

				P.	G.	V.	F.	K.	T.	S.	Sj.
24	Ὑδρου	γ Hydrae	μ	—	—	—	—	—	—		—
25	„	π „	μ	—	—	—	—	—	—		—
3	Κρατῆρος	δ Crateris		—	—	ἐ	—	—	—		(—)
4	„	ζ „	μ	—	—	—	—	—	—		—
6	„	η „	ἐ	—	—	—	—	§	—		(—)
1	Κενταύρου	2=*g* Centauri	μ	—	—	—	—	—	—		—
2	„	4=*h* „	μ	—	—	—	—	—	—		—
3	„	1=*i* „	μ	—	—	—	—	—	—		—
4	„	3=*k* „	μ	—	—	—	—	—	—		—
12	„	τ (ν) „	μ	—	?′	—	—	—	—		(—)
13	„	υ (μ) „	μ	—	?,	—	—	—	—		(—)
14	„	φ (φ) „	μ	—	?,	—	—	—	—		—
15	„	*m* (χ) „	μ	—	?,	—	—	—	—		(—)
18	„	λ (ζ) „	μ	—	—	—	′	—	—		—
13	Θηρίου	x (τ) Lupi	μ	—	′	—	—	—	*l*		—
15	„	μ (ξ) „	μ	—	—	—	—	—	—		—
18	„	ε 30 Ⅎ „	μ	—	—	—	—	—	—		÷
19	„	δ 33 Ⅎ „	μ	—	—	—	—	—	—		—
3	Θυμιατηρίου	δ (α) Arae	μ	—	—	—	—	—	—		(—)
5	„	α (ἐ¹) „	μ	—	?,	—	—	—	—		—
5	Ἰχθύος νωτίου	ε Piscis Austrini	μ	—	—	—	—	—	—		(0)
1	Ἀμ „		ἐ	—	—	—	—	—	—		*
2	Ἀμ „		ἐ	—	—	—	—	—	—		*
3	Ἀμ „		ἐ	—	—	—	—	—	0	—	*

* Mag. altogether omitted, in Sj.

The following table shows the readings of the different authorities in cases in which there is any difference in regard to the number of the magnitude. The only stars for which I find Baily's reading wrong in this respect are,

17 (Ⅱ) Geminorum . . . = γ. For 4, read 3.
16 (Ⅱ) Ceti . . . . . = η. For 5, read 3.
33 (Ⅱ) Centauri . . . . = δ Crucis. For 2, read 4.
34 (Ⅱ) „ . . . . . = α „ For 4, read 2.
6 Ἀμ (Ⅱ) Piscis austrini = 4. For 3, read 4.

			P.	G.	V.	F.	K.	T.	S.	Sj.
12	Δράκοντος	ε Draconis	4	4	4	4	4	6	4	(4)
13	„	ρ „	4	4	4	4	4	6	4	4
16	„	τ „	5	5	5	5	5	3	5	(5)
28	„	10=*i* „	4	4	4	4	3	4	3	4
5	Τοῦ ἐν γόνασιν	δ Herculis	3	3	3	3	3	3	4	(3)
11	„	ζ „	4	4	4	4	3	3	3	(3)
12	„	ε „	5	5	5	5	4	5	4	(4)
14	„	61 „	3	3	3	3	5	3	5	5
15	„	π „	4	4	4	4	3	4	3	3
7	Λυράς	β Lyrae	3	3	3	3	3	3	4	3
10	„	λ „	4	4	4	4	4	4	3	4

			P.	G.	V.	F.	K.	T.	S.	Sj.
15	Ὄρνιθος	o^1 Cygni	4	4	4	5	4	4	4	(4)
3	Περσέως	γ Persei	3	3	3	3	3	3	4	(3)
5	„	τ „	4	4	4	4	4	4	4	(5)
24	„	ξ „	4	4	4	4	4	4	3	(4)
14	Ἡνιόχου	4 Aurigae (?)	6	6	6	4	6	6	6	6
7	Ὀφιούχου	δ Ophiuchi	3	3	3	3	4	3	3	(3)
13	„		4	4	4	4	4	3	4	4
14	„	36 „	4	4	4	—	4	4	4	4
18	Ὀφέως	ϑ Serpentis	4	4	4	4	4	4	3	(4)
2	Δελφῖνος	ι Delphini	4	4	4	4	4	4	4	(6)
3	„	κ „	4	4	4	4	4	4	4	(6)
11	Ἀνδρομέδας	η Andromedae	4	4	4	4	3	4	3	4
13	„	μ „	4	4	4	4	4	4	3	(4)
14	„	ν „	4	4	4	4	4	4	3	4
10	Ταύρου	88=*d* Tauri	4	4	3	4	4	4	4	(4)
17	„	104=*m* „	4/5	5	5	5	5	4	5	(5)
21	„	β „	3	3	3	3	5	3	5	—
33	„		4	4	4	4	4	5	4	—
17	Διδύμων	γ Geminorum	3	4	4	3	3	3	3	(3)
2	Ἁμ „	κ Aurigae	4	4	4	4	5	4	4	4
1	Καρκίνου	ε Cancri	ν	ν	ν	ν	ν	ν	4	(ν)
19	Λέοντος	60=*b* Leonis	6	6	6	6	5	5	5	5
13	Παρθένου	ε Virginis	5	5	5	5	3	5	3	3
15	„	ζ „	3	3	3	3	3	3	6	3
18	„	82=*m* „	4	4	4	4	4	4	5	4
26	„	μ „	3	3	3	3	3	3	3	(4)
1	Ἁμ Σκορπίου	65 ♐ Scorpii	ν	ν	ν	ν	ν	ν	ν	(4)
1	Αἰγοκέρωτος	α^1 Capricorni	3	3	3	3	4	3	3	3
10	„	υ „	5	5	5	5	5	5	5	(6)
13	„	24=*A* „	4	4	4	4	4	4	6	4
25	„	42 „	4	4	4	4	4	4	4	(5)
18	Ὑδροχόου	δ Aquarii	3	3	3	3	4	3	3	(3)
19	Ἰχθύων	α Piscium	3	3	3	3	4	3	3	3
20	„	ο „	4	4	4	4	5	4	4	(4)
21	„	π „	5	5	5	5	3	5	5	5
22	„	γ_1 „	3	3	3	3	4	3	3	3
23	„	ρ „	4	4	4	4	5	4	4	4
25	„	τ „	5	5	5	5	6	5	5	5
26	„	68=*h* „	6	6	6	6	5	6	6	6
28	„	65=*i* „	6	6	6	6	4	6	6	6
14	Κήτους	ζ Ceti	3	3	3	3	3	3	2	3
15	„	ϑ „	3	3	3	3	4	3	2	3
16	„	γ_1 „	3	5	5	5	3	3	3	3
17	„	21 „	5	5	5	5	5	5	5	(6)
18	„	φ^2 „	5	5	5	5	5	5	5	(6)
7	Κυνός	ν^3 Canis majoris	6	6	6	6	5	6	5	(5)
37	Ἀργοῦς	δ Argûs	2	2	2	2	3	2	3	(3)

			P.	G.	V.	F.	K.	T.	S.	Sj.
41	Ἀργοῦς	φ Argûs	3	3	3	3	3	3	3	2
46	Ὑδρου	ω Hydrae	5	5	5	5	5	5	5	(6)
3	Κενταύρου	1=*i* Centauri	4	4	4	4	4	4	5	4
22	„	ξ „	* 5	5	5	5	5	5	5	(5)
33	„	δ Crucis	4	2	2	2	4	4	4	4
34	„	α „	2	4	4	4	2	2	2	(2)
37	„	ϑ Centauri	4	4	4	4	4	4	3	4
7	Στεφάνου νωτίου	γ Coronae Aust.	4	4	4	4	—	4	4	4
7	Ἰχθύος	ζ Piscis Aust.	5	5	5	5	5	5	5	4
6	Ἁμ „	4 „	4	3	3	3	4	4	4	—

* $P = 3$ afterward changed to 5.

In regard to the identifications, I have followed Baily in almost all cases, but not without reëxamination of many of the difficulties. In the present state of uranography, no doubt some of Baily's identifications could be improved, but he has certainly shown great sagacity. The first of the ἀμόρφωτοι of Cepheus has a latitude and longitude in Ptolemy corresponding to the following place for 1855,

RA.	Dec.
325°.7	58°.3.

The nearest star is,

μ Cephei	324.8	58.1	Var. 4^m to 6.5^m.

There are, also, in the neighborhood

13H Cephei	323.6	56.8	6^m
ν Cephei	325.3	60.4	5^m.

I consider Ptolemy's star to be μ.

Bootes, 10th star. I consider this to be η Coronae Borealis.

Cassiepea, 7th star. I agree with Baily in considering this to be ι Cassiepeae.

Cassiepea, 9th star. This is φ, as Baily makes it.

Ophiuchus, 6th star. Baily finds the best authorities give the Longitude $18\frac{1}{3}°$, whereas it should be $8\frac{1}{3}°$. But P. gives $\left\{\begin{matrix}18\\8\end{matrix}\right\}\frac{1}{3}°$. Baily's authorities make the latitude $33\frac{1}{2}\frac{1}{3}°$, whereas it should be $23\frac{1}{2}\frac{1}{3}°$. K. gives $23\frac{2}{3}°$. Baily is right in supposing the star to be λ.

Ophiuchus, 13th to 18th stars. These stars present one of the greatest perplexities of the whole catalogue.

No. 13. Some authorities make the longitude $26\frac{2}{3}°$ and others $23\frac{2}{3}°$. Baily says that in the former case it is 40 Ophiuchi and in the latter the *Stella Nova* of Kepler, discovered 1604. Now P. gives both readings in the original hand and ink.

No. 14, 15, 16, 17. Some authorities make the latitude north and others south.

P. gives both readings. The following are the identifications which Baily would make in the two cases.

	Lat. North.	*Lat. South.*
14	40 Ophiuchi	36 Ophiuchi
15	40 „	42 „
16	—	44 „
17	—	51 „

It appears, therefore, that the southern latitude is to be preferred and consequently the upper reading of *P*, and consequently $26\frac{2}{3}^{\circ}$ for the longitude of No. 13, which is therefore 40 Ophiuchi. The 16th star seems, however, to be Behrmann's 7 Ophiuchi. Baily has altered the latitude.

Andromeda, 23rd star. Baily strangely says that Ptolemy's place does not agree with o Andromedae, and therefore supposes the star to be 2 Andromedae. But here are the places for 1855

	RA.	Dec.
Ptolemy's place	344°5	42°1
o Andromedae	343.8	41.6
2 „	344.0	42.0

I prefer to consider o to be Ptolemy's star.

Taurus, 24th and 25th stars. I find these to be 43 and ω Tauri.

Taurus, 6th of the ἀμόρφωτοι. I prefer to consider Ptolemy's star to be 129 Tauri, not 128 as Baily does. Perhaps it would be better to suppose Π's No. 6 to be 126 Tauri, and to suppose that Π's No. 5 has disappeared.

Gemini, 9th star. Π's place contradicts his description. I subtract 5° from the longitude and then identify it with 52 Tauri.

Virgo, 6th of the ἀμόρφωτοι. I identify this star with 89 Virginis.

Scorpio, 3rd of the ἀμόρφωτοι. This is 44 Ophiuchi.

Hydrus, 2nd of the ἀμόρφωτοι. This may perhaps be 15 Sextantis.

Putting $\psi_\pi l$ for the number of stars north of the equator brighter than a certain magnitude, according to Ptolemy, I found

$$10 + \log\left(1 - \frac{\psi_\pi l}{\psi l}\right) = 12.78 + 2.6 \log \psi_\pi l.$$

The following table shows how nearly this equation satisfies the facts.

DM. Mag.	log ψl.	Observed ratio omitted.	Calculated ratio omitted.
3.0	1.90	.01	.01
3.1	1.92	.01	.01
3.2	1.98	.02	.01

DM. Mag.	log ψl.	Observed ratio omitted.	Calculated ratio omitted.
3.3	2.03	.02	.01
3.4	2.05	.02	.01
3.5	2.16	.01	.02
3.6	2.16	.02	.02
3.7	2.19	.03	.03
3.8	2.23	.03	.03
3.9	2.24	.03	.04
4.0	2.36	.07	.06
4.1	2.39	.09	.08
4.2	2.44	.11	.10
4.3	2.47	.12	.11
4.4	2.48	.12	.12
4.5	2.56	.14	.17
4.6	2.58	.16	.18
4.7	2.62	.18	.21
4.8	2.67	.22	.25
4.9	2.68	.24	.26
5.0	2.83	.39	.40
5.1	2.85	.40	.41
5.2	2.90	.45	.46
5.3	2.95	.48	.49
5.4	2.97	.50	.50

The reduction of Ptolemy's magnitudes is shown in the following table.

H. Mag.	$\psi_\pi l$.	ψl.	C.S.P. Limit.	C.S.P. Mean.
1	6	6	1.14	0.32
1ε	8	8	1.38	1.26
2μ	10	10	1.56	1.47
2	26	26	2.35	2.01
2ε	31	31	2.49	2.42
3μ	39	39	2.68	2.59
3	128	130	3.67	3.26
3ε	140	143	3.75	3.71
4μ	185	194	4.00	3.88
4	382	555	4.86	4.43
4ε	394	594	4.92	4.89
5μ	396	603	4.93	4.92
5	504	1396	5.62	5.25
6	540	2302	6.03	5.81

Sûfi. M. Schjellerup has lately published a translation of the description of the heavens by the Persian Astronomer Abd-al-rahman al-sûfi, who was born *anno domini* 903 and died in 986. This work is of inestimable value because of the almost modern accuracy of the estimations of magnitudes, and also because of its date. The stars are described by their configurations, usually so minutely as to leave no room for doubt

about their identifications. There are, however, a few cases in which as it seems to me the identifications made by the learned editor are capable of rectification. These I proceed to notice.

Cassiepea, 9th star. Schjellerup identifies this star with θ, but I prefer φ. In the first place, Ptolemy's star, which Sûfi professed to observe, is certainly φ, for we have for 1855,

	RA.	Dec.
H's place	16°8	+ 57°1
φ	17.7	+ 57.5
θ	15.6	+ 54.4

The following is Sûfi's description. "La 9e vient après la 8e [which is μ. This is true of both θ and φ] et se trouve dans le quart méridional de la Voie lactée [true of both], entre la 5e [δ Cassiepeae] et la 8e, [true only of φ not of θ], et est placée sur le coude gauche. Elle est de la cinquième grandeur, et entre elle et l'etoile du genou [δ] il y a environ une coudée [2½°. The real distance of θ is about ¾°, that is ¼ of a cubit. That of φ is 3½°, or 1½ cubits.] Ces quatres étoiles, savoir les 8e, 9e, 5e et 6e, [μ, the 9th, δ, and ε] font une ligne droite un peu courbée [suits φ well enough, but not θ.]"

Aquila, 5th star. Schjellerup makes thus ν. I prefer γ. Between these stars there is a distance of 11° of arc. Ptolemy's place indicates γ. Sûfi says, "La 5e de la troisième grandeur [true of γ, but ν is of the 5th] est la plus boréale des trois connues [γ, α, β. But ν is the southernmost star in the Eagle.] et se trouve sur l'épaule gauche [true of γ, not of ν]. La 6e [φ Aquilae] suit d'une coudée après la 5e [γ is distant about 1½ cubits and ν 5 cubits]".

Taurus, 6th star without the figure. Schjellerup makes this Σ 730 = 167 ♋ Tauri. I rather incline to make it 122 Tauri: otherwise, it is some star which has disappeared. There is some difficulty in determining which star Ptolemy intends. We may therefore consider at once Sûfi's description. What Sûfi says, amounts to this: that the star in question is South of 119 Tauri inclining to the East, and distant more than a span (⅞°) and that it has following it at a distance of more than a cubit (2¼°) a star of the 5½ magnitude. This description will not apply to any star in the heavens, at present. The star which Schjellerup adopts, is due South of 119 Tauri, and is distant 1½° — fully two spans, or nearly a cubit, and as a cubit is Sûfi's favorite unit he would so have described it. It seems to me, therefore, considering the accuracy of Sûfi's descriptions, that we must do one of two things; either suppose the words *cubit* and *span* transposed in the description, in which case 122 Tauri would be the 6th star, and 126 Tauri, called by Argelander 5m and by Heis 5⅔m, would be the star which

follows it; or we must suppose that Sûfi's star has disappeared from the heavens, its place for 1855 being RA. = 81°3, Dec. = + 17°5; and then the following star would be 130 Tauri. The magnitude of 130 Tauri is, however, rather too faint: for both Argelander and Heis make it 6. The hypothesis of the existence of such a star may be considered as deriving some support from Ptolemy, who has a star [the 5th of the *Externi* of Taurus] which there is great difficulty in identifying, and whose place for 1855 would be RA. 83°7. Dec. + 17°0.

The agreement is tolerable, but I prefer the former hypothesis. Schjellerup omits the *following* star from his synopsis of Sûfi's stars.

Leo, 25th star. Schjellerup makes this star p^5. I prefer to consider it as φ. Ptolemy's star is, no doubt, τ. Sûfi's description is as follows. "La 25e de la quatrième grandeur est située au sud de la 24e [σ. This applies better to φ than to p^5] dans une des jambes de derrière, entre elle et la 24e, il y a environ quatre coudées. [The distance from σ to φ is 4 cubits, from σ to p^5 2½ cubits.] La 26e [υ] de la cinquième grandeur suit la 25e, s'inclinant vers le nord [true if φ is taken for the 25th, but quite the reverse if we take p^5], * * * * entre ces deux étoiles, il y a environ deux coudées et demie [true for either φ or p^5]."

Virgo, 8th star. Schjellerup marks this as z, by a misprint for *k*. But I prefer 46 Virginis. The stars are very near together, but Ptolemy's place indicates 46 and not *k*. Sûfi says that the star lies between γ and ϑ Virginis, but is a little to the north of the line joining them. This will not suit *k* which is precisely on the line but indicates either 46 or 48. There is nothing to show which of these stars was intended, but 46 agrees better with Ptolemy's position, and we may suppose that Sûfi saw the same star.

Capricornus. A star not in Ptolemy is inserted in the synopsis as 42 Capricorni. I see nothing in the text to justify this.

Cetus, 19th star. I prefer to consider this as 28 (Heis) Ceti, rather than 17 Ceti, because the former is nearer to 18 Ceti, and Sûfi says that they are very near.

Cetus. An extra star is identified by Schjellerup with 28 Ceti, but I prefer 30 Ceti.

Orion, 30th star. This is plainly *e* Orionis, not υ.

Eridanus, 8th star. Schjellerup makes this o^2; I prefer 98 (♑) Eridani. Sûfi says, "La 8e se trouve devant la 7e [ξ Eridani] vers le sud; il y a entre elles un intervalle d'une coudée et un tiers à peu près [this is true of 98, but o^2 is distant 2 cubits]."

Eridanus, 31st, 32nd, and 33rd stars. Schjellerup calls these *i*, *g*, *h*. I have not at hand the means of finding out what stars *i*, *g*, *h* are, but the description very plainly refers to $υ^6$, Behrmann's 58, and $υ^7$.

Eridanus, 34th star. Schjellerup makes this α Eridani; I insist upon θ Eridani. It is difficult to appeal to the description because we unfortunately disagree about the three stars which go before; but we agree that the 30th is υ¹. Now Sûfi says that the 31st precedes 30th by about 3 cubits. υ⁶ is precisely 3 cubits from υ¹ and in the right direction. Its magnitude according to Behrmann is too faint, but all Sûfi's southern magnitudes in this neighborhood are too bright. The 32nd is said to be south preceding the 31st, at a distance of ½ a cubit. Behrmann's 58 is the only star possible on his map. It is in the right direction, at a distance of ⅔ of a cubit, and has the right relative magnitude. The 33rd is said to be preceding and a little north of the 32nd, and at a distance of half a cubit. This perfectly describes υ⁷. Now, Sûfi says that the 34th precedes these stars at a distance of about 4 cubits from the nearest. This is true of θ, the distance being 4¼ cubits; but to α the distance is about 13 cubits. Add together all the successive distances from the 30th star, about which we agree, and they amount only to 8 cubits, while α is distant from that star by 16 cubits. Besides this, α Eridani has now a declination of 58° S. (and it was further south in Sûfi's time) so that at Baghdad which has a North Latitude of 33° it is never visible. And Sûfi, a little further on, mentions some extreme southern stars which he was only able to see at Schiraz. What were these stars? According to Schjellerup, α, β, γ, ε, μ Phoenicis, — bright stars situated 12° to 15° north of α Eridani. In conclusion, Ptolemy's place suits θ very well. Sûfi was not so careless as to assign it without remark to a star distant 25° from that place.

Since I disagree with M. Schjellerup as to the 34th star, I naturally disagree also in regard to certain extra stars noticed by Sûfi in its neighborhood. In the first place, Sûfi says that preceding the 34th there are two stars one to the south and one to the north. Schjellerup identifies these with α Hydri and ζ Phoenicis, although α Hydri *follows* α Eridani. Then Sûfi that there is another star which follows the 34th at a distance of 2 cubits, and Schjellerup does not attempt to identify this. Supposing, the 34th to be θ, however, the three stars are evidently, ε Eridani, 19 (Behrmann) Eridani, and 38 (Behrmann) Eridani.

Canis major. Without the figure Nos. 5, 6, 7, 8, 9. No. 5 is λ Canis majoris, No. 6 = μ Columbae, No. 7 = λ Columbae, No. 8 = γ Columbae, and No. 9 = β Columbae.

Argo. Nos. 13, 14, 15, 16, 22, 23, 24, 27. Not identified by Schjellerup. No. 13 = (*f*) Puppis. No. 14 = (*d*) Puppis (?). No. 15 = (*c*) Puppis. No. 16 = (*b*) Puppis. No. 22 = 52 (Behrmann) Velorum. No. 23 = (*d*) Velorum. No. 24 = (*e*) Velorum. No. 27 = (*b*) Mali.

Argo. Nos. 28, 29, 30. Schjellerup makes these υ², υ³, υ⁴ Navis; I incline to prefer (*a*) Mali, (*c*) Mali, and (*d*) Mali.

I have only examined a small proportion of M. Schjellerup's identifications, selecting those which I was inclined to question *a priori*. I think the above list contains the majority of his errors, of this sort. The work which the learning of M. Schjellerup has brought to light is so important that the smallest errors of detail become interesting.

The omissions in Sûfi's catalogue, north of the equator, are expressed by the following equations, where $\psi_s l$ denotes the number of stars as bright as l in Sûfi's catalogue, north of the equator:

$$\text{When } \log \psi_s l < 2.661 \qquad 10 + \log \frac{\psi l - \psi_s l}{\psi l} = 3.6 \log \psi_s l$$

$$\text{When } \log \psi_s l > 2.661 \qquad 10 + \log \frac{\psi l - \psi_s l}{\psi l} = 4.79 + 1.8 \log \psi_s l$$

DM.'s mag.	ψl.	$\psi_s l$.	Ratio omitted. Obs.	Calc.
3.9	175	174	.01	.01
4.0	229	221	.03	.03
4.2	275	261	.05	.05
4.4	303	281	.07	.07
4.5	361	326	.10	.11
4.6	379	335	.12	.12
4.7	415	353	.15	.15
4.9	484	380	.21	.19
5.0	679	442	.35	.33
5.1	715	453	.37	.36
5.2	801	469	.41	.40
5.3	896	502	.44	.45
5.4	933	510	.45	.46

The following table exhibits the reduction of Sûfi's magnitudes.

S.'s mag.	$\psi_s l$.	ψl.	Limit.	Mean.
1	7	7	1.27	0.41
1.2	8	8	1.38	1.32
2.1	9	9	1.47	1.43
2	21	24	2.28	1.91
2.3	31	31	2.49	2.39
3.2	37	37	2.63	2.56
3	86	86	3.33	3.03
3.4	142	143	3.74	3.55
4.3	185	187	3.97	3.86
4	341	393	4.58	4.28
4.5	380	474	4.73	4.65

S.'s mag.	$\psi_s l.$	$\psi l.$	Limit.	Mean.
5.4	400	521	4.81	4.77
5	516	977	5.33	5.07
5.6	547	1146	5.46	5.39
6.5	550	1161	5.47	5.46
6	614	1718	5.79	5.63
6.7	619	1795	5.83	5.81

I use S as the abbreviation for Sûfi.

Ulugh Beg. The magnitudes of this observer, whom I designate by U., have been extracted from Hyde's edition of his catalogue (1665). It was impossible to use Baily's reprint because he has omitted the thirds of a magnitude. But I have generally followed his identifications. The epoch of the catalogue is 1437.

The following table shows this observers omissions.

DM. mag.	$\psi l.$	$\psi_3 l.$
3.6	145	141
3.7	155	153
3.8	169	166
3.9	175	171
4.0	229	211
4.1	216	223
4.2	275	246
4.3	296	261
4.4	303	265
4.5	364	308
4.6	379	315
4.7	415	334
4.8	468	355
4.9	484	358
5.0	679	406
5.1	715	415
5.2	801	431
5.3	896	443
5.4	933	462

In this case I drew an empirical curve to represent these numbers, taking $\psi_3 l$ and $\psi l - \psi_3 l$ as the two rectangular coördinates. The following table exhibits the reduction of Ulugh's magnitudes.

U.'s mag.	$\psi_3 l.$	$\psi l.$	P. mag. Limit.	P. mag. Mean.
1	8	8	1.38	0.55
2	24	24	2.28	1.91
$2p$	30	30	2.46	2.37
$3m$	37	37	2.64	2.55
3	85	85	3.32	3.02

U.'s mag.	$\psi_3 l.$	$\psi l.$	P. mag. Limit.	P. mag. Mean.
$3p$	135	136	3.71	3.53
$4m$	170	173	3.90	3.81
4	330	415	4.62	4.26
$4p$	364	498	4.77	4.70
$5m$	382	553	4.86	4.81
5	465	1000	5.35	5.10
$5p$	492	1193	5.49	5.42
$6m$	493	1203	5.50	5.49
6	537	1827	5.84	5.66
$6p$	539	1862	5.86	5.85

Tycho Brahe. There are two catalogues of this observer, whom I designate by T. One is contained in his "De Nova Stella", and contains the magnitudes of 777 stars, expressed to thirds of magnitudes, and the other is contained in Kepler's Rudolphine Tables, and contains 1005 stars expressed only to whole magnitudes.

For the catalogue in the Rudolphine tables, putting $\psi_\tau l$ for the number that Tycho has north of the equator, I find

$$40 + \log\left(1 - \frac{\psi_\tau l}{\psi l}\right) = 4.74 + 1.7\,\psi_\tau l.$$

The following table shows the agreement of this with observation.

DM. mag.	$\psi l.$	Ratio omitted. Obs.	Ratio omitted. Calc.
3.6	145	.01	.03
3.7	155	.01	.03
3.8	169	.01	.03
3.9	175	.01	.03
4.0	229	.05	.05
4.1	246	.06	.06
4.2	275	.07	.06
4.3	296	.08	.08
4.4	303	.08	.08
4.5	364	.10	.10
4.6	379	.11	.11
4.7	415	.13	.12
4.8	468	.18	.14
4.9	484	.19	.14
5.0	679	.18	.26
5.1	715	.20	.27
5.2	801	.25	.29
5.3	896	.30	.31
5.4	933	.32	.32

The following table exhibits the reduction of Tycho's magnitudes.

T. Mag.	$\psi_\tau l$.	ψl.	C.S.P. Mag. Limit.	Mean.
1	6	6	1.14	0.32
2	39	39	2.66	2.14
3	142	145	3.78	3.32
4	387	450	4.68	4.26
5	533	700	5.04	4.86
6	661	1005	5.34	5.19

For the catalogue in the "*De Nova Stella*", the magnitudes of which are to be preferred, I find

$$10 + \log\left(1 - \frac{\psi_\tau l}{\psi l}\right) = 6.35 + 1.2 \log \psi_\tau l.$$

This equation was not obtained by a separate count of the omitted stars, but its constants were simply so taken as to make the increased numbers of the catalogue in the De Nova Stella agree with those of the Rudolphine Catalogue. Thus we have

Mag.	Increased Nos. of each magnitude. In *De Nova Stella*.	In Rudolphine Tables.
1	6	6
2	34	33
3	107	106
4	300	305
5	259	250
6	299	305

The following table exhibits the reduction of the magnitudes in the *De Nova Stella*.

T mag.	$\psi_\tau l$.	ψl.	P. mag. Limit.	Mean.
1	5	5	0.99	0.17
1.	6	6	1.14	1.07
2:	11	11	1.64	1.41
2	35	36	2.60	2.23
2.	39	40	2.69	2.65
3:	48	49	2.87	2.78
3	121	130	3.67	3.33
3.	135	147	3.77	3.72
4:	153	169	3.88	3.83
4	305	388	4.57	4.23
4.	338	447	4.68	4.63
5:	354	476	4.74	4.71
5	443	665	5.01	4.87
5.	459	706	5.06	5.03
6:	466	724	5.07	5.06
6	550	973	5.32	5.20
6.	559	1005	5.35	5.34

The agreement between the limits for the two catalogues is satisfactory.

Hevelius. The numbers of this observers stars were increased by the formula

$$\log\left(1 - \frac{d\psi_{\text{II}} l}{d\psi l}\right) = -2.339 - .277 \log \psi l + .303 (\log \psi l)^2.$$

The agreement of this formula with the facts is shown in the following table.

DM. mag.	ψl.	Ratio omitted. Calc.	Obs.
3.6	145	.00	.50
3.7	155	.03	.10
4.0	229	.04	.04
4.1	246	.05	.06
4.3	296	.06	.04
4.5	364	.08	.04
4.6	379	.09	.07
4.7	415	.10	.03
4.8	468	.11	.15
4.9	484	.12	.12
5.0	679	.16	.24
5.1	715	.21	.22
5.2	801	.25	.26
5.3	896	.28	.28
5.4	933	.33	.33

The following reduction of the magnitudes of Hevelius.

H. Mag.			C.S.P. Mag. Limit.	Mean.	Mean by comp. with DM.
1			1.38	0.55	0.50
1½			1.56	1.47	2.34
2		42	2.74	2.29	2.07
2½	44	43	2.76	2.80	2.48
3	144	145	3.76	3.38	3.30
3½	148	149	3.78	3.79	3.94
4	372	383	4.56	4.22	4.24
4¼	378	389	4.57	4.56	4.78
4½	384	396	4.58	4.57	4.61
5	725	840	5.20	4.89	5.09
5¼	726	842	5.20	5.20	4.89
5½	729	843	5.20	5.20	5.38
6	1086	1362	5.60	5.40	5.36
6½	1092	1371	5.61	5.60	5.77
7	1097	1377	5.61	5.61	5.31

The last column shows the mean magnitude of Hevelius's stars according to the DM., after the latter has been reduced to the equable scale. It is evident that the method of counting gives the most trustworthy reduction of Hevelius's scale.

I designate this observer by the abbrieviation H.

The reduction of Sir W. Herschel's Observations. The observations of Sir W. Herschel on the comparative brightness of stars in the Phil. Trans. of 1796, 1797 and 1799 were reduced in the following manner. He puts a comma between two stars to signify that the first is a little brighter than the second, a dash to signify that the first is considerably brighter than the second and a period between them to denote their equality and he combines commas, dashes and periods in various ways. I have supposed that the dash indicates the difference of brightness equal to double that of the comma, and that the other differences of brightness are related to that indicated by the comma as shown in the following table

· = 0	— = 2
·· = 0	·— = 2
·, = ½	·— = 2½
, = 1	—, = 3
·· = 1	—— = 3½
·— = 1½	—— = 4

Greater differences than two dashes were excluded from the calculation. Stars connected together by comparisons were placed in one group and there were thus formed 276 groups and the comparisons of the stars of each group were reduced by the method of least squares. The next step was to obtain the value of the comma by comparison with the DM. mags. I first sought to ascertain whether the value of the comma was the same in all of Herschel's four papers and I found that for his

first paper	1796	it was	0.36
second „	1796	„ „	0.37
third „	1797	„ „	0.29
fourth „	1799	„ „	0.30

which was sufficient to show that there was no material change. I then assumed that the value of the comma was a function of the mean magnitude of the group and calculation showed that for stars brighter than 3.75 the comma was equal to .17 mag. and for fainter stars was equal to .27 mag. These values were then used to obtain the differences of magnitude, and the mean magnitude of each group was assumed from mean of the best mags. of the same stars, which were available, being generally the corrected DM. mags. The following table gives all of Sir W^m^ Herschel's stars arranged in groups. Certain of the groups were divided into subgroups in the calculation, owing to the rejection of certain comparisons. In this table the scale of magnitudes is that which is one which I used at the time the calculation was made; for which,

$$\rho' = 2.47 \qquad m' = 1.7 \log \tfrac{1}{2} l.$$

To reduce magnitudes of this scale to that at present in use, the following formula may be used

$$m = -\frac{1}{3} + \frac{10}{9} m'.$$

The second column gives the differences of Herschel's magnitude from the mean of the group, the comma being taken as unity. The third column is the concluded magnitude (m').

I designate this observer by the abbrieviation H.

Group 1.		
ε Aquarii	— 5	3.9
μ „	— 1.25	4.8
7 „	+ .75	5.3
8 „	+ 4.75	6.3
9 „	+ 5.75	6.5
ν „	— 2.5	4.5
*18 „	— .25	5.0
ξ „	— 2.75	4.4
Group 2.		
3 Aquarii	— 1.7	4.6
4 „	+ 1.3	5.3
5 „	+ .3	5.1
Group 3.		
10 Aquarii	+ 1	5.7
11 „	0	5.5
12 „	+ 1	5.3
Group 4.		
14 Aquarii	+ 1.7	5.8
17 „	— .3	5.3
19 „	— 1.3	5.1
Group 5.		
15 Aquarii	— 0.75	5.4
16 „	+ 0.25	5.7
20 „	+ 1.25	5.9
21 „	— 0.75	5.4

Group 6.

This group has been divided into ten by rejection of comparisons between

δ and π Herculis
γ „ α „
α Trianguli and 99 Piscium
α and γ Trianguli
α Arietis and β Tauri
γ Cyeni and α Cephei
ι and ζ Draconis (ζ is rejected)
β Herculis and β Draconis
ζ Herculis and 37 Serpentis
λ and υ Pegasi
υ „ 56 „
ι „ 9 Aurigae (ι is rejected)
ρ „ ν Persei
38 „ 39 „ (two different stars probably taken for 38)
γ Persei and β Trianguli (γ Persei rejected)
γ and 41 Arietis
γ Arietis and ι Aurigae
γ and ε Persei
ζ Persei and ι Aurigae.

Sub-Group A.

β Aquarii	— 4.9	3.0
α „	— 4.9	3.0
79 „ = α Piscis austrini	— 6.6	2.7
β Capricorni	— 1.9	3.5
γ „	+ 2.1	4.4
δ „	— 3.9	3.1
β Cyeni	— 4.9	3.0
δ „	— 5.2	2.9
γ „	— 8.7	2.3
ε „	— 6.7	2.7
ζ „	— 3.9	3.1
χ Herculis	+ 1.0	4.1
2 „	+ 3.5	4.7
4 „	+ 3.2	4.6

* Suspected variable.

υ	Herculis	+ 1.0	4.1	ν	Pegasi	+ 3.8	4.7
φ	"	− 0.5	3.8	23	"	+ 4.7	4.9
14	"	+ 5.8	5.3	υ	"	+ 0.3	3.8
7	"	− 0.6	3.7	30	"	+ 5.4	5.1
τ	"	− 1.8	3.5	31	"	+ 3.5	4.6
25	"	+ 4.9	5.1	32	"	+ 2.7	4.4
30=*g*	"	+ 2.2	4.4	34	"	+ 6.6	5.3
34	"	+ 3.2	4.6	35	"	+ 4.6	4.9
σ	"	0	3.9	36	"	+ 7.4	5.5
42	"	+ 1.8	4.3	37	"	+ 5.6	5.1
η	"	− 3.2	3.2	38	"	+ 3.7	4.6
52	"	+ 1.3	4.2	ζ	"	− 0.7	3.6
ε	"	− 0.9	3.7	ο	"	+ 2.0	4.2
59=*d*	"	+ 4.2	4.9	[illegible]	"	− 5.5	2.8
61=*c*	"	+ 5.2	5.1	ξ	"	+ 2.3	4.3
62	"	+ 7.4	5.7	λ	"	− 0.7	3.6
63	"	+ 4.8	5.0	μ	"	− 1.7	3.4
δ	"	− 4.7	3.0	σ	"	+ 4.7	4.9
68=*a*	"	+ 3.4	4.7	ρ	"	+ 3.8	4.7
69=*e*	"	+ 1.7	4.3	52	"	+ 5.7	5.1
70	"	+ 3.4	4.7	β	"	− 6.5	2.7
72=*w*	"	+ 4.8	5.0	α	"	− 6.5	2.7
73	"	+ 5.4	5.2	γ	"	− 4.9	2.8
ρ	"	− 0.2	3.8	β	Arietis	− 3.9	2.9
λ	"	+ 0.5	4.0	α	"	− 8.6	2.4
78	"	+ 5.8	5.3	β	Ceti	− 6.3	2.7
ι	"	− 0.9	3.7	α	Andromedae	−10.1	2.1
μ	"	− 2.5	3.3	δ	"	+ 0.4	3.3
90=*f*	"	+ 4.1	4.9	β	"	− 9.5	2.2
θ	"	− 0.8	3.7	γ	"	− 8.8	2.3
ξ	"	− 1.2	3.6	γ	Lyrae	− 1.9	3.5
93	"	+ 6.8	5.5	α	Persei	− 9.3	2.3
ν	"	+ 0.7	4.0	α	Trianguli	+ 2.5	3.9
99=*b*	"	+ 3.0	4.6	β	"	− 1.3	3.2
100	"	+ 1.4	4.2				
ο	"	− 1.5	3.5				
104=*A*	"	+ 5.0	5.1				
105	"	+ 5.0	5.1				
106	"	+ 3.0	4.6				
107=*t*	"	+ 2.0	4.4				
108	"	+ 3.0	4.6				
109	"	− 0.8	3.7				
110	"	+ 1.2	4.2				
111	"	+ 0.2	3.9				
112	"	+ 3.2	4.6				
113	"	+ 2.2	4.4				
ε	Pegasi	− 7.6	2.5				
18	"	+ 6.0	5.2				
19	"	+ 5.6	5.1				

Sub-Group B.

β	Hercules	− 7.5	2.8
ζ	"	− 7.4	2.8
α	"	− 4.2	var.
π	"	− 5.7	3.0
ι	Draconis	− 5.7	3.0
κ	Ophiuchi	− 4.7	3.2

Sub-Group C.

51	Pegasi	+ 4.2	5.6
56	"	+ 2.2	5.1
60	"	+ 5.2	5.8

* Suspected variable.

61	Pegasi	+ 6.2	6.0	β	Orionis	− 19.4	0.4
τ	"	+ 0.9	4.7	γ	"	− 6.7	1.9
63	"	+ 5.2	5.8	δ	"	− 2.2	2.6
64	"	+ 3.2	5.3	ε	"	− 5.4	2.1
65	"	+ 5.2	5.8	ζ	"	− 4.8	2.2
66	"	+ 3.6	5.1	κ	"	− 1.7	2.6
67	"	+ 3.2	5.3	α	"	− 19.0	0.5
υ	"	+ 0.6	4.4	β	Aurigae	− 8.3	1.7
69	"	+ 4.2	5.6	θ	"	− 5.0	2.4
70	"	+ 1.6	4.7	α	Tauri	− 15.5	0.9
71	"	+ 3.2	5.3	β	"	− 8.6	1.6
72	"	+ 2.2	5.1	α	Hydrae	− 3.6	2.4
73	"	+ 5.2	5.8	α	Virginis	− 10.5	1.5
74	"	+ 5.1	5.5				
75	"	+ 3.9	5.3				
76	"	+ 6.1	5.8				
77	"	+ 3.6	5.1				
78	"	+ 1.8	4.9				
79	"	+ 3.8	5.4				
80	"	+ 5.6	5.6				
φ	"	+ 2.9	5.0				
82	"	+ 2.6	4.9				
83	"	+ 5.2	5.8				
ψ	"	+ 0.9	4.5				
85	"	+ 4.2	5.6				
86	"	+ 4.6	5.4				
87	"	+ 4.2	5.6				
χ	"	+ 1.9	4.9				
23	Piscium = 1635 Pegasi	+ 3.2	5.4				

Sub-Group D.

γ	Arietis	− 10.4	3.4
41	"	− 11.4	3.2
δ	Cassiepeae	− 10.8	3.3
ε	"	− 6.8	4.1
α	Ceti	− 11.4	3.2
α	Cephei	− 11.3	3.2
ο	Piscium	− 7.4	4.0
ν	"	− 5.4	4.5
μ	"	− 4.4	4.7

Sub-Group E.

α	Canis minoris	− 17.6	0.6
γ	Geminorum	− 5.0	2.3
α	"	− 8.1	1.8
β	"	− 11.5	1.4
α	Leonis	− 8.8	1.7

* Suspected variable.

Sub-Group F.

η	Draconis	− 11.9	2.7
ζ	"	− 8.1	3.3
β	"	− 9.3	3.1
δ	"	− 9.2	3.1
α	Ursae minoris	− 13.9	2.3

Sub-Group G.

ι	Aurigae	− 8.0	3.0
ρ	Persei	− 2.2	var.
δ	"	− 3.6	3.7
ζ	"	− 7.5	3.0
ε	"	− 7.1	3.1

Sub-Group H.

ο	Persei	+ 1.3	3.8
40	"	+ 6.3	5.0
ν	"	− 0.2	3.6
42=*n*	"	+ 6.8	5.1
ξ	"	+ 1.3	3.8
λ	"	+ 4.3	4.5
48=*c*	"	+ 3.3	4.3
49	"	+ 7.3	5.3
50	"	+ 6.3	5.0
μ	"	+ 4.3	4.5
52=*f*	"	+ 5.3	4.8
53=*d*	"	+ 5.3	4.8
54	"	+ 6.3	5.0
55	"	+ 8.8	5.6
56	"	+ 9.3	5.7
58=*e*	"	+ 3.3	4.3

Sub-Group I.

57=*m*	Persei	+ 2.3	6.51
59	"	+ 2.3	6.51

Sub-Group K.

1	Trianguli	+ 12.5	5.5
ε	"	+ 12.5	5.5
5	"	+ 12.5	5.5
ι	"	+ 9.5	4.8
η	"	+ 10.5	5.0
δ	"	+ 8.5	4.5
γ	"	+ 5.5	3.8
10	"	+ 10.5	5.0
11	"	+ 11.9	5.4
12	"	+ 11.0	5.1
*13	"	+ 12.7	5.6
14	"	+ 11.0	5.1
15	"	+ 11.5	5.3

Group 7.

24	Aquarii	+ 2	5.9
25 = *d*	"	− 1	5.2
26	"	0	5.4
27	"	− 1	5.2

Group 8.

*28	Aquarii	− 2.5	4.8
30	"	− 1.5	5.0
o	"	− 5.5	4.0
*32	"	− 3.5	4.5
36	"	+ 2.5	6.0
37	"	+ 1.5	5.8
38 = *e*	"	− 0.5	5.3
39	"	+ 0.5	5.5
40	"	+ 1.7	5.8
42	"	+ 0.5	5.5
45	"	+ 1.5	5.8
ρ	"	− 2.5	4.8
50	"	+ 1.3	5.7
53	"	+ 1.5	5.8
56	"	+ 2.1	5.9
60	"	+ 0.5	5.5
61	"	+ 2.9	6.1

Group 9.

29	Aquarii	+ 2.3	5.9
*35	"	+ 1.3	5.6
41	"	+ 0.3	5.4
47	"	− 0.7	5.1
49	"	+ 2.3	5.9
υ	"	− 1.7	4.9
66 = *g*	Aquarii	− 3.2	1.5
*68	"	− 0.7	5.1

Group 10.

ι	Aquarii	− 3.1	4.0
9	"	− 2.7	4.1
σ	"	− 1.9	4.2
*69	"	− 0.5	4.6
τ	"	− 3.5	3.9
χ	"	− 4.3	3.7
77	"	+ 1.0	4.9
86 = c^1	"	− 0.7	4.5
88 = c^2	"	− 3.3	3.9
89 = c^3	"	− 0.7	4.5
98 = b^1	"	− 2.1	4.2
99 = b^2	"	− 1.4	4.4
*100	"	+ 1.3	5.1
101 = b^3	"	− 0.7	4.5
103	"	+ 0.3	4.8
*104 = *A*	"	+ 0.3	4.8
106 = i^1	"	+ 0.3	4.8
107	"	+ 0.3	4.8
108 = i^2	"	+ 0.3	4.8
ψ	Capricorni	+ 0.1	4.7
17	"	+ 3.1	5.4
ω	"	+ 1.1	5.0
19	"	+ 1.0	4.9
20	"	+ 2.1	5.2
21	"	+ 2.0	5.2
η	"	− 0.2	4.7
9	"	− 3.1	4.0
24 = *A*	"	+ 1.6	5.1
χ	"	+ 2.3	5.3
26	"	+ 5.3	6.0
27	"	+ 5.3	6.0
ε	Piscis Austrini	− 1.0	4.5

Group 11.

44	Aquarii	+ 1	5.4
51	"	0	5.2
κ	"	+ 1	5.0

Group 12.

γ	Aquarii	0	3.8
π	"	+ 2	4.2
ζ	"	− 1	3.6
η	"	0	3.8
δ	"	− 1	3.6

* Suspected variable.

Group 13.

54	Aquarii	− 0.3	6.0
58	"	− 0.3	6.0
64	"	+ 1.7	6.5
65	"	+ 1.7	6.5
70	"	− 2.3	5.5
74	"	− 2.3	5.5
75	"	+ 1.7	6.5

Group 14.

67	Aquarii	+ 0.8	5.9
78	"	+ 0.8	5.9
80	"	− 1.2	5.4
		+ 0.8	5.9
81	"	− 0.2	5.7
82	"	− 1.2	5.4

Group 15.

83 = *h*	Aquarii	− 2	4.8
84	"	+ 6	6.7
85	"	+ 3	6.0
87	"	+ 4	6.3
φ	"	− 3	4.6
ψ^1	"	− 4	4.3
χ	"	− 1	5.1
ψ^2	"	− 3	4.6
96	"	0	5.3

Group 16.

94	Aquarii	− 1	4.8
ψ^3	"	0	5.0
97	"	+ 1	5.2

Group 17.

ω^1	Aquarii	+ 0.5	4.6
*ω^2	"	− 0.5	4.4

Group 18.

7	Aquilae	+ 0.5	
8	"	− 0.5	

Group 19.

10	Aquilae	+ 1.7	5.4
11	"	+ 0.7	5.2
ε	"	− 2.8	4.3
18	"	− 0.8	4.8
19	"	+ 0.2	5.0
21	"	− 0.3	4.9
22	Aquilae	+ 1.2	5.3
23	"	− 0.3	4.9
*24	"	+ 0.7	5.2

Group 20.

20	Aquilae	+ 0.3	5.2
26 = *f*	"	+ 0.3	5.2
37	"	+ 0.3	5.2
κ	"	− 0.7	4.9
51	"	+ 1.3	5.4
56	"	+ 1.3	5.4
57	"	− 2.7	4.5

Group 21.

ω	Aquilae	− 0.5	5.1
28 = *A*	"	− 0.5	5.1
29	"	+ 2.5	5.8
31 = *b*	"	− 1.5	4.8

Group 22.

36 = *e*	Aquilae	− 1.0	5.3
42	"	+ 0.3	5.6
45	"	+ 0.7	5.7
58	"	− 0.6	5.4
62	"	− 0.5	5.4
64	"	+ 0.9	5.7
66	"	+ 0.3	5.6

Group 23.

*46	Aquilae	+ 0.4	5.4
χ	"	− 0.6	5.2
ψ	"	+ 1.4	5.6
π	"	− 0.6	5.2
φ	"	− 0.6	5.2

Group 24.

1	Capricorni	+ 0.3	5.8
ξ	"	− 0.7	5.5
3	"	+ 0.3	5.8

Group 25.

4	Capricorni	+ 2.5	5.5
α^1	"	− 4.3	3.9
α^2	"	− 6.3	3.4
σ	"	+ 1.5	5.3
ν	"	− 1.3	4.6
π	"	+ 0.7	5.1
ρ	"	− 0.3	4.8

* Suspected variable.

o	Capricorni	+ 2.2	5.4
13	„	+ 3.6	5.8
τ	„	+ 0.3	5.0
υ	„	+ 1.5	5.3

Group 26.

29	Capricorni	— 1	5.1
30	„	+ 1	5.5
31	„	+ 3	6.0
υ	„	— 3	4.6

Group 27.

42	Capricorni	— 1.9	4.8
44	„	+ 0.6	5.4
45	„	+ 1.1	5.6
χ	„	— 1.4	5.0
50	„	+ 2.6	5.9
φ	„	— 1.2	5.0

Group 28.

46 = σ^1	Capricorni	— 1.5	4.7
47 = σ^2	„	+ 1.5	5.5

Group 29.

ι	Cyeni	— 2.1	4.2
ι	„	— 2.4	4.1
20	„	+ 0.6	4.8
20	„	+ 1.6	5.1
23	„	+ 1.6	5.1
ψ	„	+ 1.6	5.1
26 = e	„	+ 1.6	5.1
o^1	„	+ 2.1	5.2
o^2	„	— 1.7	4.3
32	„	— 0.4	4.6
33	„	— 0.4	4.6
λ	„	— 0.9	4.5
ν	„	— 0.9	4.5
ξ	„	— 2.3	4.1
τ	„	— 3.4	3.9
υ	„	— 1.6	4.3
σ	„	— 0.9	4.5
78	„	— 0.5	4.6
1	Pegasi	— 2.9	4.0
2	„	— 0.4	4.6
3	„	+ 0.5	4.8
4	„	+ 1.5	5.1
5	„	+ 2.3	5.3
6	„	+ 1.5	5.1
7	„	+ 1.5	5.1
9	Pegasi	— 3.7	3.8
χ	„	— 3.2	3.9
11	„	+ 3.5	5.5
12	„	+ 1.1	5.0
13	„	+ 0.7	4.9
14	„	+ 1.5	5.1
15	„	+ 2.5	5.3
16	„	+ 0.5	4.8
17	„	+ 1.7	5.1
20	„	+ 3.2	5.5
21	„	+ 2.2	5.2
ι	„	— 4.9	3.6

Group 30.

2	Cyeni	— 0.2	5.0
4	„	+ 0.3	5.1
8	„	— 1.1	4.7
9	„	+ 1.3	5.3
11	„	+ 1.6	5.4
φ	„	— 0.6	4.9
14	„	— 0.1	5.0
15	„	— 0.8	4.8
χ	„	— 0.2	5.0
19	„	+ 1.2	5.3
η	„	— 3.7	4.1
22	„	0	5.0
25	„	+ 0.3	5.1
27 = b^1	„	+ 3.2	5.8
28 = b^2	„	+ 0.3	5.1
29 = b^3	„	+ 0.1	5.0
31	„	+ 0.1	5.0
35	„	+ 0.3	5.1
36	„	+ 1.2	5.4
39	„	— 1.7	4.6
40	„	+ 1.1	5.3
41	„	— 3.7	4.1
42	„	+ 3.1	5.8
44	„	+ 4.1	6.0
*47	„	— 3.7	4.1
52	„	— 2.7	4.4

Group 31.

3	Cyeni = 11 Vulpeculae	+ 1.3	5.9
3	Vulpeculae	— 2.7	5.0
8	„	+ 1.3	5.9

Group 32.

7	Cyeni	+ 0.5	5.7
16 = a	„	— 0.5	5.5

* Suspected variable.

Group 33.

43 = ω^1	Cyeni	+ 1	5.4
45 = ω^2	„	— 1	5.0
46 = ω^3	„	0	5.2

Group 34.

48	Cyeni	+ 0.8	5.9
		+ 1.5	0.1
49	„	— 2.2	5.2

Group 35.

51	Cyeni	+ 0.4	5.1
55	„	— 0.6	4.9
56	„	— 0.6	4.9
57	„	— 1.6	4.6
59 = f^1	„	+ 0.2	5.0
60	„	+ 1.2	5.3
63 = f^2	„	— 0.3	4.9
68 = A	„	+ 1.6	5.4

Group 36.

61	Cyeni	+ 0.2	5.2
69	„	+ 1.2	5.5
70	„	+ 0.2	5.2
72	„	— 0.8	5.0
74	„	— 1.5	4.8
75	„	— 1.5	4.8
76	„	+ 1.5	5.6
77	„	+ 0.5	5.3
79	„	+ 0.2	5.2

Group 37.

71 = g	Cyeni	+ 1.5	4.9
ρ	„	— 2.5	3.9
π^1	„	+ 1.5	4.9
π^2	„	— 0.5	4.4

Group 38.

1	Delphini	+ 3.6	5.4
ε	„	— 2.4	3.9
η	„	+ 1.6	5.1
ζ	„	+ 0.6	4.6
ι	„	+ 1.6	4.9
β	„	— 4.4	3.6
κ	„	+ 0.6	4.6
θ	„	+ 2.6	5.1
α	„	— 3.4	3.7
10	„	+ 4.6	5.6
δ	„	— 1.4	4.2
7	„	— 3.4	3.7

Group 39.

13	Delphini	+ 2.4	5.5
14	„	+ 4.4	6.0
1	Equulei	+ 0.4	5.0
7	„	— 1.6	4.5
6	„	+ 0.4	5.0
δ	„	— 1.6	4.5
α	„	— 5.6	3.7
9	„	+ 1.4	5.2
β	„	— 0.6	4.8

Group 40.

15	Delphini	+ 1.5	5.5
16	„	— 0.5	5.0
17	„	— 1.5	4.7
18	„	+ 0.5	5.2

Group 41.

2	Equulei	+ 0.7	5.8
3	„	— 0.3	5.5
4	„	— 0.3	5.5

Group 42.

3	Herculis = 52 Serpentis	— 1.7	5.3
9	„	+ 0.3	5.8
43	Serpentis	+ 1.3	6.0

Group 43.

5 = r	Herculis	— 0.25	5.2
*z	„	— 4.25	4.3
[a]8 = q	„	+ 2.75	6.0
16	„	+ 1.75	5.7

[a] Heis's star is not 8 Herculis.

Group 44.

10	Herculis	— 1.9	5.8
17	„	+ 0.6	6.4
18	„	+ 1.6	6.7
19	„	— 0.4	6.2

Group 45.

12	Herculis	+ 1.2	6.3
13	„	+ 3.2	6.8
*15	„	+ 3.2	6.8
21	„	— 2.8	5.3
*28	„	— 4.8	4.8

* Suspected variable.

Group 46.

23	Herculis	− 1.1	5.8
26	„	+ 0.9	6.3
31	„	+ 2.9	6.8
ν^1	Coronae	− 3.1	5.3
ν^2	„	+ 0.4	6.2

Group 47.

ω	Herculis	− 2.5	4.3
29 = h	„	− 0.3	4.8
49	„	+ 2.4	5.5
54 or 55	„	+ 0.4	5.0
60	„	− 1.6	4.5
72 = w	„	+ 0.4	5.0
37	Ophiuchi	+ 1.4	5.2

Group 48.

32	Herculis	+ 3.1	6.5
39	„	− 1.9	5.3
46	„	+ 4.1	6.8
48	„	+ 2.1	6.3
50	„	+ 0.1	5.8
51	„	− 3.9	4.9
53	„	− 3.9	4.9
56	„	+ 0.1	5.8
57	„	+ 0.1	5.8

Group 49.

*33	Herculis	+ 1.75	6.3
36 } = m	„	+ 2.35	6.5
37 }	„	− 2.05	5.4
38	„	+ 0.65	6.1
41	„	+ 0.75	6.1
43 = i	„	− 1.15	5.6
45 = l	„	− 2.15	5.4
47 = k	„	− 0.15	5.9

Group 50.

74	Herculis	+ 0.5	5.6
77 = x	„	− 0.5	5.4
82 = y	„	− 0.5	5.4
88 = z	„	+ 0.5	5.6

Group 51.

79	Herculis	0	5.5
83	„	+ 2	6.0
84	„	+ 2	6.0
87	„	− 3	4.8
89	„	− 1	5.3

* Suspected variable.

Group 52.

95	Herculis	− 0.1	4.6
96	„	− 0.1	4.6
*97	„	+ 0.7	4.8
98	„	+ 0.5	4.7
101	„	+ 0.3	4.7
102	„	− 1.1	4.3

Group 53.

25	Pegasi	− 1	5.2
28	„	+ 1	5.6

Group 54.

27	Pegasi	+ 1	4.7
π	„	− 1	4.3

Group 55.

33	Pegasi	+ 0.4	5.8
39	„	+ 0.4	5.8
40	„	− 1.6	5.3
41	„	− 0.6	5.6
45	„	+ 1.4	6.0

Group 56.

55	Pegasi	− 1.5	4.7
57	„	+ 1.5	5.5
58	„	+ 0.5	5.2
59	„	− 0.5	5.0

Group 57.

1	Sagittae	+ 1	5.4
1	Vulpeculae	− 1	5.0
2	„	0	5.2

Group 58.

2	Sagittae	− 0.5	5.7
3	„	+ 0.5	5.9

Group 59.

10	Sagittae	+ 0.4	5.1
11	„	− 0.6	4.9
13	„	+ 0.4	5.1
14	„	− 0.6	4.9
15	„	+ 0.4	5.1

Group 60.

ε	Sagittae	+ 2.0	5.3
α	„	− 1.5	4.4
β	„	− 1.0	4.6
δ	„	− 4.5	3.8

* Suspected variable.

ζ	Sagittae	+ 0.5	4.9
*9	„	+ 2.5	5.4
γ	„	− 6.5	3.5
η	„	+ 0.5	4.9
θ	„	+ 3.5	5.6
18	„	+ 5.5	6.1
9	Vulpeculae	0	4.8
12	„	− 0.5	4.7

Group 61.

2	Arietis	− 0.1	5.4
3	„	+ 2.7	6.2
4	„	+ 1.7	6.0
7	„	− 0.9	5.4
ι	„	− 0.3	5.5
λ	„	− 3.9	4.7
10	„	− 1.9	5.1
11	„	+ 1.1	5.9
κ	„	− 2.6	5.0
14	„	− 2.9	4.9
15	„	0	5.6
16	„	+ 1.1	5.9
η	„	− 0.6	5.5
*18	„	+ 3.0	6.3
19	„	+ 1.1	5.9
20	„	+ 0.1	5.6
21	„	+ 0.1	5.6
9	„	0	5.6
23	„	+ 3.0	6.3

Group 62.

ξ	Arietis	+ 0.2	4.8
*25	„	+ 3.7	5.7
31	„	− 0.8	4.6
*38	„	− 2.8	4.1
54	Ceti	+ 2.7	5.4
64	„	+ 1.7	5.2
ξ^1	„	− 1.3	4.5
ξ^2	„	− 1.5	4.4
μ	„	− 2.5	4.2
λ	„	+ 0.3	4.9

Group 63.

26	Arietis	0	6.0
27	„	+ 1	6.2
29	„	− 1	5.8

* Suspected variable.

Group 61.

30	Arietis	− .55	4.7
ν	„	+ 1.65	5.2
33	„	− .55	4.7
ρ	„	+ 3.15	5.6
35	„	− .55	4.7
39	„	− 1.55	4.4
47	„	− .15	4.8
ε	„	− .85	4.6
δ	„	− .55	4.7
ζ	„	− .75	4.6
τ	„	− .45	4.7
63	„	− .15	4.8
65	„	+ .85	5.0
16	Trianguli = 47 ♑ Arietis	+ .45	4.9

Group 65.

36	Arietis	+ 1.5	6.2
ο	„	− 1.5	5.4
40	„	− 1	5.6
π	„	− 2.5	5.2
σ	„	− 1.5	5.4
44	„	+ 2.5	6.4
45	„	0	5.8
ϱ	„	− 1	5.6
50	„	+ 1.5	6.2
53	„	+ 1.	6.0
54	„	+ 1.5	6.2

Group 66.

49	Arietis	+ 0.5	5.6
51	„	+ 0.5	5.6
52	„	− 1	5.3
55	„	0	5.5
56	„	0	5.5

Group 67.

59	Arietis	− 1	5.3
60	„	0	5.5
62	„	− 3	4.8
64	„	0	5.5
66	„	+ 2	6.0
7	Tauri	+ 2	6.0

Group 68.

ζ	Canis majoris	0	3.1
σ^2	„	0	3.1

* Suspected variable.

Group 69.

β	Canis majoris	−6.3	2.8
ε	"	−7.6	2.6
δ	"	−5.3	2.9
η	"	−4.3	3.1
1	Navis = 96 Ⱳ Puppis	+6.7	5.6
3	" = *l* "	+1.2	4.2
7	" = 106 Ⱳ "	−1.3	3.7
11	" = 119 Ⱳ "	+2.2	4.4
12	" = 124 Ⱳ "	+4.7	5.0
14	"	+4.7	5.0
15	" = ρ Argûs	−5.3	2.9
16	"	+2.2	4.4
665 *C.*		+3.7	4.8
686 *C.*		+4.7	5.0

Group 70.

λ	Canis majoris	−0.6	4.1
10	"	+1.4	4.5
κ	"	−0.6	4.1
8	δ² Columbae	−0.1	4.2

Group 71.

ξ¹	Canis majoris	−0.5	4.6
ξ²	"	+0.5	4.8

Group 72.

ν¹	Canis majoris	+1.25	4.7
ν²	"	−1.75	4.0
ν³	"	−0.75	4.2
11	"	+1.25	4.7
9	"	−0.75	4.2
μ	"	+1.25	4.7
ι	"	+0.25	4.5
γ	"	−0.75	4.2

Group 73.

α Canis majoris.

Group 74.

12	Canis majoris	+1	5.7
15	"	−1	5.3
17	"	+1	5.7
19	"	−1	5.3

Group 75.

o¹	Canis majoris	−1.9	4.5
22	"	−2.9	4.3

* Suspected variable.

26	Canis majoris	+2.1	5.5
27	"	+1.1	5.3
28	"	−0.9	4.8
29	"	+1.1	4.8
30	"	+1.1	5.3

Group 76.

1	Canis minoris	+0.9	5.2
ε	"	+0.6	5.1
β	"	−4.4	3.9
γ	"	−0.4	4.9
η	"	+1.6	5.4
6	"	−0.4	4.9
δ¹	"	−0.1	5.0
δ²	"	+0.9	5.2
9	"	+0.9	5.2
11	"	+0.3	5.1

Group 77.

ζ	Canis minoris	−0.2	5.1
14	"	+1.8	5.5
13	Navis = 12 H Canis minor.	−1.7	4.7

Group 78.

1	Cassiepeae	−1.6	4.6
2	"	+2.4	5.6
3	"	−2.6	4.4
4	"	+0.4	5.1
τ	"	−0.6	4.9
6	"	+2.4	5.6
ρ	"	−2.6	4.4
σ	"	−2.6	4.4
9	"	+2.4	5.6
10	"	+2.4	5.6
12	"	+1.4	5.3
13	"	+3.4	5.8
λ	"	+0.4	5.1
χ	"	−1.8	4.6
16	"	+3.4	5.8
ζ	"	−3.5	4.2
η	"	−4.9	3.8
μ	"	+2.7	5.6
9	"	−0.8	4.8

Group 79.

β	Cassiepeae	0.75	2.5
α	"	0	2.4
γ	"	0.25	2.4

* Suspected variable.

Group 80.

ξ	Cassiepeae	−1	4.7
ν	"	+1	5.1

Group 81.

π	Cassiepeae	+0.5	4.5
ο	"	−0.5	4.3

Group 82.

24	Cassiepeae	−0.25	5.1
23	"	+0.25	5.3

Group 83.

o¹	Cassiepeae	+0.25	4.8
o²	"	−0.25	4.6

Group 84.

31	Cassiepeae	+0.5	5.4
ψ	"	−1.5	4.9
43	"	+1.5	5.7
ω	"	−0.5	5.2

Group 85.

32	Cassiepeae	−1	5.6
35	"	+1	6.0

Group 86.

φ	Cassiepeae	+1.7	6.0
χ	"	−1.3	5.3
41	"	−0.3	5.5

Group 87.

38	Cassiepeae	+2.2	5.4
40	"	+1.2	5.2
42	"	+0.2	4.9
48	"	−1.8	4.5
50	"	−3.8	4.0
54	"	+2.2	5.4

Group 88.

47	Cassiepeae	−0.5	4.7
49	"	+0.5	4.9

Group 89.

52	Cassiepeae	−0.25	5.4
53	"	−1.5	5.1
55	"	+1.75	5.9

* Suspected variable.

Group 90.

1	Ceti	+1.8	5.1
2	"	−2.2	4.2
6	"	+0.8	4.9
7	"	−1.2	4.4
9	"	+0.8	4.9

Group 91.

3	Ceti	−1.8	4.9
φ¹	"	−2.5	4.7
18	"	+2.8	6.0
φ²	"	−1.2	5.0
φ³	"	+0.8	5.5
φ⁴	"	+1.8	5.7

Group 92.

4	Ceti	0.0	
5	"	0.0	

Group 93.

ι	Ceti	−3.5	3.1
η	"	−3.5	3.1
8	"	−3.5	3.4
46	"	+3.5	5.0
47	"	+1.5	4.6
48	"	+3.5	5.0
49	"	+2.5	4.8
50	"	+2.5	4.8
τ	"	−2.5	3.6
χ	"	+1.5	4.6
ζ	"	−2.5	3.6

Group 94.

10	Ceti	+1.1	6.0
11	"	+2.1	6.2
12	"	−0.4	5.6
13	"	−1.7	5.3
15	"	−0.1	5.6
20	"	−2.7	5.0
26	"	−0.7	5.5
29	"	+2.3	6.3
33	"	+0.3	5.8
35	"	+1.3	6.0
44	Piscium	−0.9	5.5

Group 95.

21	Ceti	+0.1	5.5
25	"	−1.9	5.0

* Suspected variable.

27	Ceti	+ 0.1	5.5
28	„	— 0.9	5.3
30	„	+ 0.1	5.5
32	„	+ 1.1	5.8
36	„	+ 1.1	5.8
37	„	— 1.9	5.0
41	„	+ 1.1	5.8
44	„	+ 1.1	5.8

Group 96.

31	Ceti	— 1.3	5.3
38	„	— 0.3	5.5
39	„	— 0.3	5.5
40	„	+ 0.7	5.8
42	„	+ 0.7	5.8
43	„	+ 0.7	5.8

Group 97.

56	Ceti	0	4.8
57	„	+ 1	5.0
υ	„	— 2	4.3
74	„	+ 1	5.0

Group 98.

58	Ceti	— 2.2	5.3
60	„	— 3.2	5.0
*61	„	— 1.2	5.5
62	„	+ 1.8	6.2
63	„	— 0.2	5.8
66	„	— 1.2	5.5
71	„	+ 1.8	6.2
*79	„	+ 3.8	6.7
81	„	+ 0.8	6.0

Group 99.

67	Ceti	+ 0.7	5.7
77	„	— 0.3	5.4
80	„	— 0.3	5.4

Group 100.

o	Ceti	var.	

Group 101.

69	Ceti	— 0.75	5.1
70	„	— 0.75	5.1
75	„	+ 0.25	5.4
84	„	+ 1.25	5.6

* Suspected variable.

Group 102.

ρ	Ceti	+ 0.6	4.5
σ	„	+ 0.6	4.5
ε	„	+ 0.6	4.5
π	„	— 1.4	4.1
90	„	— 0.4	4.3

Group 103.

ν	Ceti	— 0.5	5.2
*85	„	+ 1.5	5.7
88	„	— 1.5	4.9
*93	„	+ 0.2	5.3
94	„	— 0.5	5.2
95	„	+ 1.5	5.7
κ	„	— 1.5	4.9
97	„	+ 0.8	5.5

Group 104.

δ	Ceti	+ 1	3.9
γ	„	— 1	3.5

Group 105.

α	Corvi	— 1.1	3.3
ε	„	— 3.1	3.0
*3	„	+ 4.9	4.7
γ	„	— 5.1	2.6
ζ	„	+ 2.9	4.2
6	„	+ 7.9	5.4
δ	„	— 3.1	3.0
η	„	+ 0.9	3.7
β	„	— 4.1	2.8

Group 106.

τ^1	Eridani	— 1.1	4.0
τ^2	„	+ 0.9	4.5
4	„	+ 1.9	4.8
6	„	+ 2.9	5.0
τ^3	„	— 2.1	3.8
*12	„	— 0.1	4.3
15	„	+ 0.4	4.4
τ^4	„	— 3.1	3.6
τ^5	„	— 1.1	4.0
20	„	+ 2.4	4.9
τ^6	„	— 2.1	3.8
τ^7	„	+ 0.9	4.5
τ^8	„	— 0.1	4.3
τ^9	„	— 0.1	4.3

* Suspected variable.

Group 107.

*η	Eridani	— 1.15	4.0
*ε	„	— 1.15	4.0
*δ	„	— 2.15	3.8
24	„	+ 2.85	5.0
25	„	+ 4.1	5.3
π	„	+ 0.85	4.5
29	„	+ 4.85	5.5
30	„	+ 3.85	5.2
32	„	+ 1.85	4.7
γ	„	— 5.15	3.3
35	„	+ 4.85	5.5
53	„	— 3.15	4.6
*66	„	— 2.15	
β	„	— 6.15	3.1
*68	„	— 2.15	

Group 108.

5	Eridani	— 1.5	5.0
7	„	+ 1.5	5.8

Group 109.

8	Eridani	+ 0.6	5.2
ρ	„	— 0.4	5.0
10	„	+ 0.6	5.2
ζ	„	— 2.4	4.5
14	„	+ 1.6	5.5

Group 110.

17	Eridani	— 1.5	4.9
21	„	+ 1	5.5
22	„	+ 0.5	5.4

Group 111.

37	Eridani	+ 1.2	5.1
o^1	„	— 1.3	4.5
39 = A	„	— 0.3	4.7
o^2	„	— 0.3	4.7
ξ	„	+ 0.7	5.0

Group 112.

o^4	Eridani	— 0.5	3.7
o^3	„	+ 0.5	3.9

Group 113.

44	Eridani	0	5.0
45	„	0	5.0
49	„	0	5.0

* Suspected variable.

Group 114.

46	Eridani	+ 0.2	5.3
47	„	+ 0.2	5.3
55	„	+ 2.2	5.8
56	„	+ 1.2	5.6
63	„	— 0.8	5.1
*64	„	— 2.8	4.6

Group 115.

ν	Eridani	— 1	3.8
51 = c	„	+ 2	4.5
μ	„	— 1	3.8

Group 116.

o^1	Eridani	+ 0.25	3.9
o^2	„	— 0.25	3.8

Group 117.

54	Eridani	— 2	4.9
58	„	+ 1	5.6
59	„	+ 1	5.6
60	„	0	5.4

Group 118.

ω	Eridani	— 1	4.3
62 = b	„	+ 3	5.2
ψ	„	0	4.5
λ	„	— 2	4.0

Group 119.

This group divided into three by rejection of comparisons

τ and ι Geminorum

κ Aurigae to be entirely rejected

Sub-Group A.

1	Geminorum	— 3.7	4.5
ν	„	— 5.7	4.0
τ	„	— 4.7	4.3
ι	„	— 6.9	3.7
υ	„	— 5.7	4.0
125	Tauri	+ 0.3	5.5
131	„	— 0.4	5.3
132	„	— 0.7	5.2
133	„	— 1.4	5.1
134	„	— 2.9	4.7
135	„	— 1.1	5.1
136	„	— 1.7	5.0
137	„	+ 0.8	5.6
139	„	— 1.7	5.0

* Suspected variable.

Sub-Group B.

53	Geminorum	− 2.9	5.8
59	„	− 2.9	5.8
ρ	„	− 6.1	5.0
64	„	− 4.9	5.3
65 = *b*	„	− 4.9	5.3

Sub-Group C.

25	Geminorum	+ 9.3	6.3
28	„	+ 6.3	5.6
36 = *d*	„	+ 5.3	5.4
37	„	+ 6.3	5.6
39	„	+ 7.3	5.9
40	„	+ 7.3	5.9
48	„	+ 4.3	5.1
49	„	+ 1.3	4.4
53	„	+ 6.3	5.6
54	„	+ 7.3	5.9

Group 120.

2	Geminorum	+ 1	6.6
3	„	− 2	5.9
4	„	+ 1	6.6
5	„	− 1	6.2
6	„	+ 1	6.6

Group 121.

η	Geminorum	− 5.5	3.4
μ	„	− 6.5	3.2
ε	„	− 6	3.3
30	„	− 1	4.4
ξ	„	− 3	3.9
32	„	+ 1.5	5.0
33	„	+ 0.5	4.7
35	„	+ 0.5	4.7
38 = *e*	„	− 1	4.4
o	Tauri	− 5.5	3.4
ξ	„	− 3.5	3.8
4 = *s*	„	+ 3	5.3
5 = *f*	„	− 0.4	4.5
6 = *t*	„	+ 4.5	5.7
10	„	− 1.6	4.2
12	„	+ 5.5	5.9
29 = *u*	„	+ 2	5.1
30 = *e*	„	+ 1.4	4.9
31	„	+ 4	5.6
λ	„	− 2.5	4.0
ν	„	− 2.5	4.0

* Suspected variable.

40	Tauri	+ 3.0	5.3
45	„	+ 1.0	5.6
46	„	+ 2.5	5.2
47	„	+ 1.9	5.1
48	„	+ 1.1	4.9
μ	„	− 0.7	4.4
τ	„	− 2.6	4.0
55	„	+ 5.9	6.0
57 = *h*	„	+ 2.2	5.1
58	„	+ 1.4	4.9
60	„	+ 2.1	5.1
δ	„	− 1.1	4.3
63	„	+ 4.9	5.8
61	„	+ 1.9	5.1
66 = *r*	„	+ 0.1	4.7
68	„	− 0.1	4.6
π	„	− 0.6	4.5
ε	„	− 3.1	3.9
76	„	+ 4.1	5.7
81	„	− 4.1	3.6
82	„	− 4.1	3.6
79 = *b*	„	− 0.8	4.4
83	„	+ 2.5	5.2
β	„	− 1.6	4.2
88 = *d*	„	− 1.5	4.2
96 = *c*	„	− 2.3	4.0
93	„	+ 2.1	5.1
ζ	„	− 4.0	3.6

Group 122.

8	Geminorum	− 0.6	6.4
9	„	− 0.6	6.4
10	„	+ 0.4	6.6
11	„	+ 0.4	6.6
12	„	+ 0.4	6.6

Group 123.

14	Geminorum	− 0.25	6.5
15	„	− 0.25	6.5
16	„	− 0.25	6.5
17	„	+ 0.75	6.8

Group 124.

19	Geminorum	− 0.5	6.3
20	„	− 1.3	6.1
22	„	+ 1.4	6.7
23	„	+ 0.4	6.5

Group 125.

26	Geminorum	+ 0.6	5.6
56	„	− 0.9	5.3

61	Geminorum	+ 0.6	5.6
63	„	− 0.4	5.4

Group 126.

a	Geminorum	+ 0.3	3.8
ζ	„	+ 2.3	4.1
λ	„	− 1.2	3.5
δ	„	− 0.7	3.6
κ	„	− 0.7	3.6

Group 127.

41	Geminorum	− 0.4	5.4
45	„	− 1.9	5.0
50	„	+ 0.6	5.6
51	„	− 3.9	4.6
67	„	+ 0.1	5.5
68	„	− 2.9	4.8
74 = *f*	„	− 0.9	5.3
79	„	+ 3.1	6.2
81 = *g*	„	− 1.9	5.0
82	„	+ 3.1	6.2
84	„	+ 4.1	6.5
85	„	+ 1.1	5.8

67 Geminorum excluded from the mean.

Group 128.

ω	Geminorum	− 1.5	5.6
44	„	+ 1.5	6.4

Group 129.

47	Geminorum	− 0.3	5.6
48	„	− 1.3	5.4
49	„	+ 2.7	6.3
52	„	+ 0.7	5.9
57 = *A*	„	− 3.3	4.9
58	„	+ 1.7	6.1

Group 130.

70	Geminorum	+ 1.3	5
o	„	− 0.7	4
π	„	− 0.7	4

Group 131.

σ	Geminorum	− 1.3	4.8
76 = *e*	„	− 0.3	5.0
φ	„	+ 1.7	5.5

Group 132.

κ	Leonis	− 2.1	4.5
λ	„	− 3.6	4.1

* Suspected variable.

15 = *f*	Leonis	+ 0.9	5.2
20	„	+ 3.9	5.9
22 = *g*	„	+ 0.9	5.2

Group 133.

ω	Leonis	+ 1.9	5.6
3	„	+ 3.4	5.9
ξ	„	− 2.1	4.6
6 = *h*	„	− 0.1	5.1
10	„	− 3.1	4.4

Group 134.

7	Leonis	+ 0.5	6.1
8	„	− 0.5	5.9
11	„	+ 1.5	6.4
ψ	„	− 1.5	5.6

Group 135.

9	Leonis	− 0.3	6.0
12	„	+ 1.7	6.5
13	„	− 1.3	5.8

Group 136.

o	Leonis	+ 1.5	3.6
ε	„	− 0.2	3.3
μ	„	+ 3.5	3.9
η	„	+ 2.0	3.7
ξ	„	+ 2.1	3.7
τ	„	− 7.7	2.0
ρ	„	+ 2.5	3.7
δ	„	− 3.7	2.6
θ	„	+ 2.1	3.7
β	„	− 5.7	2.3
46	Leonis minoris	+ 4.1	4.0

Group 137.

18	Leonis	− 1	5.9
19	„	0	6.1
21	„	+ 1	6.3
23	„	0	6.1

Group 138.

ν	Leonis	+ 0.5	5.4
π	„	− 1.5	4.9
31 = *A*	„	− 3.5	4.5
40	„	− 1.5	4.9
10	Sextantis	+ 2.0	5.8
11	„	+ 4.0	6.3

* Suspected variable.

Group 139.

33 Leonis	+ 4.5	7.1
34 „	+ 0.5	6.3
37 „	− 3.5	5.4
42 „	− 1.5	5.8

Group 140.

35 Leonis	+ 0.25	6.1
39 „	− 0.25	5.2

Group 141.

43 Leonis	+ 0.7	6.1
44 „	− 0.3	5.8
45 „	+ 0.7	6.1
46 = *i* „	− 0.3	5.8
48 „	− 2.3	5.3
49 „	− 1.3	6.6
50 „	+ 1.7	6.3
51 = *m* „	− 0.3	5.8
47 Leonis minoris	+ 1.7	6.3

Group 142.

52 = *k* Leonis	+ 1	5.3
53 = *l* „	0	5.1
41 Leonis minoris	− 1	4.9

Group 143.

54 Leonis	− 3.1	4.1
60 = *b* „	− 3.1	4.1
64 „	+ 4.6	6.0
67 „	+ 2.6	5.5
72 „	− 0.4	4.8

Group 144.

55 Leonis	− 1.9	5.7
57 „	+ 0.1	6.2
62 = p^3 „	− 0.9	6.0
34 Sextantis	+ 0.1	6.2
35 „	− 1.9	5.7
36 „	+ 1.1	6.5
37 „	+ 1.1	6.5
38 „	+ 2.1	6.7

Group 145.

56 Leonis	+ 3	5.7
58 = *d* „	− 0.5	4.9
59 „	0	5.0
χ „	− 2.5	4.4

Group 146.

61 = p^2 Leonis	− 1.3	4.9
66 „	+ 1.7	5.6
69 = p^5 „	− 0.3	5.1

66 Leonis excluded from the mean.

Group 147.

65 = p^1 Leonis	+ 1	5.6
75 „	− 1	5.2
76 „	+ 2	5.9
79 „	0	5.4
? „	− 2	4.9

Group 148.

73 = *n* Leonis	− 1.1	5.1
84 „	− 0.6	5.3
85 „	+ 1.1	5.7
86 „	− 2.6	4.8
88 „	+ 2.4	6.0
90 „	+ 0.6	5.5

Group 149.

φ Leonis	+ 0.4	4.5
π „	− 2.6	3.8
ρ „	− 1.6	4.0
87 = *e* „	+ 3.4	5.2
σ „	+ 0.1	4.5

Group 150.

80 Leonis	− 0.75	6.3
82 „	+ 2.25	7.0
83 „	+ 1.25	6.8
89 „	− 2.75	5.8

Group 151.

92 Leonis	+ 1.7	5.2
93 „	− 2.3	4.2
95 „	+ 0.7	5.0

Group 152.

o Andromedae	− 5.1	3.6
2 „	+ 2.7	5.3
4 „	+ 3.7	5.6
6 „	+ 4.7	5.8
λ „	− 3.1	4.0
? „	− 1.1	4.4
18 „	+ 0.7	4.9
ι „	− 1.3	4.4
φ „	+ 0.7	4.9
22 „	− 1.3	4.4
23 „	+ 2.7	5.4

26 Andromedae	+ 3.7	5.6
51 „ = υ Persei	− 7.1	3.3

Group 153.

3 Andromedae	− 0.3	4.9
5 „	+ 2.2	5.5
7 „	− 2.3	4.4
8 „	− 0.3	4.9
11 „	+ 0.7	5.2

Group 154.

9 Andromedae	+ 2	6.0
10 „	+ 2	6.0
12 „	− 1	5.3
13 „	0	5.5
14 „	− 2	5.0
15 „	− 1	5.3

10 Andromedae excluded from the mean.

Group 155.

θ Andromedae	− 0.3	4.5
σ „	− 1.3	4.3
ρ „	+ 1.7	5.0

Group 156.

28 Andromedae	− 0.1	4.9
π „	− 2.5	4.3
ε „	− 2.9	4.2
32 „	+ 0.3	5.0
ζ „	− 0.3	4.8
ν „	− 1.3	4.6
36 „	+ 3.7	5.8
μ „	− 5.4	3.7
γ_1 „	+ 0.7	5.1
39 „	+ 2.3	5.5
40 „	+ 1.4	5.2
ξ „	+ 1.7	5.3
ω „	+ 0.7	5.1
49 = *A* „	+ 2.7	5.5
υ „	− 3.9	4.0
χ „	+ 0.7	5.1
π „	− 0.3	4.8
55 „	+ 0.7	5.1
56 „	+ 1.7	5.3
58 „	− 1.3	4.6
59 „	+ 1.7	5.3
60 = *b* „	+ 0.7	5.1

* Suspected variable.

Group 157.

41 Andromedae	+ 1.1	5.3
φ „	− 1.9	4.5
44 „	+ 1.1	5.3
45 „	+ 1.1	5.3
47 „	+ 1.1	5.3
54 „ = φ Persei	− 2.4	4.4

Group 158.

61 Andromedae	+ 1.4	5.8
62 = *c* „	− 0.6	5.4
63 „	+ 0.4	5.6
64 „	+ 0.4	5.6
65 „	− 1.6	5.1
66 „	+ 0.6	5.6
6 Persei	− 0.6	5.4

Group 159.

1 Bootae	+ 2	5.2
2 „	+ 3	5.4
τ „	− 2	4.2
υ „	− 2.5	4.1
6 = *e* „	0	4.7
7 „	+ 1	4.9
10 „	+ 3	5.4
26 „	+ 3	5.4
π „	− 0.5	4.6
ζ „	− 5.5	3.5
31 „	+ 0.5	4.8
32 „	+ 2.5	5.3
34 „	− 1	4.5
o „	− 1.5	4.3
ξ „	− 1.5	4.3

7, π, 34 excluded from the mean.

Group 160.

3 Bootae	+ 5.2	5.6
9 „	+ 3.2	5.2
11 „	+ 5.2	5.6
12 = *d* „	+ 1.7	4.8
ρ „	− 4.8	3.4
σ „	+ 1.2	4.7
μ „	− 0.8	4.2
β Coronae	− 3.9	3.6
9 „	− 1.4	4.1
ζ „	− 0.2	4.4
γ „	− 4.4	3.5
δ „	+ 1.1	4.7
ε „	− 1.9	3.9

* Suspected variable.

Group 161.

η	Bootae	+ 0.8	2.7
γ	„	+ 2.3	3.0
ε	„	− 1.2	2.4
β	„	+ 5.1	3.4
δ	„	+ 5.5	3.5
α	Coronae	− 3.2	2.1
α	Ophiuchi	− 4.0	2.0
β	Ursae minoris	− 3.8	2.0
ζ	Virginis	+ 0.3	2.6
γ	Draconis	− 1.8	2.3

Group 162.

13	Bootae	− 1	5.0
24 = g	„	+ 1	5.4

Group 163.

14	Bootae	+ 0.7	5.8
15	„	+ 1.7	6.0
18	„	− 0.3	5.5
20	„	− 1.3	5.3
22 = f	„	− 0.8	5.4

Group 164.

α	Bootae	− 1.3	0.4
α	Lyrae	+ 2.7	0.7
α	Aurigae	− 1.3	0.4

Group 165.

κ	Bootae	+ 0.5	4.2
λ	„	− 0.5	4.0
ι	„	+ 0.5	4.2
θ	„	− 0.5	4.0

Group 166.

33	Bootae	− 0.5	4.9
38 = h	„	+ 1.5	5.4
39	„	− 0.5	4.9
40	„	+ 1.5	5.4
44	„	− 1.5	4.6
47 = k	„	− 0.5	4.9

Group 167.

ω	Bootae	0	4.9
ψ	„	− 2.5	4.3
45 = c	„	− 0.5	4.8
46 = b	„	+ 2	5.4
χ	„	+ 1	5.1
50	„	0	4.9

* Suspected variable.

Group 168.

ν^1	Bootae	− 0.2	4.5
ν^2	„	− 0.7	4.3
φ	„	+ 0.8	4.7

Group 169.

1	Canori	+ 2.9	5.9
3	„	+ 1.9	5.7
5	„	+ 2.9	5.9
8	„	− 0.1	5.2
12	„	+ 2.9	5.9
ζ	„	− 1.1	4.9
β	„	− 5.8	3.8
γ	„	− 1.1	4.9
45 = A^1	„	+ 0.9	5.4
46	„	+ 1.7	5.6
δ	„	− 4.1	4.2
ι	„	− 5.0	4.0
50 = A^2	„	+ 1.9	5.7
53	„	+ 1.7	5.6
ρ^1	„	+ 1.2	5.5
57	„	− 0.6	5.1
ρ^2*	„	− 1.0	5.0
60	„	+ 0.9	5.4
61	„	+ 1.7	5.6
α	„	− 4.1	4.2
67	„	+ 1.2	5.5
70	„	+ 2.2	5.7
75	„	0	5.2
$\varkappa$	„	− 0.1	5.2

Group 170.

ω	Canori	+ 0.75	5.9
4	„	+ 1.75	6.1
6	„	− 2.5	5.1
7	„	+ 0.75	5.9
9	„	− 0.25	5.6
μ	„	− 2.25	5.0
11	„	+ 1.0	5.9
13	„	+ 1.75	6.1
ψ	„	− 0.25	5.6
15	„	− 0.75	5.5
χ	„	− 1.0	5.5
φ^1	„	+ 1.0	5.9
φ^2	„	0	5.7

Group 171.

λ	Canori	− 2.6	5.0
24	„	+ 1.4	5.9
28	„	+ 0.4	5.7

ρ^1	Canori	− 0.6	5.5
ρ^2	„	+ 1.4	5.9

Group 172.

20 = d^1	Canori	+ 0.25	5.7
25 = d^2	„	+ 1.25	5.9
9	„	− 0.75	5.4
η	„	− 0.75	5.4

Group 173.

21	Canori	+ 0.9	6.1
27	„	− 1.6	5.5
29	„	− 1.1	5.6
31	„	+ 1.9	6.4
36 = e	„	+ 1.9	6.4
37	„	− 0.1	5.9
49 = b	„	− 2.1	5.4

Group 174.

35	Canori	+ 0.4	6.8
38	„	+ 0.4	6.8
39	„	− 1.1	6.4
40	„	+ 0.4	6.8
ζ	„	− 0.1	6.7
42	„	− 0.1	6.7

Group 175.

σ^1	Canori	0	5.5
σ^2	„	0	5.5
σ^3	„	0	5.5
66	„	0	5.5

Group 176.

52*	Canori	+ 1.1	6.7
54	„	− 0.9	6.2
56	„	− 3.9	5.5
63	„	− 3.9	5.5
68	„	+ 1.6	6.8
71	„	+ 2.8	7.1
78	„	+ 2.1	6.9
80	„	+ 0.8	6.6
81	„	+ 0.1	6.4
π	„	− 1.9	5.9
83	„	+ 1.6	6.8

Group 177.

ν	Canori	− 1	4.9
ξ	„	− 1	4.9
79	„	+ 2	5.6

* Suspected variable.

Group 178.

1 = i	Centauri	− 0.2	4.3
2 = g	„	+ 1.8	4.7
3 = k	„	− 0.2	4.3
4 = h	„	− 1.2	4.0
9	„	− 0.2	4.3

Group 179.

$\varkappa$	Cephei	− 2.5	4.4
9	„	− 0.5	4.9
10	„	− 2.5	4.4
11	„	− 1.5	4.6
12	„	+ 1.5	5.4
ξ	„	− 2.5	4.4
ε	„	− 3.5	4.2
25	„	+ 7.5	6.8
26	„	+ 4.5	6.1
30	„	+ 1.5	5.4
π	„	− 1.5	4.6

Group 180.

θ	Cephei	+ 4.1	4.3
η	„	+ 1.1	3.7
β	„	− 2.1	3.1
ζ	„	+ 1.1	3.7
ι	„	+ 0.9	3.7
γ	„	− 5.1	2.5

Group 181.

4	Cephei	+ 0.7	5.3
6	„	− 1.3	4.8
7	„	+ 0.7	5.3

Group 182.

μ	Cephei	− 2.4	5.5
14	„	− 1.9	5.6
ν	„	+ 1.1	6.4
		+ 3.1	6.8

Group 183.

16	Cephei	+ 1.2	5.0
24	„	+ 0.2	4.7
50	Draconis	+ 2.2	5.2
υ	„	+ 1.2	5.0
55	„	+ 1.2	5.0
π	„	− 3.6	3.8
59	„	+ 3.7	5.6
τ	„	− 0.8	4.5
σ	„	− 1.8	4.3

* Suspected variable.

ε Draconis	— 6.6	3.2
ρ "	— 2.9	4.0
73 "	+ 1.2	5.0
77 "	+ 3.2	5.5
78 "	+ 2.2	5.2

55, π, and ρ Draconis excluded from the mean.

GROUP 184.

18 Cephei	+ 0.4	5.5
19 "	— 0.6	5.3
20 "	+ 0.9	5.6
λ "	— 0.6	5.3

GROUP 185.

δ Cephei.

GROUP 186.

28 Cephei	— 1	5.2
ρ "	+ 1	5.6

GROUP 187.

o Cephei	— 1	4.8
31 "	+ 1	5.2

GROUP 188.

o Coronae	+ 2	5.7
η "	— 2	4.7

GROUP 189.

μ Coronae	+ 0.25	5.2
π "	+ 1.25	5.4
κ "	— 1.75	4.7
λ "	+ 0.25	5.2

GROUP 190.

ι Coronae	— 1.7	4.5
υ "	+ 2.3	5.5
ξ "	— 0.7	4.7

GROUP 191.

ρ Coronae	+ 1.3	5.3
τ "	— 1.7	4.6
σ "	+ 0.3	5.1

GROUP 192.

1 H. Lacertae	— 0.6	4.7
1 "	— 1.6	4.4
2 "	— 1.3	4.5
3 "	— 0.4	4.7
4 "	— 0.4	4.7
5 "	— 0.4	4.7
6 "	— 0.6	4.7
7 Lacertae	— 2.6	1.2
S "	— 0.6	4.7
9 "	+ 0.6	4.9
10 "	— 0.6	4.7
11 "	+ 0.4	4.9
12 "	+ 0.4	4.9
13 "	+ 1.4	5.1
14 "	+ 2.4	5.4
15 "	+ 1.4	5.1
16 "	+ 2.4	5.4

GROUP 193.

1 Leporis	+ 2	5.6
ι "	— 2.2	4.6
κ "	— 1.2	4.8
λ "	— 2.2	4.6
ν "	0	5.1
S "	— 0.4	5.0
10 "	+ 2	5.6
12 "	+ 2	5.6

GROUP 194.

ε Leporis	— 1	3.8
μ "	— 2	3.6
β "	— 3	3.5
α "	— 5	3.1
γ "	0	4.0
ζ "	— 1	3.8
δ "	+ 1	4.2
η "	0	4.0
17 "	+ 3	4.7
θ "	+ 2	4.5
19 "	+ 6	5.4

GROUP 195.

2 Navis	0	5.5
4 "	— 1	5.3
5 "	0	5.5
6 "	0	5.5
8 "	+ 1	5.7
9 "	0	5.5
10 "	0	5.5

GROUP 196.

17 Navis	+ 0.6	5.8
18 "	— 0.4	5.6
19 "	— 2.4	5.1
20 "	— 1.4	5.4
21 "	+ 1.1	6.0
22 "	+ 2.6	6.3

GROUP 197.

π^3 Orionis	— 7.8	3.3
π^2 "	— 0.8	4.7
π^4 "	— 4.8	3.8
o^1 "	— 1.3	4.6
5 "	+ 0.2	4.9
6 = *g* "	+ 3.2	5.7
π^1 "	+ 1.2	5.2
π^5 "	— 5.8	3.6
o^2 "	— 3.3	4.1
π^6 "	— 2.8	4.2
11 "	— 2.3	4.3
13 "	+ 6.7	6.5
14 = *i* "	+ 4.2	5.9
15 "	— 0.3	4.8
16 = *h* "	+ 4.7	6.0
18 "	+ 4.7	6.0
35 "	+ 1.7	5.3
96 Tauri	+ 2.7	5.5
97 = *i* "	— 0.3	4.8

11 Orionis excluded from the mean.

GROUP 198.

ρ Orionis	+ 0.1	5.2
21 "	+ 2.1	5.7
23 = *m* "	+ 0.6	5.3
25 "	— 0.9	5.0
ψ "	— 0.9	5.0
32 = *A* "	— 2.9	4.5
33 = *n* "	+ 2.6	5.8
38 "	+ 1.6	5.6
ω "	— 1.9	4.7

ρ Orionis excluded from the mean.

GROUP 199.

τ Orionis	— 1.1	3.9
τ_1 "	— 3.1	3.5
29 = *e* "	+ 0.6	4.3
υ "	+ 1.6	4.6
ι "	— 5.1	3.2
σ "	— 2.1	3.7
49 = *d* "	+ 3.6	5.1
55 "	+ 5.6	5.5

GROUP 200.

Separated into two groups, 15 Monocerotis excluded from both means.

SUB-GROUP A.

22 Orionis	— 5.6	4.9
27 "	— 4.6	5.1
31 "	— 5.6	4.9

SUB-GROUP B.

5 Monocerotis	— 1.6	4.2
7 "	+ 3.4	5.4
S "	— 0.6	4.5
9 "	+ 4.4	5.7
10 "	+ 1.4	4.9
11 "	— 3.6	3.7
12 "	+ 4.2	5.6
13 "	+ 1.4	4.9
14 "	+ 5.1	5.8
15 "	+ 0.9	4.8
16 "	+ 3.9	5.5
17 "	+ 1.9	5.1
18 "	+ 0.4	4.7
26 "	— 2.1	4.1
27 "	+ 1.4	4.9
28 "	+ 0.4	4.7
29 "	— 0.6	4.5
30 "	— 4.1	3.7
31 "	+ 0.4	4.7

GROUP 201.

φ^1 Orionis	+ 1.75	4.8
λ "	— 2.25	3.9
φ^2 "	— 0.25	4.3
μ "	+ 0.75	4.6

GROUP 202.

ϑ^1 Orionis	0
ϑ^2 "	0

GROUP 203.

42 = c^1 Orionis	— 0.5
45 = c^2 "	+ 0.5

GROUP 204.

51 = *b* Orionis	— 1.2	5.1
52 "	— 0.7	5.2
56 "	— 1.7	5.0
59 "	+ 3.3	6.2
60 "	+ 0.3	5.5

GROUP 205.

χ^1 Orionis	— 2.5	4.7
57 "	+ 1.5	5.7
χ^2 "	— 0.5	5.2

64	Orionis	+ 1.5	5.7
68	„	+ 0.5	5.4
71	„	− 0.5	5.2

GROUP 206.

63	Orionis	+ 0.1	5.7
66	„	− 0.1	5.7

GROUP 207.

	ν Orionis	− 2.4	4.6
69	$=f^1$ „	+ 0.3	5.3
	ξ „	− 1.7	4.8
72	$=f^2$ „	+ 0.3	5.3
73	„	+ 1.8	5.6
74	$=k$ „	+ 1.3	5.5
75	$=l$ „	+ 0.3	5.2

GROUP 208.

77	Orionis	− 1	5.4
78	„	+ 1	5.8

GROUP 209.

1	Aurigae	+ 0.7	5.2
2	„	− 0.2	5.0
ω	„	− 0.5	4.9

GROUP 210.

5	Aurigae	− 2	5.8
6	„	0	6.3
12	„	+ 2	6.8

GROUP 211.

ε	Aurigae	− 4.9	3.1
ζ	„	− 0.4	4.0
η	„	− 2.4	3.5
ξ	„	+ 2.6	4.7
δ	„	− 1.2	3.8
10	Camelopardali	− 0.7	3.9
31	„	+ 4.6	5.2
42	„	+ 2.6	4.0

10 Camelopardali excluded from the mean.

GROUP 212.

9	Aurigae	− 0.1	5.3
7	Camelopardali	− 1.6	4.9
11	„	− 1.1	5.0
12	„	+ 2.9	6.0

GROUP 213.

μ	Aurigae	− 0.6	4.9
λ	„	− 1.6	4.6
φ	„	+ 1.2	5.3
σ	„	+ 1.1	5.3

GROUP 214.

14	Aurigae	0	5.4
16	„	− 1	5.2
17	„	0	5.4
18	„	+ 1	5.6
19	„	0	5.4

18 Aurigae excluded from the mean.

GROUP 215.

22	Aurigae	+ 1.3	5.5
ψ	„	+ 0.3	5.3
χ	„	− 1.2	4.9
26	„	+ 0.3	5.3
ρ	„	− 0.7	5.0

GROUP 216.

28	Aurigae	+ 1.8	5.5
τ	„	− 1.2	4.8
υ	„	− 0.2	5.1
ν	„	− 2.2	4.6
π	„	− 1.2	4.8
36	„	+ 1.8	5.5
40	„	− 0.2	5.1
41	„	+ 1.8	5.5
45	„	− 0.2	5.1
ν^1	„	− 0.2	5.1

ν Aurigae excluded from the mean.

GROUP 217.

38	Aurigae	− 0.1	5.9
39	„	− 0.1	5.9
42	„	+ 0.6	6.1
43	„	+ 0.6	6.1
47	„	− 0.1	5.9

GROUP 218.

ψ^2	Aurigae	− 0.75	5.1
51	„	+ 3.25	6.1
ψ^3	„	+ 1.25	5.6
ψ^4	„	− 2.75	4.6
ψ^5	„	+ 0.25	5.4
ψ^6	„	+ 0.25	5.4
ψ^7	„	− 0.75	5.1
16	Lyncis	− 0.75	5.1

GROUP 219.

59	Aurigae	0	5.8
60	„	0	5.8
61	„	0	5.8
62	„	0	5.8

GROUP 220.

63	Aurigae	− 0.6	5.1
64	„	+ 0.9	5.4
65	„	− 0.6	5.1
66	„	+ 0.4	5.3

GROUP 221.

λ	Draconis	− 1.1	3.7
2	„	+ 2.9	4.6
3	„	+ 2.4	4.5
κ	„	− 1.1	3.7
α	„	− 3.1	3.3
9	„	− 0.1	3.9

GROUP 222.

4	Draconis	0	4.5
6	„	0	4.5

GROUP 223.

7	Draconis	+ 1.75	5.3
8	„	− 0.25	4.8
9	„	+ 0.75	5.1
10	$=i$ „	− 2.25	4.4

GROUP 224.

15	$=A$ Draconis	− 0.2	5.0
18	$=g$ „	− 0.2	5.0
19	$=h$ „	− 1.2	4.7
20	„	+ 2.8	5.7
μ	„	− 0.2	5.0
30	„	− 1.2	4.7

GROUP 225.

16	Draconis	+ 1	5.0
17	„	− 1	4.6

GROUP 226.

ν^1	Draconis	− 0.7	4.9
ν^2	„	− 0.7	4.9
26	„	+ 1.3	5.4

GROUP 227.

27	$=f$ Draconis	− 1.1	5.0
ω	„	− 3.1	4.6
29	„	+ 4.9	6.5
ψ	„	− 5.1	4.1
34	„	− 0.1	5.3
35	„	− 0.1	5.3
36	„	− 1.1	5.0
37	„	+ 1.9	5.8
38	„	+ 2.9	6.0
42	Draconis	− 1.1	5.0
40 + 41	„	+ 1.9	5.8
40	„	+ 1	6.2
41	„	− 1	5.8

GROUP 228.

ξ	Draconis	+ 0.3	4.0
φ	„	+ 2.3	4.4
χ	„	− 2.7	3.4

GROUP 229.

39	$=b$ Draconis	− 2.1	4.2
45	$=d$ „	− 3.1	4.0
46	$=e$ „	− 2.1	4.2
o	„	− 1.6	4.3
48	„	+ 2.4	5.3
49	„	+ 2.4	5.3
51	„	+ 2.4	5.3
53	„	+ 1.4	5.0
54	„	+ 0.4	4.8

45 and 46 Draconis excluded from the mean.

GROUP 230.

64	$=e$ Draconis	− 3.25	4.9
65	„	− 0.25	5.6
69	„	+ 0.75	5.9
70	„	+ 2.75	6.4

GROUP 231.

66	Draconis	0	5.1
68	„	0	5.1
71	„	0	5.1

GROUP 232.

74	Draconis	+ 0.7	5.7
75	„	− 1.3	5.2
76	„	+ 0.7	5.7

GROUP 233.

79	Draconis	+ 1	6.3
80	„	− 1	5.9

GROUP 234.

1	Lyncis	− 0.8	5.3
2	„	− 3.8	4.6
4	„	+ 2.2	6.0
5	„	+ 0.2	5.5
6	„	+ 2.2	6.0

GROUP 235.

3	Lyncis	+ 0.5	6.5
8	„	− 1.5	6.0

10 Lyncis	+ 1.5	6.8
41 Camelopardali	− 0.5	6.3

Group 236.

9 Lyncis	+ 3.6	6.3
11 „	+ 1.6	5.8
12 „	− 2.4	4.8
13 „	− 0.4	5.3
14 „	+ 0.6	5.5
15 „	− 2.9	4.7
19 „	− 0.4	5.3
23 „	+ 2.6	6.0
24 „	− 2.4	4.8

Group 237.

17 Lyncis	+ 1	5.8
18 „	− 2	5.1
47 Camelopardali	+ 1	5.8

Group 238.

20 Lyncis	+ 3.3	6.5
21 „	− 0.7	5.8
22 „	− 2.7	5.4

21 Lyncis excluded from the mean.

Group 239.

25 Lyncis	+ 2	5.7
26 „	− 1	5.0
28 „	+ 1	5.4
34 „	0	5.2
36 (35) „	− 1	5.0
37 (36) „	− 1	5.0
42 (41) „ = 22 H Ursae majoris	0	5.2
45 (44) „ = 63 Ⅎ „	0	5.2
14 Leonis minoris	+ 4	6.2
15 „	0	5.2
υ Ursae majoris	− 4	4.2

Group 240.

27 Lyncis	+ 0.3	5.0
31 „	− 2.7	4.3
50 Camelopardali	+ 2.3	5.5

Group 241.

29 Lyncis	− 1	5.6
30 „	0	5.8
56 Camelopardali	+ 1	6.0
58 „	0	5.8

Group 242.

32 Lyncis	+ 0.25	5.9
33 „	− 0.25	5.7

Group 243.

38 37 Lyncis = 40 Ⅎ Ursae maj.	− 1	6.0
40 39) „	+ 1	6.4

Group 244.

39 38 Lyncis	+ 0.5	3.8
41 (40) „	− 2.5	3.2
10 Leonis minoris	+ 1.5	4.0
10 Ursae majoris	+ 3.5	4.4
κ „	− 0.5	3.6
θ „	− 2.5	3.2

Group 245.

43 (42) Lyncis	− 1	5.0
44 (43) „	+ 1	5.4

Group 246.

ε Lyrae	0	4.5
ζ^1 „	+ 3	5.2
ε^1 „	− 1.5	4.1
ε^2 „	− 1.5	4.1
ζ^1 „	− 2	4.0
ζ^2 „	+ 2	5.0

Group 247.

δ Lyrae	+ 2.3	5.6
ν „	+ 1.3	5.3
δ^1 „	+ 1.3	5.3
δ^2 „	− 1.7	4.6
π „	− 1.7	4.6
λ „	+ 0.3	5.1
ρ „	+ 0.3	5.1
17 „	+ 0.3	5.1
ι „	+ 0.3	5.1
19 „	+ 1.8	5.4
γ_1 „	− 2.7	4.4
θ „ Σ	− 2.2	4.5

Group 248.

β Lyrae.

Group 249.

1 Monocerotis	− 1.6	5.7
2 „	+ 1.4	6.1
3 „	− 4.6	5.0
4 „	+ 2.4	6.7
6 „	+ 2.4	6.7

2 Monocerotis excluded from the mean.

Group 250.

19 Monocerotis	− 1	5.1
20 „	0	5.3

21 Monocerotis	0	5.3
22 „	− 3	4.6
23 „	+ 2	5.8
24 „	+ 1	5.5
25 „	+ 1	5.5

Group 251.

1 Persei	− 1.6	5.1
2 „	+ 1.4	5.8
3 „	+ 0.4	5.6
4 = *g* „	− 3.1	4.8
5 „	+ 1.3	5.8
7 „	+ 1.4	5.8
8 „	+ 1.5	5.9
9 = *i* „	− 1.1	5.2

Group 252.

11 Persei	− 0.8	3.9
θ „	+ 1.2	4.4
τ „	+ 0.9	4.3
χ „	− 1.3	3.8

11 Persei excluded from the mean.

Group 253.

12 Persei	+ 0.3	5.0
14 „	+ 0.55	5.0
16 „	− 1.95	4.4
17 „	− 0.95	4.9
20 „	+ 2.05	5.4
21 „	+ 0.05	4.9
π „	− 0.95	4.7
24 „	− 0.2	4.9
ω „	+ 0.05	4.9
43 = *A* „	+ 1.05	5.2

Group 254.

β Persei.

Group 255.

29 Persei	+ 2.2	5.3
31 „	+ 2.2	5.3
34 „	+ 0.2	4.8
σ „	− 1.8	4.4
ψ „	− 2.8	4.1

ψ Persei excluded from the mean.

Group 256.

30 Persei	+ 1.4	5.6
32 = *l* „	− 1.3	5.0
36 „	0	5.3

* Suspected variable.

Group 257.

1 Sextantis	− 1	4.8
2 „	+ 1	5.2

Group 258.

3 Sextantis	+ 0.75	5.8
5 „	+ 1.75	6.0
6 „	− 0.25	5.5
8 „	− 2.25	5.1

Group 259.

4 Sextantis	− 1.4	5.8
7 „	− 2.4	5.5
9 „	+ 1.6	6.5
12 „	+ 0.6	6.2
13 „	+ 1.6	6.5

Group 260.

14 Sextantis	− 0.3	6.2
16 „	+ 0.7	6.5
19 „	− 0.3	6.2

Group 261.

15 Sextantis	− 4.5	4.4
23 „	+ 2.5	6.1
24 „	+ 0.5	5.6
25 „	+ 3.5	6.3
26 „	+ 3.5	6.3
27 „	− 0.5	5.4
28 „	+ 0.5	5.6
29 „	− 1.5	5.1
30 „	− 3.5	4.7
31 „	+ 5.5	6.8
32 „	+ 2.5	6.1
33 „	− 0.5	5.4
τ^2 Hydrae	− 2.5	4.9
ι „	− 5.5	4.2

25 and 32 Sextantis excluded from the mean.

Group 262.

17 Sextantis	− 0.8	5.6
18 „	− 0.8	5.6
20 „	+ 1.2	6.1
21 „	+ 2.2	6.3
22 „	− 1.8	5.4

Group 263.

39 Sextantis	+ 1.3	5.5
40 „	+ 0.3	5.2
41 „	− 1.7	4.7

Group 264.

11 Tauri	+ 1.9	5.7
16 „	+ 1.9	5.7
17 „	− 3.1	4.5
18 „	+ 1.9	5.7
19 = *q* „	− 2.1	4.7
20 „	− 3.1	4.5
21 „	+ 1.9	5.7
22 „	+ 2.9	5.9
23 „	− 2.1	4.7
24 „	+ 4.9	6.4
η „	− 6.1	3.7
26 „	+ 3.9	6.1
27 „	− 3.1	4.5
28 „	+ 0.9	5.4

21 and 24 Tauri excluded from the mean.

Group 265.

13 Tauri	− 0.75	5.1
14 „	+ 0.75	5.5

Group 266.

32 Tauri	+ 1.5	5.6
33 „	+ 1.5	5.6
34 „	+ 4.9	6.4
36 „	+ 2.9	5.9
37 = *A* „	− 2.0	4.7
39 „	+ 2.9	5.9
43 „	+ 1.4	5.5
ω „	− 1.9	4.7
51 „	+ 1.2	5.5
53 „	+ 0.5	5.3
56 „	− 0.2	5.2
62 „	− 1.5	4.9
χ „	− 3.3	4.4
67 „	− 1.2	4.9
υ „	− 2.3	4.6
70 „	− 2.3	4.6
τ „	− 2.3	4.6
95 „	+ 0.7	5.1

Group 267.

41 Tauri	− 0.2	5.1
ψ „	− 0.7	4.9
44 = *p* „	+ 0.3	5.2

* Flamsteed's 34 was the planet Uranus. Herschel states, in a note, that he observed a telescopic star in nearly the place given by Flamsteed; but no star of the right position and magnitude can be found in the DM.

φ Tauri	− 0.7	4.9
χ „	+ 1.3	5.4

Group 268.

70 Tauri	+ 2.2	6.0
71 „	− 2.3	4.9
75 „	− 1.8	5.1
80 „	+ 0.2	5.5
81 „	− 0.8	5.3
84 „	+ 2.2	6.0
85 „	+ 0.2	5.5

Group 269.

89 Tauri	+ 0.7	5.5
σ^1 „	+ 0.2	5.3
σ^2 „	− 0.8	5.1

Group 270.

98 = *k* Tauri	+ 0.55	5.6
99 „	+ 2.05	6.0
101 „	+ 4.15	6.5
ι „	− 4.65	4.4
103 „	+ 0.55	5.6
104 = *m* „	− 3.85	4.6
105 „	+ 0.35	5.6
106 = *l* „	− 0.95	5.3
107 „	+ 2.65	6.1
108 „	+ 2.05	6.0
109 = *n* „	− 0.45	5.4
114 „	− 2.45	4.9

Group 271.

110 Tauri	+ 0.9	6.0
111 „	− 2.1	5.3
113 „	+ 0.9	6.0
115 „	− 1.1	5.5
116 „	− 1.1	5.5
117 „	+ 0.9	6.0
119 „	− 4.1	4.8
120 „	+ 0.9	6.0
122 „	− 1.1	5.5
126 „	− 3.1	5.1
127 „	+ 3.9	6.7
128 „	+ 2.9	6.5
129 „	+ 1.9	6.3
130 „	− 0.1	5.8

Group 272.

118 Tauri	+ 1	5.4
121 „	− 1	5.0

Group 273.

140 Tauri	0	6.4
141 „	0	6.4

Group 274.

α Aquilae	+ 0.3	1.3
α Cygni	+ 3.5	1.7
α Scorpii	− 3.7	0.9

Group 275.

Divided into four Sub-Groups.

σ Sagittarii suspected and *γ* Aquilae known to be variable.

Sub-Group A.

φ Capricorni	+ 0.2	5.0
33 „	+ 0.5	5.1
ζ „	− 4.8	3.9
35 „	+ 1.5	5.4
36 = *b* „	− 1.1	4.7
37 „	+ 2.9	5.7
38 „	+ 3.9	5.9
ε „	− 3.5	4.2
41 „	− 0.5	4.9
χ „	− 0.1	5.0

Sub-Group B.

34 Sagittarii	− 2.9	3.0
λ Aquilae	− 2	3.1
ζ „	− 2.9	3.0
δ „	− 0.1	3.5
γ „	− 3.9	2.8
η „	+ 0.9	3.9
β Aquilae	+ 0.9	3.7
θ „	− 0.9	3.3

Sub-Group C.

θ Serpentis	0	4.5
1 Aquilae	− 1	4.3
2 „	0	4.5
3 „	0	4.5
4 „	+ 2	5.0
5 „	+ 3	5.2
6 „	0	4.5
9 „	+ 1	4.7
12 „	0	4.5
14 = *g* „	+ 2	5.0
15 = *h* „	+ 3	5.2

Sub-Group D.

27 = *d* Aquilae	+ 1.1	5.2
ν „	− 0.9	4.7
35 = *c* „	+ 3.1	5.6
μ „	− 1.3	4.5
ι „	− 3.1	4.2
σ „	− 0.1	4.9
υ „	+ 1.5	5.3
o „	+ 0.5	5.0
ξ „	− 0.8	4.7
τ „	+ 1.5	5.3
p „	− 1.8	4.5
68 „	+ 2.9	5.6
69 „	+ 1.4	5.2
70 „	+ 0.4	5.0
71 „	− 1.6	4.5
1 Aquarii	+ 1.4	5.2

The reduction of Zöllner's measures. Zöllner, whom I denote by Z., in his Photometry of the Heavens gives what he calls an astrophotometric catalogue which consists of comparisons of a considerable number of stars upon different nights entirely unconnected together. I have divided these stars into sixteen groups, each group being connected by Zöllner's measures, have reduced them by least squares, and have then taken the mean magnitude of each group from the mean of the best magnitudes obtainable for the same stars, those of Seidel being preferred and for stars that Seidel had not measured, those of the DM. The ratio of light assumed is the same as that derived from the theory of equable distribution. The following table shows Zöllner's groups, expressed in the preliminary scale of magnitudes already referred to.

Group I.

Z.'s No.		Name.	DM. corr.	Seidel's corr.	Z.
1	ω	Persei	4.89		5.10
2	ρ	„	var.		3.86
8	β	„	var.	2.18	
9	κ	„	4.29		4.23
10	δ	„	3.58	2.99	3.14
11	γ	„	3.31	3.08	3.08
12	α	„	2.06	1.99	1.98
13	ε	„	3.31	2.87	3.07
22	ν	„	4.04		4.33
23	π	„	4.83		5.56
24	λ	„	4.11		4.43
25	μ	„	4.11		4.24
26	c	„	4.69		4.32
40	ι	Aurigae	3.04	3.08	2.63
41	β	Tauri	2.06	1.73	1.85

Group II.

3	β	Cassiopeiae	2.53	2.45	2.25
4	α	„	var.	2.55	2.12
5	γ	„	2.63	2.17	2.17
6	δ	„	3.04	2.81	2.74
7	ε	„	3.41	3.27	3.55
226	κ	„	4.29		4:19

α Cassiopeiae rejected from the mean.

Group III.

14	θ	Aurigae	3.04	2.50	2.64
15	β	„	2.06	1.95	2.00
16	ε	„	var.	3.19	2.96
17	α	„	∞	0.60	0.12
34	9	Camelopardis	4.69		4.32
35	10	„	4.69		3.91
36	δ	Aurigae	3.91		3.69
37	θ	Geminorum	3.31	3.54	3.65
38	α	„	1.70	1.62	1.59
39	β	„	1.43	1.52	1.63

Group IV.

18	δ	Ursae majoris	3.41	3.24	3.37
19	α	„	2.06	2.17	1.85
20	β	„	2.63	2.31	2.47
21	γ	„	2.74	2.40	2.50
31	ε	„	2.06	1.84	1.85
32	ζ	„	2.06	2.09	2.08
33	η	„	2.06	1.82	2.00
69	ξ	„	3.70		3.81
70	χ	Ursae majoris	3.91		3.63
71	ν	„	3.47		3.38
72	43	Comae Ber.	4.69		4.31
73	12	Canum Venat.	3.04	2.72	2.97
74	8	„	4.29		4.29
75	κ	Draconis	3.41	3.68	3.85
76	α	„	3.47	3.73	3.62
77	t	„	3.04	3.01	3.14
83	ι	Bootis	4.29		4.68
84	χ	„	4.29		4.40
85	θ	„	3.91		4.03
86	84	Ursae majoris	6.01		5.76
87	83	„	5.12		4.42
88	86	„	5.52		5.63
89	λ	Bootis	3.91		4.26
90	y	„	2.83	3.17	3.20
91	β	„	3.04	3.54	3.53
98	6	Draconis	4.49		4.86
99	4	„	4.42		4.83
100	δ	Bootis	3.04	3.56	3.55
101	ρ	„	3.91		3.63
102	ε	„	2.63	2.64	2.41
112	31	Comae Ber.	4.69		4.97
113	41	„	4.83		4.73
114	37	„	4.89		4.81
120	σ	Bootis	4.69		4.61
121	31	„	5.32	5.26	4.83
129	θ	Coronae	4.11		4.31
130	γ	Lyrae	3.31	3.15	3.32
131	β	„	var.	3.27	3.65
135	ψ	Bootis	4.42		4.58
136	b	„	5.82		5.72
137	w	„	4.42		4.81
138	c	„	4.49		5.07
139	δ	Coronae	4.29		4.63
140	β	„	3.91	3.61	3.77
141	α	„	2.06	2.21	2.43
142	γ	„	3.91	3.68	4.01
143	ε	„	3.91		4:15
144	ν	Bootis	4.29 / 4.49		4.92
145	μ	„	3.91	4.08	4.30
146	ζ	Coronae	4.17		4.62
147	χ	Herculis	4.17		4.62
148	υ	„	4.04		4.68
149	φ	„	3.76		4.37
150	τ	„	3.58	3.71	4.03

Z.'s No.		Name.	DM. corr.	Seidel's corr.	Z.
153	ε	Herculis	3.58	3.78	4.08
154	η	„	3.04	3.64	3.59
155	π	„	3.04	3.44	3.22
156	ζ	„	3.04	2.96	2.93
157	∂	„	3.04	3.15	3.34
158	σ	„	3.91		4.26
159	ν	„	4.29	4.42	4.37
160	μ	„	3.58	3.58	3.60
161	ξ	„	3.91	3.89	3.71
162	o	„	3.80	3.73	3.91
163	θ	„	3.58	4.11	3.86
164	ν	Draconis	4.89 / 4.89		4.52
165	ι	Herculis	3.91	3.88	4.14
166	y	Draconis	2.53	2.65	2.32
167	β	„	3.04	3.06	2.96
168	ξ	„	3.58	3.95	3.97
169	ζ	Lyrae	4.29 / 5.12		4.18
170	δ	„	4.29		4.18
171	η	„	4.29		4.47
172	θ	„	4.37		4.38
173	η	Cygni	4.11		3.97
174	δ	„	3.04	2.89	2.94
175	γ	„	2.83	2.48	2.18
176	ε	„	2.80	2.84	2.40
182	μ	„	4.29		4.59
183	ζ	„	3.04	3.38	3.24
184	κ	Pegasi	4.04		4.24
185	ι	„	3.91		3.87
58	ω	Ursae majoris	4.89		4.91
59	ψ	„	3.58	3.24	2.97
80	54	Leonis	3.91		4.37
81	χ	Ursae majoris	3.91		3.63
82	46	Leonis minor.	3.91		3.71
103	16	Comae Ber.	4.89		5.12
104	8	Canum Venat.	= No. 74		4.29
105	15	Comae Ber.	4.42		4.37
106	14	„	4.83		4.97
107	17	„	5.52		5.39
108	13	„	4.98		5.37
109	12	„	4.69		4.98
110	7	„	4.98		5.02
111	23	„	4.49		5.67
122	15 / 17	Canum Venat.	5.12		5.98
123	14	„	5.12		5.33
124	11 H	Can. Venat.	4.69		4.98
125	20	„	4.17		4.74
126	17 H	„	4.83		5.02
127	25	„	4.98		4.87
128	23 H	„	5.32		4.83
132	5 H	„	5.03		5.52
133	6	„	5.03		5.04
134	2 H	„	4.69		5.00

Group V.

27	υ	Aurigae	4.69		4.68
28	η	„	3.47		3.35
29	ζ	„	3.67		3.72
30	ν	„	3.91		3.94
55	τ	„	4.42		4.49
56	ν	„	= No. 30		
57	40	„	5.52		5.51

Group VI.

42	o	Ursae majoris	3.41	3.49	3.26
43	θ	„	3.04	3.22	3.19
44	ι	„	3.24	3.12	3.08
45	κ	„	3.57	3.39	3.59
46	10	„	4.04		3.84
47	38	Lyncis	4.11	3.78	3.79
48	40	„	3.24	3.50	2.92
49	31	Leonis minor.	3.91		4.26
50	λ	Ursae majoris	3.58	3.20	3.45
51	μ	„	3.24	3.30	2.91
52	μ	Leonis	3.91		3.59
53	ζ	„	3.58	3.35	3.37
54	ε	„	3.31	3.21	2.79
60	31	„	4.69		4.71
61	33 H	Ursae maj.	4.89		6.03
62	38	Leonis minor.	5.24		6.04
63	35	„	5.82		6.62
64	φ	Ursae majoris	4.69		4.69
65	υ	„	3.91		3.72
66	α	Leonis	1.43	1.41	1.60
67	α	Canis minoris	∞	0.74	0.59
68	α	Aurigae	= No. 17		
78	h	Ursae majoris	3.58		3.60
79	λ	Draconis	3.41	4.01	3.57
92	τ	Ursae majoris	4.69		4.60
93	c	„	4.69		4.99
94	b	„	5.24		5.57
95	σ^2	„	4.98		4.79

Z.'s No.	Name.	DM. corr.	Seidel's corr.	Z.
96	σ^1 Ursae majoris	4.89		4.91
97	ρ „	4.83		4.65
115	ζ Leonis. . .	3.58	3.35	3.40
116	β „ . . .	2.06	2.15	2.48
117	δ „ . . .	2.63	2.53	2.89
118	ϑ „ . . .	3.41	3.22	3.68
119	γ „ . . .	2.06	2.42	2.28
	Group VII.			
151	α Lyrae . . .	∞^0	0.42	0.39
152	α Bootis . . .	∞	0.62	0.22
186	α Cygni . . .	1.62	1.46	1.61
187	α Aquilae . . .	1.26	1.05	1.33
	Group VIII.			
177	α Delphini . .	3.58		4.64
178	β „ . . .	3.58	3.79	3.74
179	γ „ . . .	3.58	4.18	3.96
180	δ „ . . .	4.11		4.01
181	ε „ . . .	3.76	4.00	4.26
	Group IX.			
182	υ Cygni . . .	4.11		4.49
183	β „ . . .	4.04		4.00
184	σ „ . . .	4.29		4.32
185	ι „ . . .	3.91		3.73
186	ξ „ . . .	3.91		3.61
187	ω „ . . .	3.91		4.03
	Group X.			
188	ι Andromedae .	4.11		4.26
189	λ „ . .	3.58		3.76
190	$\varkappa$ „ . .	4.42		4.14
191	o „ . .	3.70		3.64
	Group XI.			
192	ψ Draconis . .	4.49		4.33
193	δ „ . .	3.47	2.96	2.94
194	φ „ . .	4.42		4.18
195	χ „ . .	3.76		3.44

Z.'s No.	Name.	DM. corr.	Seidel's corr.	Z.
196	α Cephei . .	3.04	2.53	2.42
197	β „ . .	3.04	2.99	3.12
	Group XII.			
198	59 B Cassiopeiae	5.52		5.02
199	υ^1 „	4.69		4.53
200	υ^2 „	4.69		4.58
201	γ_1 „	3.76	3.44	3.34
202	ζ „	4.04	3.57	3.69
203	λ „	4.89		4.68
	Group XIII.			
204	ξ Andromedae .	4.89		4.74
205	φ „ .	4.21		4.24
206	ω „ .	4.69		4.73
207	ϑ Cassiopeiae .	4.49		4.27
208	υ Persei . . .	3.67	3.90	3.90
209	φ „ . . .	4.04		3.50
	Group XIV.			
210	c Cygni . . .	5.82 / 5.73		5.84
211	o^1 „ . . .	3.92		4.07
212	o^2 „ . . .	4.69		4.21
213	ϑ „ . . .	4.56		4.79
214	ι „ . . .	3.92		4.16
215	$\varkappa$ „ . . .	4.30		4.09
	Group XV.			
216	e Draconis . .	4.98		4.90
217	o „ . .	4.17		4.32
218	d „ . .	5.03		4.59
219	b „ . .	4.49		4.97
	Group XVI.			
220	b Cygni . . .	5.12		5.39
221	P „ . . .	4.98		4.90
222	36 „ . . .	5.67		5.36
223	35 „ . . .	5.03		5.19
224	b^3 „ . . .	5.03		4.98
225	b^2 „ . . .	5.12		5.13

Sir John Herschel observed by the method of sequences as fully described in his Cape Observations, and determined his scale of magnitudes photometrically. According to him the light of a star is connected with its magnitude by the formula,

$$(m_h - .41)^2\, l = 1.$$

In fact, however, Herschel's own photometric observations are better satisfied by a logarithmic equation of the usual form. The formula I have used for reducing his magnitudes is

$$m = -0.69 + 1.14\, m_h.$$

I designate this observer by the abbrieviation *h*.

Seidel. The formula

$$\log l_s = +.057 - .44\, m$$

which I have obtained by the comparison of Seidel with the Durchmusterung serves to reduce his measures to my scale of magnitudes.

I designate this observer by the abbrieviation S.

Behrmann. Abbrieviation, B. This observer of southern stars has closely imitated the scale of the Uranometria. I have, therefore, ascertained by what constant it was necessary to multiply the numbers of his stars as bright as any numerical magnitude in order to get the number as bright as the same magnitude in A., north of the equator. This constant I find to be 1.48. As Behrmann's stars are contained in two thirds of a hemisphere, it follows that the density of the southern heavens observed by him is the same as that of the northern hemisphere. The following table exhibits the reduction of Behrmann's magnitudes:

	Number.	Limit.	Mean.	Mean for A.
1	5	1.4	0.6	0.3
1.2	7	1.7	1.0	1.3
2.1	12	2.1	1.9	1.7
2	17	2.4	2.3	2.1
2.3	28	2.8	2.6	2.6
3.2	35	3.0	2.9	2.8
3	57	3.4	3.3	3.2
3.4	84	3.7	3.6	3.6
4.3	106	3.9	3.8	3.8
4	142	4.1	4.0	4.1
4.5	207	4.5	4.3	4.4
5.4	306	4.8	4.6	4.6
5	448	5.1	4.9	4.9
5.6	670	5.4	5.3	5.2
6.5	1077	5.8	5.6	5.3
6	2324	6.4	6.2	5.8

Abbrieviations of the Names of Astronomers etc.,

used in this Book.

A. Argelander.
B. Bode.
𝔅. Behrmann.
DM. Durchmusterung.
F. Flamsteed.
H. Hevelius.
H. Sir Wm. Herschel.
h. Sir John Herschel.
ℌ. Heis.
LC. Lacaille (Henderson's Edition).
LL. Lalande (Baily's Edition).
M. Messier.
P. Piazzi; Peirce.
Pt. Ptolemy.
Σ. F. G. W. Struve.
ɔ. Otto Struve.
S. Sûfi.
Ⓢ. Seidel.
Sj. Schjellerup.
Se. Secchi.
T. Tycho Brahe.
U. Ulugh Beg.
Z. Zöllner.

CHAPTER III.

ORIGINAL OBSERVATIONS.

My observations have been made with a Zöllner's Astrophotometer. Though the principle of this instrument is well-known, I may remind the reader that an artificial star is thrown into the field of a telescope, and that its brightness is reduced by the rotation of a Nicol prism, until it matches, in brightness, any real star which is in the field at the same time. The Nicol prism being furnished with a graduated circle, the ratio of the reduction of the light is calculated from the reading. A third Nicol with an interposed quarz plate, cut perpendicular to its axis, makes it possible to alter the color of the artificial star, and this third Nicol is also furnished with a graduated circle. I suppose it was Zöllner's intention that the observer should match the real star in color as well as in brightness, and so determine two of the three constants which are necessary to define any appearance of light. My experience, however, soon led me to attach a clamp to the "color-circle", and to keep it fixed in position, except when I desired to make a special observation upon the color of a star. For the color of a Kerosene lamp varies very much, even during one evening, so that the change of brightness of the artificial star produced by turning the color-circle was by no means constant. Moreover, the color-circle was only graduated to every 5° and an alteration of its position equal to tenth of this interval would make a difference of a twentieth of a magnitude, so that an error of reading might have a decided effect. Another important reason for not using the color-circle in measures of brightness was that, owing to the lens which serves as objective to the artificial star not being achromatic, it had

to be focussed every time the color-circle was moved, and this focussing was not only difficult, but also, as I shall explain presently, tended to introduce another sort of error.

There is not, as is generally supposed, any great difficulty in comparing two lights of different colors and deciding which is the brighter, and the impossibility of accurately comparing the brightness of two lamps or two stars does not lie in the difficulty of the immediate observation. It lies chiefly in the fact that a change in the objective light produces less change in the sensation of blue than in those of red and green, so that the warmer colored stars appear relatively brighter on fine, clear nights. If we could keep our artificial star constant in color and could easily modify the color in a known way, all difficulty in comparing stars of different color could be overcome. But since this cannot be done, and since the error in the immediate comparison of light of different colors is not great, I should prefer, in constructing a photometer, to leave the artificial star fixed in brightness and only alter the light of the real star. In that way, we should compare the stars of different colors at a fixed relation to one another. Further on, I shall mention another reason for preferring such an arrangement.

I have specially observed the color of about 150 stars. In the comparative color-catalogue which is appended to this memoir, it will be observed that my italicized observations, which are those made with the instrument, never make a star red, or even orange, — never more than decidedly yellow, — while I frequently call stars blue which other observers consider to be yellow. This is because I have attached names to the colors by lighting the photometer lamp in the daytime, setting the color-circle to the various readings and then assigning such names to the colors as seemed appropriate. Owing to the relative faintness of the light in the daytime, all the colors appear much bluer.

The quartz plate which produced the color of the photometer star was measured with an accurate plate gauge (tested in the office of weights and measures), and was found to have thickness of from 0.1960 to 0.1961 inches. The thickness was also determined by the amount of deviation of the plane of polarization of the light formed by impregnating the wick of an alcohol lamp with salt. The thickness calculated from this experiment was 0.1953 inches. The following shows the comparative thickness of the quartz plates used by three observers.

	mm.		mm.		mm.
Zöllner	5.150	Rosén	4.966	Peirce	4.976

I observed the relative brightness of the photometer star in different positions of the color-circle and found it well represented by the empirical formula

$$m = -4.374 + .095288\,\psi - .0004526\,\psi^2$$

where ψ is the reading of the color-circle and m is the magnitude on a scale for which $\rho = 2.25$. The following table shows the comparison of the formula with the observations.

Color-circle.	Color.	MAGNITUDE. Observed.	Calculated.	Obs.—Calc.
		m.	m.	m.
15°	Grey	— 3.01	— 3.04	+ .04
30	Intense blue green	— 1.87	— 1.92	+ .05
45	Decidedly greenish white	— 0.91	— 1.10	+ .09
60	Straw	— 0.33	— 0.29	— .04
75	Decided yellow	+ 0.12	+ 0.22	— .10
90	Intense yellow	+ 0.51	+ 0.53	— .02
105	Yellow orange	+ 0.66	+ 0.65	+ .01
120	Very red orange	+ 0.57	+ 0.55	+ .02
135	Scarlet	+ 0.30	+ 0.24	+ .06
150	Cherry	— 0.19	— 0.27	+ .08
165	Crimson	— 1.07	— 0.97	— .10
180	Very red purple	— 1.94	— 1.89	— .05
195	Grey	— 3.01	— 3.00	— .01

These numbers were obtained by the comparison of one artificial star of fixed color with another whose color was varied. They suffice of themselves to dispel the idea, if anybody still has it, that the observation of two lights of different colors, to say which is the brighter, is devoid of all certainty, because they exhibit so much regularity and concordance among themselves. My color equation differs perceptibly but not greatly from those of Zöllner and Rosén.

The flame which produces the artificial star is that of a kerosene lamp. It was necessary to take the greatest pains to keep this clean and well filled, and to have the wick trimmed with perfect neatness and parallelism to the top of the wick-tube, except at the corners, which were cut off, to prevent the expansion of the flame to each side. This lamp ought to have been made with great nicety, but it was, in fact, a poor affair, not nearly so well constructed as those we use in our houses. The height of the flame could be kept constant by means of a little sight attached, but when the lamp worked well this did not vary during the evening. The lamp was prevented from being blown by the wind by a shield which would have shut off the draught, if it had not been for some tubes by which the air was conveyed to the lamp. As this came from the maker, it was badly constructed, the tubes not supplying enough air nor at the right points. I soon had it altered, after which it gave no trouble, except when there was a considerable breeze, or when the weather was hot. In the summer of 1872 in Washington when the temperature was generally about 90° F. in the observatory, the lamp did not behave well*. The accompanying table shows the fluctuations of the light on the adopted scale of magnitudes.

* The observations were also made difficult, that season, by the swarms of insects.

Table showing the magnitude of the Photometer Star, with no cap, and pinhole 5, and with the circle at 90°, diminished by 4m.10.

No of set.	Date.	Value.	Remarks.
		m.	
1	1872 March 15	1.04	*Cambridge.*
2	„ „ 16	.93	Obs. thinks lamp may have altered during set.
3	„ „ 18	.92	Bright moon. Thin, light, clouds.
4	„ „ 21	1.00	Moon very near. Very windy. Seeing very bad.
5	„ „ „	1.14	
6	„ „ 22	0.74	Very bright moonlight, windy and hazy.
7	„ May 4	— 0.03	*Washington.*
8	„ „ „	+ .08	
9	„ „ 5	.37	Sky slightly hazy.
10	„ „ 21	.11	Full moon; sky very bright and clear.
11	„ „ 26	— .54	Very fine night.
12	„ „ 28	.74	Clear still night; no moon. Interrupted by clouds.
13	„ „ 30	.47	Night clear, no moon. Stars twinkling, somewhat windy, clouds.
14	„ „ 31	.37	Night clear, no moon, and still. Later, windy.
15	„ June 2	.74	Still, cloudless, no moon. Stopped by haze.
16	„ „ 10	.38	Moon about 3 days old. Gentle breeze, light clouds.
17	„ „ 11	.38	
18	„ „ 12	.10	Clear. Moon 6 days old. Gentle breeze.
19	„ „ „	.29	
20	„ „ 13	.01	Moon quartered. Sky moderately clear. Windy.
21	„ „ „	.37	Windy; light clouds illuminated by moon. Stopped by haze.
22	„ „ 18	.26	Moon nearly full, pretty windy. Light clouds repeatedly interrupted.
23	„ „ 19	.19	Pretty calm, moon full. Light clouds.
24	„ „ 29	.37	
25	„ „ 30	.13	Clear calm, moonless.
26	„ „ „	.16	Superb night.
27	„ July 1	.11	Pretty clear, gentle wind, no moon. Later, clouds.
28	„ „ 2	.16	Nearly clear, breeze, no moon.
29	„ „ 8	— .12	Nearly clear, moon 2 days old.
30	„ „ 19	.18	Gentle wind, full moon. Haze near horizon.
31	„ „ 20	.10	Very clear, full moon, moderate breeze.
32	„ „ 28	.24	Strong wind blowing lamp, no moon.
33	„ „ 30	.01	Moonless, gentle wind, stopped by clouds.
34	„ „ 31	.22	
35	„ Aug. 6	.18	Lamp in some way out of order. Clear, pretty strong wind. No moon. Affected by irregularities of lamp or of atmosphere.
36	„ „ „	.06	
37	„ „ 7	.14	Clear; moon about 2 days old; pretty strong wind.
38	„ „ „	— .15	
39	„ „ 8	.29	Clear; moderately bright aurora; pretty strong wind; moon about 3 days old.

No of set.	Date.			Value. m.	Remarks.
40	1872	Aug.	7	.14	
41	„	„	9	— .08	Clear, moon about 5 days old; brisk wind blowing lamp badly.
42	„	„	17	— .01	Clear, nearly calm, moon nearly full. Interrupted by clouds.
43	„	„	18	— .15	Clear, slight haze, very calm, v. slight breeze.
44	„	Sep.	7	— .42	Moderately clear, v. hazy, no moon.
45	„	„	14	— .25	Clear and moderate wind. Moon about 11 days old.
46	„	„	16	— .30	Clear, calm, and full moon.
47	„	„	19	— .40	Clear, moon about 17 days old, strong wind.
90	„	Oct.	15	.18	Full moon, very windy.
48	1873	Feb.	28	— .37	Clear, hazy and windy.
49	„	March	6	— .02	Clear, gentle wind. Moon about 8 days old.
50	„	„	7	.32	Rather hazy, light wind, moon about 9 d. old. Haze increased, then decreased.
51	„	„	9	.26	Clear. Moon-light. Stopped by increasing wind.
52	„	„	10	— .08	Bright moonlight, clear, pretty windy; lamp went out once; light clouds close to place of observation.
53	„	„	„	.27	
54	„	„	17	.22	Clear moonless, moon rose 3d round; suspicion of clouds.
55	„	„	22	.31	Clear, moonless, moderate wind; seeing bad, stars varying.
56	„	May	26	.22	Night still, moonless, clear.
57	„	„	31	.18	Clear, light west wind, moon about 3 d. old.
58	„	June	1	.26	Night clear, pretty strong wind, moon about 5 d. old.
59	„	„	7	.14	Clear, calm, moonlight night. Clouds came up.
60	„	„	8	.19	Moderate wind, moonlight. Images bad. Clouds in the neighborhood.
61	„	„	13	.45	Clear moonless night, rather strong wind.
62	„	„	18	.74	
63	„	„	20	.45	Pretty hard northwind.
64	„	„	21	1.17	Light haze.
65	„	„	25	r	Cool, clear, moderate wind, stars are steady.
66	„	„		1.03	
67	„	July	1	1.20	Calm, very clear fine night.
68	„	Sep.	6	.40	*Hoosac mountain.* Remarkably clear and calm: moon nearly full.
69	„	„	8	.31	
70	„	„	„	.35	
71	„	„	9	.40	Interruption by clouds. Lamp blown by wind.
72	„	„	12	.35	Cap wind blows lamp.
73	„	„	„	.61	Compⁿ. vitiated by clouds.
74	„	„	16	.06	Strong wind; lamp badly blown; once blown out.
75	„	„	17	.0	Superb night.
76	„	„	21	.69	
77	„	„	24	.34	Seeing excessively bad.
78	„	„	25	.63	Wind interfered.
79	„	„	26	.66	Seeing bad.

No of set.	Date.			Value. m.	Remarks.
80	1873	Sep.	26	.64	
81	„	„	27	.65	
82	„	„	„	.56	
83	„	„	28	.26	
84	„	„	„	.41	Lamp blown a little; light wind.
85	„	„	30	.80	
86	„	„	„	.42	
87	„	„	„	.62	Observ. stopped by clouds.
88	„	Oct.	1	—	
89	„	„	8	.59	
91	„	„	„	.54	
92	„	„	9	.66	
93	„	„	„	.58	
94	„	„	„	.61	
95	„	„	11	.47	
96	„	„	„	.11	
97	„	„	13	.16	Very hazy, seeing not very good.
98	„	„	15	— .17	
99	„	„	„	— .22	
100	1874	Feb.	4	.62	*Washington.*
101	„	„	„	.36	
102	„	„	„	r	
103	„	„	10	.66	
104	„	„	„	.80	
105	„	„	„	.90	
106	„	„	11	1.12	Seeing very bad.
107	„	„	12	1.47	Hazy, perhaps cloudy, no moon.
108	„	„	18	.52	
109	„	March	24	.58	
110	„	„	„	.50	
111	„	„	26	.29	Bright moonlight.
112	„	„	„	.46	
113	„	„	„	.82	
114	„	„	29	.86	
115	„	Apr.	13	.17	*Cambridge.*
116	„	„	30	.51	
117	„	May	2	.26	Very clear.
118	„	„	„	.46	
119	„	„	„	.22	
120	„	„	3	.43	Began in the twilight; clouds susp. in the neigborhood; moon not up; lamp commenced making a noise.
121	„	„	„	.55	
122	„	„	28	.25	*Hoosac mountain.* Very clear, bright moonlight.
123	„	June	13	.08	
124	„	„	21	.11	
125	„	„	„	— .02	
126	„	„	72	.06	Lamp considerably blown.

No of set.	Date.	Value.	Remarks.
		m.	
127	1874 June 27	.11	
128	„ „ „	.12	
129	„ „ „	— .42	
130	„ „ 28	.02	
131	„ „ „	.11	
132	„ July 14	*r*	Night very hazy.
133	„ „ „	.81	Seeing especially bad.
134	„ „ 15	.23	
135	„ „ „	.45	
136	„ „ 17	.30	
137	„ „ „	.40	
138	„ „ 22	.45	
139	„ „ „	.74	
140	„ „ „	.69	
141	„ „ 30	.90	
142	„ „ „	.73	
143	„ „ „	.70	
144	„ „ 31	.95	Bright moonlight and light clouds continually passing over the field.
145	„ Aug. 2	.54	
146	„ „ „	.66	
147	„ „ „	.66	
148	„ „ 3	.88	
149	„ „ „	.95	Very light passing clouds.
150	„ „ „	.98	
151	„ „ „	1.62	Growing light rapidly.
152	„ „ 4	.91	Thin flying clouds during all these observ.
153	„ „ „	.91	
154	„ „ „	.93	
155	„ „ 22	.85	
156	„ „ „	1.08	
157	„ „ „	1.33	
158	„ „ „	.99	
159	„ Oct. 21	.88	*Northampton.*
160	„ „ „	.93	
161	„ „ „	.91	After 2nd round lamp filled again.
162	„ „ 22	.94	Bright moonlight.
163	„ Nov. 26	.82	*Cambridge.*
164	„ „ „	.82	
165	„ „ „	.94	
166	„ „ „	.86	
167	„ „ „	1.01	
168	„ „ „	.94	
169	„ „ „	.84	
170	„ „ „	.88	
171	„ „ „	.85	
172	„ „ 30	.74	

No of set.	Date	Value.	Remarks.
		m.	
173	1874 Nov. 30	.63	
174	„ „ „	.89	
175	„ „ „	.99	
176	„ „ „	.99	
177	„ „ „	1.04	Presence of clouds suspected.
178	„ „ „	.87	
179	„ „ „	.88	
180	„ Dec. 4	.75	Wind blows lamp a little.
181	„ „ 21	.80	
182	„ „ „	.91	
183	„ „ „	.98	
184	„ „ „	.98	
185	„ „ „	.95	
186	1875 Jan. 10	.78	
187	„ „ „	.86	
188	„ „ „	1.04	Extreme cold interferes.
189	„ „ 14	.78	Lamp very much blown by wind.

The light shone through a small glass window in the metallic chimney of the lamp, on to a disk, perforated with minute holes which I have called the pinholes. There were six pinholes, either of which could be so placed that the light would pass through it. I measured the longest and shortest diameter of each pinhole and found them to be as follows. The measures are expressed in turns of a micrometer screw each of which is equal to about $\frac{1}{40}$ of an inch.

Pinhole.	Longest Diameter.	Shortest Diameter.
1	1.039	1.038
2	.817	.805
3	.599	.597
4	.396	.315
5	.315	.301
6	.219	.217

Pinhole 4 is very irregular, (thus ◇). The measure of its short diameter was accommodated, so as to make what it was estimated would be an ellipse of equal area. I made experiments to determine the effect of the different pinholes upon the magnitude of the photometer star, and found it to be such as would be obtained by multiplying the theoretical logarithms of the light by 0.894. The following tables exhibit the results of these experiments. They are expressed in a scale of magnitudes for which $\rho = 2.25$, this being the scale used in the reduction of my observations.

1872 March 8.

Pinhole.	Diff. from. mean. Obs.	Theory × .894
3	+ .31	+ .25
2	+ .94	+ .92
5	— 1.25	— 1.19

1872 March 18.

Pinhole.	Obs.	Diff. from mean. Theory × .894
1	+ 1.68	+ 1.71
2	+ 1.14	+ 1.16
3	+ 0.51	+ 0.49
4	− 0.61	− 0.67
5	− 1.03	− 0.97
6	− 1.67	− 1.73

In the reduction of my observations, I have used the theoretical values multiplied by .894. The pinhole was not often changed.

From the pinhole, the light passes through a small double concave lens of short focus, then through a diaphragm with an aperture of about a millimeter, then through the first Nicol, then through the quartz plate, then through the other two Nicols. It will be observed that, unless these Nicols are placed with their axes in coincidence with the optical axis, the law of the diminution of light

$$l = a\,(\sin\varphi)^2,$$

where φ is the photometer reading, does not hold. The Nicols which came with the instrument were replaced by others in the winter of 1872—73, and these later ones were, I think, successfully put into position. They seem absolutely to extinguish the brightest light I have been able to throw in. To avoid, in some degree, the error due to malposition of the prisms, it was my habit to make four observations in the four quadrants of the photometer circle.

From the last Nicol the light passes through an unachromatic double concave lens. This lens is capable of being moved so as to bring its focus into coincidence with that of the objective of the telescope, and the screw by means of which this is effected, is furnished with a divided scale. But this motion is excessively bad and wobbling; and as the last Nicol is moved with this lens, the parallelism of the Nicols cannot be kept perfect. The motion of this lens by means of the screw is so slow that I found it excessively difficult to find the focus. For if I attempted to do this by means of the appearance of distinctness of the image, the involuntary accommodation of the eye greatly troubled me; while owing to the low magnifying power of the telescope (about 11) and its want of fixity, it was quite impossible to do so by the parallax between the photometer-star and the real star. In fact, I found the only way was to get it as near as I could and then alter it as I got the impression of one or the other image appearing more distinct.

From this lens, the light passes through a diaphragm with an aperture of about a millimeter, and then strikes upon a plate of glass in the middle of the telescope, from which it is reflected to the eye-piece (an ordinary negative ocular), thence passes through a totally reflecting prism, which turns it at right angles, and thence through the diaphragm of the eye-piece.

The light of the lamp therefore strikes the following surfaces.

Part of the Instrument.	No. of surfaces.
Window of chimney	2
Concave lens	2
First Nicol	4
Quartz plate	2
Two last Nicols	8
Convex lens	2
Glass plate	1
Eye-piece	4
Reflecting prism	3
Total	28

The plate of glass which reflects the photometer star down to the eye-piece has its two surfaces inclined to one another. The effect of the interposition of this plate is to make the image of the real star excessively bad.

Owing to the photometer star being reduced in light to match the real star, instead of the real star being reduced to match the photometer star, it is necessary, for bright stars, to put a cap on the object-glass of the telescope. The aperture of the telescope I have used is 37 millimetres, the aperture of cap 1 is 26 millimetres and the aperture of cap 2 is 13 millimetres. I found that these caps seldom or never produced their theoretical effects. Moreover, the effects actually produced varied very much. I attribute this chiefly to the want of polish of the reflecting surface of the totally reflecting prism, which, being close up to a piece of brass, would get spotted with oxide of zinc and according as it was worse, in this respect, about where the outside or the inside of the pencil of rays fell, it would give a greater or a less value to the cap constant. The irregularity was increased by the dirt which would get on the outer surface of the prism and which the eye-piece diaphragm, fixed in its position with screws, would prevent me from entirely removing from the edges of the ocular aperture. This difficulty was at its worst in winter, when frost would have to be frequently wiped off in the midst of the observations. I have found it necessary to assume five different sets of values of the cap constants for observations made at different periods. I express these in a scale for which $\rho = 2.25$.

First Period. Observations at Cambridge from 1872 Feb. 14 to March 22, and observations in Washington from 1872 May 4 to 1873 March 22. After this, the instrument was taken to pieces and some alterations were made in it.

Second Period. Observations in Washington from 1873 May 13 to July 1, observations on the Hoosac Mountain from 1873 September 6 to October 15, and observations in Washington from 1874 January 23 to March 29.

Third Period. Observations at Cambridge from 1874 April 13 to May 3.

Fourth Period. Observations on the Hoosac Mountain from 1874 May 28 to August 22.

Fifth Period. Observations at Northampton, 1874 October 21 and 22, and observations at Cambridge 1874 November 26 to 1875 January 15.

The following are the adopted values.

Period.	Cap 1. m.	Cap 2.
1	0.46	1.79
2	0.70	2.19
3	0.80	2.11
4	0.72	2.44
5	0.74	2.15

The following table shows the separate observations for the determinations of these constants. Their disagreements are very great. I write "Cap 0" for no Cap.

Observations on Cap Constants.

Difference between Cap 1 and Cap 0.

PERIOD I.

Date.	Star.	Value.	No. of settings.	Remarks.
1872 March 8		0.55	5	Good. Pinhole 3.
„ „ „		0.43	5	Good. Pinhole 3.
„ „ 16	*g* Urs. maj.	0.56	5	Good.
„ May 4	ν_1 and ν_2 Bootae	0.42		
„ „ 8	„	0.38		
„ June 11	13 Lyrae	0.54	4	Good. Rather too bright.
1873 Feb. 28	ζ Urs. maj.	0.24	1	Much too bright. Reject.

PERIOD II.

1873 June 1	ω Urs. maj.	0.69	3	Very good.
„ „ „	47 „ „	0.77	2	Very good.
„ „ „	49 „ „	0.79	2	Very good.
„ „ 20	θ Herculis	0.64	4	Very accordant, but altogether too bright.
„ Sept. 8	*i* Herc.	0.56	3	Very accordant.
„ „ 30	66 B Cygni	0.38	4	Very good.
„ „ „	10 B „	0.47	4	Good.
„ „ „	θ „	0.56	4	Pretty good.
„ „ „	16 Lyrae	0.55	4	Pretty good.
„ Oct. 8	λ Androm.	0.59	2	Very good.
„ „ „	φ „	0.68	4	Pretty good.
„ „ 15	22 „	0.61	2	Poor.
1874 Feb. 11	γ Persei	0.38	4	Accordant, but much too bright. Reject.
„ „ 12	γ „	0.65	2	Poor.

PERIOD III.

Date.	Star.	Value.	No. of settings.	Remarks.
1874 Apr. 30	10 Urs. maj.	0.69	3	Good.
„ May 2	38 Lyncis	0.55	4	Pretty accordant, but much too bright. Reject.

PERIOD IV.

1874 June 13	τ Cygni	0.43	3	Accordant, but much too bright. Reject.
„ „ „	σ „	0.50	3	Pretty accordant, but too bright.
„ „ 27	8 Can. ven.	0.58	2	Good.
„ „ 28	θ Bootae	0.67	4	Pretty good.
„ „ „	ι „	0.71	4	Good.
„ „ „	κ „	0.89	4	Tolerable.
„ „ „	λ „	0.57	4	„
„ July 17	13 „	0.77	2	Pretty good.
„ „ „	13 „	0.69	2	Tolerable.
„ „ 22	φ „	0.91	3	„
„ „ „	ν^2 „	0.61	3	Good.
„ „ „	ν^1 „	0.75	3	Tolerable.
„ „ 30	*g* Cygni	0.72	1	
„ „ „	π^2 „	1.15	1	
„ „ „	*g* „	0.72	4	Very good.
„ „ „	π^2 „	0.54	4	Pretty good — Some doubt about one observation.
„ „ „	σ Lacertae	0.78	4	Good.
„ „ „	11 „	0.81	4	Very good.
„ „ 31	ο Androm.	0.84	1	
„ Aug. 2	ν „	0.74	4	Good.
„ „ „	18 H „	0.82	4	Pretty good.
„ „ 3	φ Persei	0.64	4	„ „
„ „ „	χ Androm.	0.80	4	„ „
„ „ „	41 H „	0.72	4	Very good.
„ „ „	υ „	0.88	4	Good.
„ „ 4	φ Persei	0.72	4	Very good.
„ „ „	υ „	0.67	4	Good.
„ „ „	ω Androm.	0.94	4	Pretty good.
„ „ „	ξ „	0.86	4	„ „
„ „ „	υ „	0.83	4	Good.
„ „ 22	θ Persei	0.90	2	Poor.
„ „ „	ι „	0.77	2	„
„ „ „	*c*	0.66	2	Pretty good. Doubtful identity.

PERIOD V.

1847 Oct. 22	δ Persei	0.74	2	Too bright.
„ Nov. 26	υ „	0.60	2	Pretty good.
„ „ „	λ „	0.66	2	„ „
„ „ 30	ν „	0.81	4	Poor.
„ „ „	ε Aurigae	0.77	3	Tolerable.
„ „ „	χ Urs. maj.	0.64	4	Poor.

Date.	Star.	Value.	No. of settings.	Remarks.
1874 Nov. 30	*e* Persei	0.58	4	Pretty good.
„ Dec. 21	*e* „	0.84	2	Poor.

Difference between Cap 2 *and Cap* 1.

PERIOD I.

1872 March 8	α Leonis	1.09	9	Pretty good.
„ „ „	ν „	1.39	9	„
„ „ 16	g Urs. maj.	1.41	6	Good.
„ „ 22	12 Can. ven.	1.52	5	Good. Involves constant for Pinhole 6.
„ May 4	ν_1 and ν_2 Bootae	1.37		
„ „ 8	τ Herculis	1.36	6	Pretty good. Involves constant for Pinhole 6.
„ „ „	υ „	1.31	6	Pretty good.
„ „ „	φ „	1.40	6	„ „
„ „ 26	52 „	1.26	2	Poor.
„ July 8	*R* Lyrae	1.54	3	Pretty accordant, too bright.
„ „ „	ε^1 „	1.52	3	Pretty good.
„ „ „	ε^2 „	1.42	3	„ „
„ „ 20	χ Herculis	1.23	4	Accordant, but too bright. Reject.
„ „ 28	o^2 Cygni	1.25	4	Accordant, but too bright.
1873 Feb. 28	ζ Urs. maj.	1.23	1	Too bright. Reject.
„ March 6	ζ „ „	1.07	4	Tolerable.
„ „ „	38 Lyncis	1.22	4	Pretty accordant, but much too bright.

PERIOD II.

1873 June 1	ω Urs maj.	1.52	3	Very good.
„ „ „	47 „ „	1.51	2	„ „
„ „ 20	η Herculis	1.36	4	Very good. Rather bright.
„ „ „	θ „	1.63	4	Very good.
„ Sept. 8	ι „	1.34	3	„ „
„ „ 16	ι „	1.22	1	Too bright. Reject.
„ „ „	b „	1.38	1	Too bright.
„ Oct. 8	λ Androm.	1.31	2	
„ „ 11	ρ Cygni	1.95	3	Accordant. Too bright.
„ „ „	π „	1.92	3	Very good. Perhaps too bright.
„ „ 13	φ Androm.	1.21	2	Good.
„ „ 15	ι „	1.05	1	Very poor.
„ „ „	χ „	1.42	2	Pretty good.
„ „ „	λ „	1.36	2	Tolerable.
1874 Feb. 11	γ Persei	1.45	4	Pretty good.
„ „ 12	γ „	1.59	2	Poor.

PERIOD III.

1874 Apr. 30	10 Urs. maj.	1.20	3	Good.

PERIOD IV.

1874 June 13	τ Cygni	1.89	3	Tolerable.
„ „ 28	θ Bootae	1.81	4	Pretty good.

Date.	Star.	Value.	No. of settings.	Remarks.
1874 June 28	κ Bootae	1.45	4	Pretty good.
„ „ „	λ „	1.32	4	Tolerable.
„ July 17	13 „	1.78	2	Very good.
„ „ „	λ „	2.01	2	Poor.
„ „ „	13 „	1.75	2	„
„ „ „	λ „	1.82	2	Pretty good.
„ „ 22	μ „	1.79	3	Very good.
„ „ 30	ρ „	1.26	1	
„ „ „	ρ „	1.81	4	Good.
„ „ 31	o Androm.	1.88	1	
„ Aug. 2	o „	1.56	4	Good.
„ „ „	ι „	1.62	4	„
„ „ „	χ „	1.88	4	Very good.
„ „ „	φ „	1.90	4	Poor.
„ „ 3	φ Persei	1.48	4	Pretty good.
„ „ „	υ „	1.68	4	„ „
„ „ „	υ Androm.	1.44	4	„ „
„ „ 4	φ Persei	1.92	4	Tolerable.
„ „ „	υ „	1.72	4	Pretty good.
„ „ „	ν Androm.	1.65	4	Good.
„ „ 22	τ Persei	1.52	2	Tolerable.

PERIOD V.

1874 Oct. 22	δ Persei	1.34	2	Poor.
„ Nov. 26	δ „	1.63	2	Tolerable.
„ „ 30	ν „	1.17	4	Pretty good.
„ Dec. 21	ε Aurigae	1.46	3	Good.
„ „ „	ζ „	1.37	3	Tolerable.
„ „ „	η „	1.45	3	Pretty good.

Difference between Cap 0 *and Cap* 2.

PERIOD I.

1872 March 18		1.83	5	Good. Involves constant for Pinhole No. 3.

PERIOD II.

1873 Sept. 26	ι Androm.	2.37	2	Poor.
„ „ „	χ „	2.58	2	„
„ „ 27	7 Lacertae	1.97	4	Tolerable.
„ „ „	λ Androm.	2.46	2	Good.
„ „ 28	7 Lacertae	2.09	3	Very accordant, but too bright.

PERIOD III.

1874 May 2	ι Urs. maj.	2.04	4	Poor. Pinholes 5 and 3.
„ „ „	χ „ „	1.90	4	Poor. „
„ „ 3	ι „ „	2.28	4	„ „
„ „ „	χ „ „	2.32	4	Tolerable. „
„ „ „	θ „ „	2.27	4	Pretty good. „

Date	Star.	Value.	No. of settings.	Remarks.
		PERIOD IV.		
1874 May 28	38 Lyncis	1.76	3	Pretty accordant. Too bright. Reject.
		PERIOD V.		
1875 Jan. 10	χ Urs. maj.	2.16	3	Tolerable.

There can be no doubt, that the errors introduced by the use of these diaphragms are by far the most serious of those by which my observations are affected.

The most difficult part of the observations with this instrument consists in putting the eye straight to the telescope. It must be remembered that two pencils of rays emerge from the eye-piece, not quite parallel to one another; and both must pass entirely into the pupil. This would be comparatively easy if the observer had a long straight tube before him to show the direction of the ocular axis, instead of the shortest possible prism; or if the telescope remained fixed, instead of constantly changing its direction. I should strongly recommend that such photometers be constructed with the eye-piece in the axis of the telescope; and if this were combined with a system of large Nicol prisms arranged so that the real star instead of the photometer star should be altered in brightness, I am sure it would result a greater accuracy in the observations. As it was, I was obliged to have a table constructed to raise and lower with a crank, and also to turn on a horizontal axis, so as to give room to rest the arm upon it, while it enabled me to put my body close to the instrument. This table had a very smooth motion and proved exceedingly advantageous. Each time that I put my eye to the telescope I had to move it about until I was confident I had got it in the best position. It was, also, of course, necessary, that both real star and photometer star should constantly fall on fixed parts of the retina. Here came in another source of error, owing to these parts being unequally fatigued by the varying lights, — a difficulty which would not have been noticed, I think, if the real star had been brought to a fixed standard. Having spent perhaps a minute in getting my eye properly placed, I found it best not to spend much time in moving the photometer circle tentatively one way and the other, but to place it pretty promptly where it ought to be, so as not to fatigue myself in the effort to reduce an element of error which would at all events be small, as compared with the others.

The instrument always stood in a little round observatory, constructed for the purpose. In Washington, the observatory was of wood and stood on the roof of the Coast Survey office. In other places, I had an observatory which is shown in a plate at the end of this volume, constructed with a wooden frame covered with canvas. It could be taken to pieces and packed in small compass, so as to be easily transported from place to place, as the nature of my duties as an officer of the Coast Survey

required. The roof rolled on three composition billiard balls on a bed of cast brass. The wooden observatory was all painted dead black in the inside; the canvas one was hung with black cambric. This was found to be an indispensible precaution. In Washington, I communicated with a recorder within the large building by means of a speaking tube. In other places, a portable wooden building stood close to the photometric observatory, and I called out to a recorder seated in that. He always repeated what he heard, but nevertheless erroneous numbers were, no doubt, sometimes set down. Wrong designations of the stars are frequent, but to these I paid less attention. It is unfortunate that the recorder could not read the instrument, so as to avoid having the observer look at any illuminated surface, and perhaps that might be contrived, in making a new photometer.

Plan of the Observations.

The design of the observations was to obtain the magnitudes of all the stars in Argelander's Uranometria between 40° and 50° of North declination (a zone having an area of about $\frac{1}{16}$ of the sphere) with a probable error of not more than $\frac{1}{16}$ of a magnitude, so that we might be provided with stars at all times at every altitude with which to compare others in forming a new uranography. The plan was to observe 368 stars, but others sufficiently bright being noticed, about one hundred more have actually been measured. The stars proposed to be observed were divided into seventy groups each consisting of neighboring stars, these groups lying in two zones, and so that the boundary between two adjacent groups of either zone should have as near as possible the same right ascension as the middle of a group in the other zone. The groups of the southern zone were numbered I, III, V, VII etc., and those of the northern zone II, IV, VI, etc. Each set of observations consisted in comparing the stars of two groups with the photometer star, thus comparing these groups with one another. Every star was observed once in turn and then I began at the first one again and so went round four times, unless clouds or some other inconvenience prevented. I should have done better to have made each alternate round in the inverse order so as to eliminate any gradual uniform change in transparency of the atmosphere. Every group was thus compared with four others adjacent to it, group number n being compared with groups $n - 2$, $n - 1$, $n + 1$, and $n + 2$. The error accumulated in completing the circle of right ascension in this way only happened to amount to $0^m.158$ on my usual scale. But, to avoid accumulation of error, I also compared groups I and XXI, XI and XXXI, etc. at times when the two distant groups were about equally high in opposite sides of the heavens.

The observations having been completed, all the photometer readings were

separately reduced to a scale of magnitudes for which $\rho = 2.25$. The table by which this was done is appended to this memoir. The mean difference of magnitude of each star from the photometer star during each set of observations was then taken, then the difference of the mean magnitude of the stars of the two groups from the photometer star, as well the difference of the means of the two groups from each other, and then the difference of each star from the mean of the group to which it belonged. The following is an example of this process.

Sunday, 1874, March 29.

Groups XIII and XV.

42 Aurigae.

Cap, 0. Readings.	Pinhole, 5. Mags.
+ 15.0	3.33
+ 165.9	3.48
— 168.7	4.02
— 12.1	3.79
Mean,	3.65

43 Aurigae. *

Cap, 0.	Pinhole, 5.
+ 15.3	3.29
+ 166.9	3.66
— 164.9	3.32
— 13.5	3.59
Mean,	3.46

ψ^1 Aurigae.

Cap, 0.	Pinhole, 5.
+ 28.7	1.81
+ 152.0	1.86
— 151.6	1.83
— 28.8	1.80
Mean,	1.82

ψ^6 Aurigae.

Cap, 0.	Pinhole, 5.
+ 23.7	2.25
+ 157.8	2.40
— 158.2	2.44
— 25.2	2.11
Mean,	2.30

ψ^9, *n*, Aurigae.

Cap. 0. Readings.	Pinhole, 5. Mags.
+ 15.6	3.24
+ 164.2	3.21
— 164 0	3.18
— 18.5	2.83
Mean,	3.11

ψ^9, *s*, Aurigae.

Cap, 0.	Pinhole, 5.
+ 16.2	3.15
+ 165.7	3.45
— 167.2	3.72
— 16.4	3.12
Mean,	3.36

ψ^{10} Aurigae.

Cap, 0.	Pinhole, 5.
+ 24.7	2.15
+ 157.3	2.35
— 158.4	2.46
— 23.2	2.30
Mean,	2.31

43 B Lyncis.

Cap, 0	Pinhole, 5.
+ 19.2	2.74
+ 160.2	2.67
— 163.9	3.16
— 19.3	2.73
Mean,	2.82

* Observer's Note. 43 Aurigae is a little brighter than 42.

8 H Lyncis.

Cap, 0. Readings.	Pinhole, 5. Mags.
+ 27.2	1.93
+ 154.7	2.10
— 156.8	2.30
— 23.1	2.31
Mean,	2.16

21 Lyncis.

Cap, 0.	Pinhole, 5.
+ 27.5	1.91
+ 150.2	1.72
— 154.0	2.03
— 26.5	1.99
Mean,	1.91

22 Lyncis.

Cap, 0.	Pinhole, 5.
+ 20.4	2.60
+ 158.4	2.46
— 159.5	2.59
— 21.0	2.53
Mean,	2.54

P. VII. 92.

Cap, 0.	Pinhole, 5.
+ 19.3	2.73
+ 161.7	2.86
— 164.0	3.18
— 17.1	3.02
Mean,	2.95

Anonyma. *

Cap, 0. Readings.	Pinhole, 5. Mags.
+ 17.2	3.00
+ 161.0	2.77
— 161.1	2.78
— 17.2	3.00
Mean,	2.86

P. VII. 156. **

Cap, 0.	Pinhole, 5.
+ 19.5	2.71
+ 160.3	2.68
— 161.7	2.86
— 18.6	2.88
Mean,	2.77

25 Lyncis. ***

Cap, 0.	Pinhole, 5.
+ 19.5	2.71
+ 159.3	2.56
— 163.2	3.06
— 19.4	2.72
Mean,	2.76

26 Lyncis.

Cap, 0.	Pinhole, 5.
+ 13.6	3.57
+ 164.3	3.22
— 167.4	3.76
— 13.1	3.66
Mean,	3.55

Subsequent Notes. The star called ψ^9, s, Aurigae was 130 Ḫ Aurigae. 25 and 26 Lyncis have their designations transposed. "Anonyma", is 26 Ḫ Lyncis.

Group XIII.

Star.	Diff. from Phot. star.	Diff. from mean of group.
ψ^{10} Aurigae	2.31	— 0.55
ψ^1 „	1.82	— 1.04
ψ^6 „	2.30	— 0.56
ψ^9 „	3.11	+ 0.25

130 Ḫ Aurigae	3.36	+ 0.50
43 „	3.46	+ 0.60
42 „	3.65	+ 0.79
Mean,	2.86	

Magnitude of Photometer star, 1.05.

* The second time round, Anonyma seems brighter. It is reddish.
** P. VII. 156 is the s. f. of two.
*** 25 Lyncis is n. of 26.

Group XV.

Star.	Diff. from Phot. star.	Diff. from mean of group.
21 Lyncis	1.91	− 0.81
8 H „	2.16	− 0.56
P. VII. 156	2.77	+ 0.05
26 Lyncis	2.76	+ 0.04
43 B „	2.82	+ 0.10
26 δ „	2.86	+ 0.14
P. VII. 92.	2.95	+ 0.23
25 Lyncis	3.55	+ 0.83
Mean,	2.72	
Extra star		
22 Lyncis	2.54	− 0.18

Magnitude of Photometer star, 1.10

Ditto, mean of two values 1.08

Group XIII − Group XV = + 0.14.

As my observations are of very various degrees of precision, I should have much diminished the probable error of my results if I had weighted the observations according to their agreement with one another during the set. But I could not have done this without taking account of the effect of clouds and without rejecting numerous observations by Peirce's criterion. Such a proceeding would have involved more labor than I was inclined to undertake, and I have, therefore, simply allowed the means of every night's observations the same weight. To this I have made a few exceptions in the case of observations which were supposed at the time to be worthless or of extremely little value, which I have sometimes rejected altogether and sometimes have allowed them ¼ weight. In two or three cases, sets of observations which I did not suspect at the time, further than to note unfavorable circumstances attending them, have been rejected on account of enormous discrepancies showing the presence of abnormal sources of error. Three or four of the sets which have been the worst vitiated by clouds so that the residuals of succesive observations would for a while have nearly the same value which would then change more or less suddenly to another value, and where the presence of clouds was mentioned in the record, have been separated into partial sets which have been separately computed, and then the mean result of the partial sets has been adopted. In a few other cases of this sort where the first residuals have been very large, with one sign, and have gradually changed until the last have been very large with the other sign, I have assumed that there was a uniform change in the transparency of the air. In every case in which the residuals have appeared to

have so marked a system in their values as to require either of these procedures, I have undertaken it without knowing what the result would be, and have always adhered to that result. But I have in no case found such a proceeding to make the agreement of the observed and calculated differences greater than it was before nor of a different sign.

Having thus obtained the observed values of the differences of the means of the different groups, I had to consider (neglecting at first the comparisons of *distant* groups) sixty-nine equations between the differences of *successive* groups considered as unknown quantities. Using the usual procedure of least squares, these equations will not be of the same form, but we may introduce a quantity X so as to give seventy equations of the form

$$\begin{aligned} &X + (\text{Group } n - \text{Gr. } (n+1)) + 3(\text{Gr. } (n+1) - \text{Gr. } (n+2)) + (\text{Gr. } (n+2) - \text{Gr. } (n+3)) \\ &\quad = \text{Obs. } (\text{Gr. } n - \text{Gr. } (n+2)) + \text{Obs. } (\text{Gr. } (n+1) - \text{Gr. } (n+2)) \\ &\quad\quad + \text{Obs. } (\text{Gr. } (n+1) - \text{Gr. } (n+3)). \end{aligned}$$

Group 71 must be considered to be Group 1, and group 72 to be group 2. Taking the successive differences of these equations, and using the notation

$$x_n = \text{Group } n - \text{Group } (n+1)$$

$$\{a, b\} = \text{Obs. (Group } a - \text{Group } b)$$

we obtain seventy equations of the form

$$f_n = x_{n-2} + 2x_{n-1} - 2x_n - x_{n+1} - \{n-2, n\} - \{n-1, n\} + \{n, n+1\} + \{n, n+2\} = 0.$$

These equations are not all independent but the quantity

$$x_{70} = \text{Group } 70 - \text{Group } 1$$

is the negative of the sum of all the other unknown quantities. Let us now write,

$$a_1 f_1 + a_2 f_2 + a_3 f_3 + \text{etc.} = 0;$$

the coefficients being supposed to be such that when the values of f_1, f_2, f_3 etc. are substituted, all the unknown quantities are eliminated except x_m and x_{70}. In order that any unknown quantity x_n (supposed not be x_{69}, x_{70}, or x_1) may disappear from this equation we must have

$$a_{n-1} + 2a_n - 2a_{n+1} - a_{n+2} = 0.$$

Considering this as an equation of finite differences, we solve it by means of the auxiliary equation

$$y^3 + 2y^2 - 2y - 1 = 0$$

The roots of the auxiliary equation being 1, $\frac{-3-\sqrt{5}}{2}$, and $\frac{-3+\sqrt{5}}{2}$ the general solution of the equation of differences is

$$a_n = c_1 + c_2\left(\frac{-3-\sqrt{5}}{2}\right)^n + c_3\left(\frac{-3+\sqrt{5}}{2}\right)^n$$
$$= c_1 + c_2\left(\frac{-3-\sqrt{5}}{2}\right)^n + c_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-n}$$

If now we take one set of values for c_1, c_2, and c_3 for all the a's from a_1 to a_{m+1} inclusive, and another set of values of these constants (which we may denote by C_1, C_2, and C_3) for all the a's from a_m to a_{70} inclusive, we shall have, in order to determine these six quantities, the following equations. First, two equations to make the two values of of a_m and a_{m+1} which are included in both sets, equal.

$$c_1 + c_2\left(\frac{-3-\sqrt{5}}{2}\right)^m + c_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-m} = C_1 + C_2\left(\frac{-3-\sqrt{5}}{2}\right)^m + C_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-m}.$$

$$c_1 + c_2\left(\frac{-3-\sqrt{5}}{2}\right)^{m+1} + c_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-m-1} = C_1 + C_2\left(\frac{-3-\sqrt{5}}{2}\right)^{m+1} + C_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-m-1}.$$

Second, an equation to make the coefficient of x_m unity,

$$+ 3c_1 + c_2\left(\left(\frac{-3-\sqrt{5}}{2}\right)^{m-1} + 2\left(\frac{-3-\sqrt{5}}{2}\right)^m\right)$$
$$+ c_3\left(\left(\frac{-3-\sqrt{5}}{2}\right)^{-m+1} + 2\left(\frac{-3-\sqrt{5}}{2}\right)^{-m}\right)$$
$$- 3C_1 - C_2\left(\left(\frac{-3-\sqrt{5}}{2}\right)^{m+2} + 2\left(\frac{-3-\sqrt{5}}{2}\right)^{m+1}\right)$$
$$- C_3\left(\left(\frac{-3-\sqrt{5}}{2}\right)^{-m-2} + 2\left(\frac{-3-\sqrt{5}}{2}\right)^{-m-1}\right) = 1.$$

And, third, two equations to make, the coefficients x_{69} and x_1 vanish,

$$c_1 + c_2 + c_3 - C_1 - C_2\left(\frac{-3-\sqrt{5}}{2}\right)^{70} - C_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-70} = 0$$

$$c_1 + c_2\left(\frac{-3-\sqrt{5}}{2}\right) + c_3\left(\frac{-3-\sqrt{5}}{2}\right)^{-1} - C_1 - C_2\left(\frac{-3-\sqrt{5}}{2}\right)^{71} - C_3\left(\frac{-3-\sqrt{5}}{2}\right)^{71} = 0.$$

These equations serve to determine $(c_1 - C_1)$, c_2, c_3, C_2, and C_3 and in this way the

equations might be solved without much labor. I have, however, found it more convenient to make use of the approximate formula,

$$\sqrt{5}.x_n = \{n,n+1\} + G(\{n,n+2\} + \{n-1,n+1\}) - G^2(\{n-1,n\} + \{n+1,n+2\})$$
$$- G^3(\{n+1,n+3\} + \{n-2,n\}) + G^4(\{n-2,n-1\} + \{n+2,n+3\})$$
$$+ G^5(\{n+2,n+4\} + \{n-3,n-1\}) - G^6(\{n-3,n-2\} + \{n+3,n+4\})$$
$$- \text{etc.}$$

Where $G = \frac{1}{2}(\sqrt{5}-1)$. I do not know whether this method of solving equations has been used before, or not; but it seems to me, at all events, to have some interest. This procedure gave values so nearly right that the solution of the complete equations involving the comparisons of distant groups was then effected by adjustment by substitutions, in a very short time. The work was carried to thousandths of magnitude. The following table exhibits the results.

Comparisons of the Groups.

Groups.	Observed values (Number of set in parenthesis)	Adopted mean.	Calc.
I—II.	$0^m.16$ (157)	— 0.16	— 0.22
I—III.	— 0.60 (107); — 0.60 (108)	— 0.60	— 0.61
II—III.	— 0.43 (160)	— 0.43	— 0.40
II—IV.	— 1.31 (161)	— 1.31	— 1.32
III—IV.	— 0.98 (151)	— 0.98	— 0.92
III—V.	— 0.10 (53); — 0.02 (104)	— 0.08	— 0.07
IV—V.	+ 0.79 (162)	+ 0.79	+ 0.85
IV—VI.	— 0.14 (163)	— 0.14	— 0.12
V—VI.	— 0.94 (164)	— 0.94	— 0.97
V—VII.	— 0.33 (109), taken at low altitude; — 0.24 (111)	— 0.24	— 0.17
VI—VII.	+ 0.83 (166)	+ 0.83	+ 0.80
VI—VIII.	+ 0.92 (172)	+ 0.92	+ 0.92
VII—VIII.	— 0.10 (180), wind blowing lamp; fluctuations of light.	0.00	+ 0.12
VII—IX.	— 0.95 (110); — 0.90 (112)	— 0.92	— 1.01
VIII—IX.	— 1.13 (173); — 1.26 (181)	— 1.19	— 1.13
VIII—X.	— 0.38 (185)	— 0.38	— 0.34
IX—X.	+ 0.78 (167)	+ 0.78	+ 0.79
IX—XI.	+ 0.22 (105)	+ 0.22	+ 0.20
X—XI.	— 0.67 (174)	— 0.67	— 0.59
X—XII.	+ 0.23 (182)	+ 0.23	+ 0.21
XI—XII.	+ 0.72 (186), clouds	+ 0.72	+ 0.80
XI—XIII.	— 0.12 (113)	— 0.12	— 0.11
XII—XIII.	— 0.95 (175)	— 0.95	— 0.91
XII—XIV.	— 0.46 (183)	— 0.46	— 0.43
XIII—XIV.	+ 0.48 (187)	+ 0.48	+ 0.48
XIII—XV.	+ 0.14 (114)	+ 0.14	+ 0.19
XIV—XV.	— 0.35 (165)	— 0.35	— 0.29
XIV—XVI.	+ 0.61 (118)	+ 0.61	+ 0.58
XV—XVI.	+ 0.74 (115), clouds.	+ 0.74	+ 0.87

Obs.		Adopted mean.	Calc.
XV—XVII.	+ 0m79 (168); + 0.69 (184)	+ 0.74	+ 0.72
XVI—XVII.	— 0.18 (116)	— 0.18	— 0.15
XVI—XVIII.	— 0.35 (101)	— 0.35	— 0.29
XVII—XVIII.	— 0.14 (119)	— 0.14	— 0.14
XVII—XIX.	+ 0.12 (121)	+ 0.12	+ 0.12
XVIII—XIX.	+ 0.20 (122)	+ 0.20	+ 0.26
XVIII—XX.	— 0.02 (48), once round; + 0.22 (49)	+ 0.17	+ 0.18
XIX—XX.	+ 0.01 (170); — 0.06 (176)	— 0.03	— 0.08
XIX—XXI.	— 0.98 (178), clouds	— 0.98	— 0.86
XX—XXI.	— 0.74 (50)	— 0.74	— 0.78
XX—XXII.	— 0.65 (10); — 0.89 (3)	— 0.77	— 0.77
XXI—XXII.	0.00 (171)	0.00	+ 0.01
XXI—XXIII.	+ 1.77 (54)	+ 1.77	+ 1.74
XXII—XXIII.	+ 1.74 (179); + 2.01 (188), low altitudes	+ 1.74	+ 1.73
XXII—XXIV.	— 0.18 (58)	— 0.18	— 0.17
XXIII—XXIV.	— 1.23 (2), not yet accustomed to inst.; — 1.80 (15)	— 1.80	— 1.90
XXIII—XXV.	— 2.76 (65)	rejected	— 2.23
XXIV—XXV.	— 0.34 (124); — 0.25 (126)	— 0.30	— 0.33
XXIV—XXVI.	+ 0.89 (55); + 1.01 (1), before I had learnt to observe	+ 0.89	+ 0.83
XXV—XXVI.	+ 1.13 (6)	+ 1.13	+ 1.16
XXV—XXVII.	+ 0.39 (67)	+ 0.39	+ 0.32
XXVI—XXVII.	— 0.92 (127)	— 0.92	— 0.85
XXVI—XXVIII.	— 0.87 (5); — 0.77 (7)	— 0.82	— 0.92
XXVII—XXVIII.	— 0.04 (128)	— 0.04	— 0.08
XXVII—XXIX.	+ 0.15 (125)	+ 0.15	+ 0.21
XXVIII—XXIX.	+ 0.36 (64)	+ 0.36	+ 0.29
XXVIII—XXX.	— 0.64 (59); — 0.68 (60)	— 0.66	— 0.74
XXIX—XXX.	— 0.97 (66)	— 0.97	— 1.03
XXIX—XXXI.	+ 0.57 (130)	+ 0.57	+ 0.61
XXX—XXXI.	+ 2.22 (132), once round; + 1.54 (134); + 1.72 (136)	+ 1.81	+ 1.64
XXX—XXXII.	+ 0.75 (61)	+ 0.75	+ 0.79
XXXI—XXXII.	— 0.85 (133); — 0.95 (69), of no value	— 0.85	— 0.85
XXXI—XXXIII.	— 1.02 (137)	— 1.02	— 0.96
XXXII—XXXIII.	— 0.15 (138)	— 0.15	— 0.11
XXXII—XXXIV.	+ 0.83 (8)	+ 0.83	+ 0.84
XXXIII—XXXIV.	+ 0.82 (139)	+ 0.82	+ 0.95
XXXIII—XXXV.	+ 0.95 (140)	+ 0.95	+ 0.94
XXXIV—XXXV.	+ 0.09 (9)	+ 0.09	— 0.01
XXXIV—XXXVI.	— 0.86 (56)	— 0.86	— 0.62
XXXV—XXXVI.	— 0.53 (31)	— 0.53	— 0.61
XXXV—XXXVII.	— 0.60 (18)	— 0.60	— 0.65
XXXVI—XXXVII.	— 0.02 (20)	— 0.02	— 0.04
XXXVI—XXXVIII.	+ 0.22 (23)	+ 0.22	+ 0.41
XXXVII—XXXVIII.	+ 0.45 (57)	+ 0.45	+ 0.45
XXXVII—XXXIX.	+ 0.01 (11); + 0.09 (72)	+ 0.05	— 0.04
XXXVIII—XXXIX.	— 0.57 (70)	— 0.57	— 0.59
XXXVIII—XL.	— 0.62 (63)	— 0.62	— 0.49
XXXIX—XL.	+ 0.03 (71)	+ 0.03	0.00

Obs.		Adopted mean.	Calc.
XXXIX—XLI.	— 1m33 (13)	— 1.33	— 1.31
XL—XLI.	— 1.36 (27)	— 1.36	— 1.31
XL—XLII.	— 0.35 (68)	— 0.35	— 0.32
XLI—XLII.	+ 0.99 (73)	+ 0.99	+ 0.99
XLI—XLIII.	+ 0.62 (19)	+ 0.62	+ 0.67
XLII—XLIII.	— 0.41 (25)	— 0.41	— 0.32
XLII—XLIV.	+ 0.37 (74); + 0.38 (85)	+ 0.38	+ 0.30
XLIII—XLIV.	+ 0.50 (92)	+ 0.50	+ 0.62
XLIII—XLV.	+ 0.35 (17)	+ 0.35	+ 0.39
XLIV—XLV.	— 0.31 (16); — 0.32 (22)	— 0.31	— 0.23
XLIV—XLVI.	+ 0.12 (21)	+ 0.12	+ 0.07
XLV—XLVI.	+ 0.23 (28)	+ 0.23	+ 0.30
XLV—XLVII.	+ 1.32 (86)	+ 1.32	+ 1.38
XLVI—XLVII.	+ 1.01 (24)	+ 1.01	+ 1.08
XLVI—XLVIII.	— 0.39 (30)	— 0.39	— 0.45
XLVII—XLVIII.	— 1.64 (93)	— 1.64	— 1.53
XLVII—XLIX.	— 1.57 (26)	— 1.57	— 1.54
XLVIII—XLIX.	— 0.06 (95)	— 0.06	— 0.01
XLVIII—L.	+ 0.87 (33)	+ 0.87	+ 0.87
XLIX—L.	+ 0.92 (35)	+ 0.92	+ 0.88
XLIX—LI.	+ 0.62 (14)	+ 0.62	+ 0.75
L—LI.	— 0.10 (39)	— 0.10	— 0.13
L—LII.	— 0.10 (29)	— 0.10	— 0.12
LI—LII.	— 0.06 (37)	— 0.06	+ 0.01
LI—LIII.	+ 0.24 (32); + 0.40 (87), once round	+ 0.27	+ 0.21
LII—LIII.	+ 0.17 (34)	+ 0.17	+ 0.20
LII—LIV.	+ 0.16 (41)	+ 0.16	+ 0.15
LIII—LIV.	+ 0.11 (43); + 0.11 (42)	+ 0.11	— 0.01
LIII—LV.	— 0.69 (94)	— 0.69	— 0.56
LIV—LV.	— 0.58 (123)	— 0.58	— 0.55
LIV—LVI.	— 0.49 (38), clouds	— 0.49	— 0.61
LV—LVI.	— 0.25 (135)	— 0.25	— 0.06
LV—LVII.	+ 0.51 (96)	+ 0.51	+ 0.48
LVI—LVII.	+ 0.58 (44)	+ 0.58	+ 0.54
LVI—LVIII.	+ 0.16 (46)	+ 0.16	+ 0.28
LVII—LVIII.	— 0.38 (141), once round; — 0.14 (142)	— 0.19	— 0.26
LVII—LIX.	+ 0.51 (36); + 0.63 (83)	+ 0.57	+ 0.55
LVIII—LIX.	+ 0.81 (81)	+ 0.81	+ 0.81
LVIII—LX.	— 0.31 (79); once round; — 0.13 (143)	— 0.13	— 0.07
LIX—LX.	— 0.77 (45), clouds	— 0.77	— 0.88
LIX—LXI.	— 0.77 (47)	— 0.77	— 0.68
LX—LXI.	+ 0.26 (40)	+ 0.26	+ 0.20
LX—LXII.	— 0.48 (145); — 0.43 (144)	— 0.46	— 0.45
LXI—LXII.	— 0.69 (84)	— 0.69	— 0.65
LXI—LXIII.	+ 0.38 (89); + 0.60 (90), bad	+ 0.49	+ 0.37
LXII—LXIII.	+ 1.08 (98)	+ 1.08	+ 1.02
LXII—LXIV.	+ 0.04 (80)	+ 0.04	+ 0.16
LXIII—LXIV.	— 0.80 (146)	— 0.80	— 0.86

	Obs.	Adopted mean.	Calc.
LXIII—LXV.	— 0m08 (82)	— 0.08	— 0.22
LXIV—LXV.	+ 0.53 (147)	+ 0.53	+ 0.64
LXIV—LXVI.	— 0.23 (75)	— 0.23	— 0.26
LXV—LXVI.	— 0.86 (91)	— 0.86	— 0.90
LXV—LXVII.	+ 0.52 (97)	+ 0.52	+ 0.44
LXVI—LXVII.	+ 1.57 (148)	+ 1.57	+ 1.44
LXVI—LXVIII.	+ 1.46 (149)	+ 1.46	+ 1.50
LXVII—LXVIII.	+ 0.07 (152)	+ 0.07	+ 0.06
LXVII—LXIX.	— 0.98 (103)	— 0.98	— 1.08
LXVIII—LXIX.	— 1.16 (150)	— 1.16	— 1.14
LXVIII—LXX.	— 0.88 (153)	— 0.88	— 0.87
LXIX—LXX.	+ 0.26 (154)	+ 0.26	+ 0.27
LXIX—I.	+ 1.81 (106)	+ 1.81	+ 1.72
LXX—I.	+ 1.53 (156)	+ 1.53	+ 1.45
LXX—II.	+ 1.13 (159)	+ 1.13	+ 1.23
I—XXI.	— 1.51 (169); — 1.49 (177); — 1.71 (189), low altitude	— 1.50	— 1.61
XXIII—XLI.	— 3.16 (62)	— 3.16	— 3.11
XLI—LXI.	+ 1.54 (76)	+ 1.54	+ 1.65
LXI—XI.	— 0.55 (158)	— 0.55	— 0.34
XI—XXXI.	+ 0.52 (117); + 0.53 (120)	+ 0.53	+ 0.71
XXXI—LI.	— 0.34 (131), clouds	— 0.34	— 0.37
LI—I.	+ 1.10 (99), too low; + 1.34 (155)	+ 1.34	+ 1.33

The last step in the computation was to compare the magnitudes of about 340 of my stars with the reduced magnitudes of the Durchmusterung, so as to reduce my scale to that scale of equable distribution used in the comparative catalogue. I found by least squares that $\log \rho = .443$ for that scale, after entirely rejecting the bright stars. But upon subsequent consideration, I concluded to use $\log \rho = 0.409$, which better represents a few bright stars.

The following table exhibits the results of the comparison of each star with the mean of its group, according to each set of observations. The first column contains the number of the set of observations. The next column shows the number of settings upon each star. When any star was observed a greater or smaller number of times a number is prefixed to the measure of that star. The other columns show the differences of the magnitude of each star from the mean of the group to which it belongs.

I.

		γ *Persei.*	ι *Persei.*	θ *Persei.*	τ *Persei.*	47 Ω *Persei.*
(99)	4.	— 1.11	+ 0.20	— 0.10	— 0.20	+ 1.20
(106)	4.	— 1.50	+ 0.11	+ 0.04	— 0.07	+ 1.44
(107)	2.	— 1.44	+ 0.38	+ 0.07	— 0.07	+ 1.07
(108)	4.	— 1.38	+ 0.10	+ 0.07	— 0.16	+ 1.37
(155)	2.	— 1.37	+ 0.18	— 0.08	— 0.13	+ 1.38
(156)	2.	— 1.29	+ 0.13	+ 0.09	— 0.16	+ 1.25
(157)	3.	— 1.59	+ 0.37	— 0.02	— 0.04	+ 1.28
(169)	2.	— 1.38	+ 0.23	+ 0.06	— 0.16	+ 1.25
(177)	2.	— 1.13	+ 0.14	— 0.10	— 0.23	+ 1.31
(189)	2.	— 1.17	+ 0.33	+ 0.10	— 0.25	+ 0.99
Means		— 1.36	+ 0.22	+ 0.04	— 0.14	+ 1.25

II.

		β *Persei.*	κ *Persei.*	ω *Persei.*	1 *Persei.*	30 *Persei.*
(157)	3.	— 2.47	— 0.56	+ 0.45	+ 0.94	+ 1.64
(159)	4.	— 2.38	— 0.53	+ 0.52	+ 0.90	+ 1.49
(160)	4.	— 2.18	— 0.59	+ 0.27	+ 0.92	+ 1.56
(161)	4.	— 2.15	— 0.55	+ 0.42	+ 0.85	+ 1.43
Means		— 2.30	— 0.56	+ 0.42	+ 0.90	+ 1.53

III.

		α *Persei.*	76 Ω *Persei.*	34 *Persei.*	29 *Persei.*	63 Ω *Persei.*	31 *Persei.*	Extra Stars. 73 Ω *Persei.*	Extra Stars. 77 Ω *Persei.*
(53)	1.	— 3.10	+ 0.63	+ 0.55	+ 0.61	+ 0.67	+ 0.62	+ 0.97	+ 1.51
(104)	3.	— 3.26	+ 0.37 rej.	+ 0.42	+ 0.66	+ 0.76	+ 1.04		
(107)	2.	— 3.17	+ 0.80	+ 0.42	+ 0.69	+ 0.53	+ 0.75		
(108)	4.	— 3.16	+ 0.58	+ 0.40	+ 0.57	+ 0.57	+ 0.68	+ 0.88	
(151)	4.	— 3.45 rej.	+ 0.89	+ 0.23	+ 0.98	+ 0.57	+ 0.77		
(160)	4.	— 3.10	+ 0.59	+ 0.24	+ 0.94	+ 0.47	+ 0.85		
Means		— 3.16	+ 0.68	+ 0.37	+ 0.79	+ 0.59	+ 0.78	+ 0.92	+ 1.51

IV.

		α *Persei.*	ψ *Persei.*	36 *Persei.*	D M. 47° 847.	D M. 47° 844.	84 Ω *Persei.*
(151)	4.	— 1.24	— 1.24	— 0.49	+ 0.44	+ 1.07	+ 1.43
(161)	4.	— 1.12	— 1.06	+ 0.07	+ 0.14	+ 0.76	+ 1.19
(162)	3.	— 1.13	— 1.18	+ 0.07	+ 0.29	+ 0.67	
(163)	4.	— 1.05	— 1.19	— 0.01	+ 0.36	+ 0.72	+ 1.17
Means		— 1.13	— 1.17	— 0.09	+ 0.31	+ 0.80	+ 1.26

V.

		δ *Persei.*	λ *Persei.*	A *Persei.*	104 Ω *Persei.*	Extra Star. 102 Ω *Persei.*
(53)	1.	— 1.68	— 0.16	+ 0.79	+ 1.05	+ 1.25
(104)	3.	— 1.71	— 0.13	+ 0.82	+ 1.01	
(109)	2.	— 1.65	— 0.22	+ 0.82	+ 1.04	
(111)	2.	— 1.62	— 0.04	+ 0.71	+ 0.97	
(162)	2.	— 1.65	— 0.25	+ 0.96	+ 0.93	
(164)	4.	— 1.58	— 0.35	+ 0.88	+ 1.06	
Means		— 1.65	— 0.19	+ 0.83	+ 1.01	+ 1.25

VI.

		ν *Persei.*	96 Ϙ *Persei.*	99 Ϙ *Persei.*	97 Ϙ *Persei.*	92 Ϙ *Persei.*
(163)	4.	— 1.92	+ 0.20	+ 0.48	+ 0.57	+ 0.71
(164)	2.	— 1.86	+ 0.28	+ 0.38	+ 0.38	+ 0.84
(166)	4.	— 2.04	+ 0.34	+ 0.28	+ 0.74	+ 0.70
(172)	4.	— 1.97	+ 0.33	+ 0.23	+ 0.69	+ 0.73
Means		— 1.95	+ 0.29	+ 0.34	+ 0.60	+ 0.74

VII.

		e *Persei.*	μ *Persei.*	b *Persei.*	d *Persei.*	120 Ϙ *Persei.*	Extra Star. 121 Ϙ *Persei.*
(109)	2.	— 0.79	— 0.45	+ 0.08	+ 0.25	+ 0.92	
(110)	2.	— 0.55	— 0.54	— 0.02	+ 0.30	+ 0.83	
(111)	2.	— 0.47	— 0.46	— 0.08	+ 0.35	+ 0.67	
(112)	2.	— 0.59	— 0.55	+ 0.04	+ 0.22	+ 0.79	
(166)	4.	— 0.66	— 0.69	+ 0.01	+ 0.13	+ 1.19	
(180)	4.	— 0.84	— 0.60	— 0.05	+ 0.39	+ 1.08	+ 1.09
Means		— 0.65	— 0.54	± 0.00	+ 0.28	+ 0.92	+ 1.09

VIII.

		ψ *Persei.*	e *Persei.*	f *Persei.*	59 *Persei.*	m *Persei.*
(172)	4.	— 1.95	— 0.58	+ 0.02	+ 0.72	+ 1.81
(173)	2.	— 1.90	— 0.56	+ 0.06	+ 0.57	+ 1.83
(180)	4.	— 1.89	— 0.60	— 0.17	+ 0.97	+ 1.70
(181)	2.	— 2.19	— 0.62	+ 0.08	+ 0.99	+ 1.75
(185)	3.	— 1.71	— 0.58	— 0.18	+ 0.79	+ 1.66
Means		— 1.93	— 0.59	— 0.02	+ 0.81	+ 1.75

IX.

		22 Ϙ *Aurigae.*	4 Ϙ *Aurigae.*	133 Ϙ *Persei.*	135 Ϙ *Persei.*	Extra Star. α Aurigae.
(105)	2.	— 0.17	— 0.08	— 0.03	+ 0.29	— 6.62
(110)	2.		— 0.12	— 0.04	+ 0.35	
(112)	2.		+ 0.01	+ 0.07	+ 0.12	— 6.46
(167)	3.	— 0.04	— 0.19	0.00	+ 0.22	— 6.34
(178)	2.	— 0.34	— 0.01	+ 0.04	+ 0.31	— 5.73 rej.
(181)	2.	— 0.22	+ 0.18	— 0.18	+ 0.21	— 6.61
Means		— 0.19	— 0.04	— 0.02	+ 0.25	— 6.51

X.

		η *Aurigae.*	ζ *Aurigae.*	λ *Aurigae.*	ρ *Aurigae.*	37 Ϙ *Aurigae.*	34 Ϙ *Aurigae.*	65 Ϙ *Aurigae.*	85 Ϙ *Aurigae.*	Extra Star. ε *Aurigae.*
(167)	3.	— 2.11	— 1.64	— 0.33	+ 0.50	+ 0.65	+ 0.76	+ 0.78	+ 1.35	— 1.54
(174)	3.	— 2.00	— 1.72	— 0.32	+ 0.48	+ 0.80	+ 0.56	+ 1.00	+ 1.20	— 1.49
(182)	3.	— 2.13	— 1.72	— 0.30	+ 0.27	+ 0.81	+ 0.69	+ 0.83	+ 0.57	— 1.65
(185)	3.	— 2.17	— 1.62	— 0.40	+ 0.41	+ 0.83	+ 0.73	+ 0.98	+ 1.24	— 1.54
Means		— 2.10	— 1.67	— 0.34	+ 0.41	+ 0.77	+ 0.66	+ 0.90	+ 1.09	— 1.55

XI.

		π *Aurigae.*	o *Aurigae.*	36 *Aurigae.*	85 Ϙ *Aurigae.*	41 *Aurigae.*
(105)	2.	— 1.08 rej.	— 0.05	0.00	+ 0.44	+ 0.79 rej.
(113)	2.	— 1.21	+ 0.04	+ 0.07	+ 0.50	+ 0.60
(117)	1.	— 1.52	+ 0.27	+ 0.29	+ 1.03 rej.	+ 0.43
(120)	3.	— 1.39	+ 0.10	+ 0.32	+ 0.64	+ 0.35
(158)	2.	— 1.28	+ 0.01	+ 0.21	+ 0.58	+ 0.46
(174)	3.	— 1.36	— 0.16	+ 0.24	+ 0.68	+ 0.30
(186)	2.	— 1.36	+ 0.03	+ 0.56 rej.	+ 0.76	+ 0.39
Means		— 1.35	+ 0.03	+ 0.19	+ 0.57	+ 0.42

XII.

		β *Aurigae.*	ψ_2 *Aurigae.*	ψ_3 *Aurigae.*	39 *Aurigae.*	38 *Aurigae.*	Extra Stars. DM. 43°1421	ψ_1 *Aurigae.*	43 *Aurigae.*	42 *Aurigae.*
(175)	3.	— 3.21	— 0.11	+ 0.72	+ 1.12	+ 1.48	+ 1.67			
(182)	3.	— 3.30	— 0.01	+ 0.57	+ 1.21	+ 1.51				
(183)	2.	— 3.31	+ 0.06	+ 0.64	+ 1.33	+ 1.26	+ 1.57			
(186)	2.	— 3.56	+ 0.09	+ 0.76	+ 1.29			— 0.21	+ 1.57	+ 1.57
Means		— 3.34	+ 0.01	+ 0.67	+ 1.24	+ 1.42	+ 1.62	— 0.21	+ 1.57	+ 1.57

XIII.

		ψ_{10} *Aurigae.*	ψ_1 *Aurigae.*	ψ_6 *Aurigae.*	ψ_9 *Aurigae.*	130 Ϙ *Aurigae.*	43 *Aurigae.*	42 *Aurigae.*
(113)	2.	— 0.81	— 0.72	— 0.45	+ 0.08	+ 0.43	+ 0.52	+ 0.97
(114)	4.	— 0.55	— 1.04	— 0.56	+ 0.25	+ 0.50	+ 0.60	+ 0.79
(175)	3.	— 0.82	— 1.09	— 0.38	+ 0.40		+ 0.57	+ 0.81
(187)	3.	— 0.81	— 1.14	— 0.56	+ 0.06		+ 0.86	+ 1.11
Means		— 0.75	— 1.00	— 0.49	+ 0.20	+ 0.46	+ 0.64	+ 0.92

XIV.

		ψ_4 *Aurigae.*	ψ_5 *Aurigae.*	ψ_7 *Aurigae.*	66 *Aurigae.*	64 *Aurigae.*
(118)	1.	— 0.50	— 0.25	— 0.19	+ 0.13	+ 0.83
(165)	4.	— 0.27	— 0.01	— 0.33	+ 0.14	+ 0.73
(183)	2.	— 0.32	— 0.01	— 0.27	— 0.09	+ 0.68
(187)	3.	— 0.41	+ 0.02	— 0.38	+ 0.07	+ 0.68
Means		— 0.35	— 0.06	— 0.29	+ 0.06	+ 0.78

XV.

		21 *Lyncis.*	8 H *Lyncis.*	32 Ϙ *Lyncis.*	26 *Lyncis.*	16 Ϙ *Lyncis.*	26 Ϙ *Lyncis.*	23 Ϙ *Lyncis.*	25 *Lyncis.*	Extra Stars. 22 *Lyncis.*	27 Ϙ *Lyncis.*
(114)	4.	— 0.81	— 0.56	+ 0.05	+ 0.04	+ 0.10	+ 0.14	+ 0.23	+ 0.83	— 0.18	
(115)	4.	— 0.99	— 0.31	+ 0.31	+ 0.03	— 0.05	— 0.03	+ 0.45	+ 0.66		
(165)	4.	— 1.02	— 0.50			+ 0.23		+ 0.21		— 0.31	
(168)	2.	— 0.96	— 0.67	+ 0.51	— 0.07	— 0.09	+ 0.09	+ 0.42	+ 0.77		+ 0.19
(184)	2.	— 0.92	— 0.54	+ 0.14	— 0.02	+ 0.10	+ 0.12	+ 0.31	+ 0.79	— 0.12	+ 0.58
Means		— 0.94	— 0.52	+ 0.25	0.00	+ 0.06	+ 0.08	+ 0.32	+ 0.76	— 0.20	+ 0.38

XVI.

		10 *Urs. maj.*	31 *Lyncis.*	17 H *Ursae maj.*	35 *Lyncis.*	36 *Lyncis.*
(101)	4.	— 0.73	— 0.55	— 0.04	+ 0.65	+ 0.69
(115)	4.	— 0.67	— 0.65	— 0.13	+ 0.50	+ 0.96
(116)	3.	— 1.01	— 0.40	— 0.03	+ 0.51	+ 0.93
(118)	1. rej.	— 0.31	— 0.48	+ 0.06	+ 0.42	+ 0.31
Means		— 0.80	— 0.53	— 0.07	+ 0.55	+ 0.86

XVII.

		ι *Urs. maj.*	κ *Urs. maj.*	34 *Lyncis.*	76 Ş *Lyncis.*	25 Ş *Urs. maj.*	38 Ş *Urs. maj.*	Extra Stars. 35 *Lyncis.*	75 Ş *Lyncis.*
(116)	3.	— 2.13	— 1.48	+ 0.60	+ 0.82	+ 0.89	+ 1.28		
(119)	4.	— 1.88	— 1.39	+ 0.46	+ 0.80	+ 0.70	+ 1.34		
(121)	4.	— 1.80	— 1.35	+ 0.55	+ 0.92	+ 0.43	+ 1.24		
(168)	2.	— 1.94	— 1.54	+ 0.61	+ 1.00	+ 0.61	+ 1.24	+ 0.36	
(184)		— 2.04	— 1.69	+ 0.67	+ 1.04	+ 0.82	+ 1.23	+ 0.36	+ 1.08
Means		— 1.96	— 1.49	+ 0.58	+ 0.92	+ 0.69	+ 1.27	+ 0.36	+ 1.08

XVIII.

		38 *Lyncis.*	85 Ş *Lyncis.*	19 *Leon. min.*	42 *Lyncis.*	43 *Lyncis.*
(48)	1.	— 1.27	— 0.35	+ 0.24	+ 0.57	+ 0.79
(49)	4.	— 1.25	— 0.12	+ 0.29	+ 0.45	+ 0.62
(101)	4.		— 0.22	+ 0.38	+ 0.49	+ 0.68
(119)	4.	— 1.27	— 0.08	+ 0.23	+ 0.38	+ 0.76
(122)	3.	— 1.44	— 0.07	+ 0.41	+ 0.38	+ 0.72
Means		— 1.31	— 0.17	+ 0.31	+ 0.45	+ 0.71

XIX.

		θ *Urs. maj.*	65 Ş *Urs. maj.*	22 H *Urs. maj.*	31 *Urs. maj.*	26 *Urs. maj.*	Extra Star. DM. 45°1728.
(121)	4.	— 1.67	+ 0.28	+ 0.70	+ 0.92	+ 0.23	
(122)	3.	— 2.13 rej.	+ 0.71 rej.	+ 0.88	+ 0.72	+ 0.19	
(170)	1.	— 1.62			+ 0.66	— 0.20	+ 2.40
(176)	3.	— 1.73	+ 0.20	+ 0.80	+ 0.66	+ 0.06	
(178)	4.	— 1.72	+ 0.44	+ 0.67	+ 0.71	— 0.09	
Means		— 1.77	+ 0.41	+ 0.77	+ 0.74	+ 0.05	+ 2.40

XX.

		λ *Urs. maj.*	31 *Leon. min.*	33 H *Urs. maj.*	DM. 41°2076.	32 *Leon. min.*	38 *Leon. min.*	μ *Urs. maj.*
(3)	5.	— 0.95 rej.	— 0.52		+ 1.16	+ 1.20	+ 1.28	— 1.91
(4)	2.	— 1.58		+ 0.09		+ 1.38		— 1.89
(10)	6.	— 1.24	— 0.30	+ 0.04	+ 0.95	+ 1.06	+ 1.17	— 1.70
(48)	1.	— 1.33	— 0.63	+ 0.10	+ 0.61 rej.	+ 1.25	+ 2.13 rej.	— 1.99
(49)	4.	— 1.51	— 0.61	+ 0.12	+ 1.24	+ 1.37	+ 1.55	— 2.14
(50)	4.	— 1.46	— 0.60	+ 0.21	+ 1.26	+ 1.34	+ 1.42	— 2.17
(170)	2.	— 1.61	— 0.74	— 0.09	+ 1.46	+ 1.49	+ 1.66	— 2.15
(176)	3.	— 1.34	— 0.57	+ 0.12	+ 1.22	+ 1.36	+ 1.15	— 1.97
Means		— 1.44	— 0.54	+ 0.10	+ 1.21	+ 1.31	+ 1.37	— 1.98

Extra Stars.

		Comp. λ.	25 *Leon. min.*	51 *Urs. maj.*	35 *Leon. min.*	foll. μ	39 H *Urs. maj.*	36 H *Urs. maj.*	χ *Urs. maj.*	136 Ş *Urs. maj.*
(3)	2	+ 2.13	2 + 2.28	+ 1.61	+ 1.02					
(4)							+ 0.33	+ 0.17	— 1.97	+ 0.88
(10)						+ 1.11				
Means		+ 2.13	+ 2.28	+ 1.61	+ 1.02	+ 1.11	+ 0.33	+ 0.17	— 1.97	+ 0.88

XXI.

		ψ *Urs. maj.*	36 H *Urs. maj.*	39 H *Urs. maj.*	90 Ş *Urs. maj.*	DM. 49°1960.	Extra Stars 84 Ş *Urs. maj.*	DM. 46°1659.
(50)	4.	— 2.92	— 0.04	+ 0.22	+ 1.21	+ 1.55	+ 0.75	
(54)	4.	— 2.85	— 0.19	+ 0.11	+ 1.28	+ 1.66	+ 0.71	+ 2.37
(169)	2.	— 3.05	— 0.15	0.00		+ 1.02 rej.		
(171)	3.	— 2.74	— 0.26	— 0.10	+ 1.41	+ 1.70		
(177)	2.	— 2.39 rej.	— 0.10	+ 0.09	+ 1.25	+ 1.61		
(178)	4.	— 2.76	— 0.27	+ 0.20	+ 1.20	+ 1.63		
(189)	2.	— 2.99	— 0.06	+ 0.12	+ 1.31	+ 1.60		
Means		— 2.88	— 0.15	+ 0.09	+ 1.19	+ 1.62	+ 0.73	+ 2.37

XXII.

		56 *Urs. maj.*	47 *Urs. maj.*	49 *Urs. maj.*	57 *Urs. maj.*	59 *Urs. maj.*	58 *Urs. maj.*	122 Ş *Urs. maj.*
(3)	5.		— 0.24	— 0.38				+ 0.52
(10)	6.	— 0.51	— 0.47	— 0.33	— 0.04	+ 0.20	+ 0.59	+ 0.55
(58)	4.	— 0.60	— 0.34	— 0.37	+ 0.08	+ 0.03	+ 0.59	+ 0.61
(171)	3.	— 0.32	— 0.48	— 0.45	— 0.24	+ 0.18	+ 0.33	+ 0.98
(179)	4.	— 0.55	— 0.53	— 0.24		+ 0.17	+ 0.55	+ 0.67
(188)	3.	— 0.55	— 0.50	— 0.39	— 0.13	— 0.03	+ 0.82	+ 0.80
Means		— 0.51	— 0.43	— 0.36	— 0.08	+ 0.11	+ 0.59	+ 0.72

Extra Stars.

		ω *Urs. maj.*	DM. 43°2069.	DM. 42°2162.	DM. 41°2093.	DM. 41°2097.	130 Ş *Urs. maj.*	DM. 40°2432.
(3)		— 0.39	+ 1.19	+ 0.53	+ 0.31	+ 1.32		
(10)		— 0.71					+ 0.49	
(58)		— 0.83	+ 1.49	+ 0.64				+ 1.99
(171)		— 0.60		+ 0.75				
(179)		— 0.83						
Means		— 0.67	+ 1.34	+ 0.64	+ 0.31	+ 1.32	+ 0.49	+ 1.99

XXIII.

		γ *Urs. maj.*	ζ *Urs. maj.*	3 *Can. Ven.*	158 Ş *Urs. maj.*	65 *p. Urs. maj.*	65 sq. *Urs. maj.*	60 *Urs. maj.*	DM. 49°2110?	DM. 49°2132?	65 *Urs. maj.*
(5)	5.	— 1.55	— 0.32		+ 2.78	+ 2.03 rej.	+ 2.73				
(15)	4. rej.	— 1.32	— 0.09	+ 1.41		+ 3.37					
(54)	8.	— 1.53	6 — 0.13	5 + 1.71	5 + 2.65	5 + 3.48		5 + 2.73	1 + 3.30	1 + 3.36	
(62)	2.	— 1.52	— 0.39	+ 1.92		+ 3.22					
(65)	4.	— 1.66	— 0.29	+ 1.94	+ 2.89	+ 3.59					
(179)	4.	— 1.50	8 — 0.37		+ 2.71						+ 2.59
(188)	3.	— 1.82	6 — 0.16	+ 1.99							+ 3.02
Means		— 1.61	— 0.27	+ 1.89	+ 2.75	+ 3.43	+ 2.73	+ 2.73	+ 3.30	+ 3.36	+ 2.80

XXIV.

		6 *Can. Ven.*	67 *Urs. maj.*	2 *Can. Ven.*	4 *Can. Ven.*	11 ♀ *Can. Ven.*
(1)	5. rej.	— 1.07	— 0.13			
(2)	5.	— 0.54	— 0.63	— 0.13	+ 0.58	+ 0.71
(15)	3.	— 0.66	— 0.16 rej.	0.00	+ 0.33	+ 0.86
(55)	4.	— 0.64	— 0.31	— 0.17	+ 0.33	+ 0.79
(58)	4.	— 0.77	— 0.44	— 0.14	+ 0.41	+ 0.96
(124)	1.	— 0.77	— 0.67	+ 0.03	+ 0.36	+ 1.03
(126)	4.	— 0.74	— 0.47	— 0.13	+ 0.39	+ 0.96
Means		— 0.69	— 0.54	— 0.08	+ 0.40	+ 0.88

Extra Stars.

	Comp. of 67 *Urs. maj.*	DM. 41°2251.	DM. 41°2252.	179 ♀ *Urs. maj.*	DM. 40°2485?	DM. 43°2221.
(2)	+ 0.89	2 + 1.13	2 + 0.57	2 + 0.71	2 + 1.48	+ 1.07
(55)			+ 1.57	+ 1.10		
(58)	+ 1.15					
Means	+ 1.15	+ 1.13	+ 1.57	+ 1.10	+ 1.48	+ 1.07

XXV.

		26 ♀ *Can. Ven.*	42 ♀ *Can. Ven.*	32 ♀ *Can. Ven.*	36 ♀ *Can. Ven.*	23 ♀ *Can. Ven.*	11 *Can. Ven.*	29 ♀ *Can. Ven.*	Extra Star. 21 ♀ *Can. Ven.*
(6)	5.	— 0.77	— 0.41	— 0.09	+ 0.03	+ 0.29	+ 0.38	+ 0.58	
(65)	4.	— 1.09	— 0.40	— 0.30	+ 0.15	+ 0.40	+ 0.61	+ 0.66	
(67)	4.	— 0.87	— 0.53	— 0.27	+ 0.14	+ 0.43	+ 0.53	+ 0.58	
(124)	1.	— 0.66	— 0.09	— 0.41	+ 0.28	— 0.11 rej.	+ 0.36		+ 1.05
(126)	4.	— 0.97	— 0.29	— 0.24	+ 0.24	+ 0.37	+ 0.28		
Means		— 0.87	— 0.34	— 0.26	+ 0.17	+ 0.37	+ 0.43	+ 0.61	+ 1.05

XXVI.

		12 *Can. Ven.*	8 *Can. Ven.*	6 H *Can. Ven.*	10 *Can. Ven.*	Extra Stars. 9 *Can. Ven.*	40 ♀ *Can. Ven.*
(1)						+ 1.38	
(5)	5.	— 1.71	— 0.41	+ 1.14	+ 0.97	+ 1.55	
(6)	5.	— 1.73	— 0.38	+ 0.98	+ 1.15	+ 1.50	
(7)	3.	— 1.50 rej.	— 0.26	+ 1.03	+ 1.18	+ 1.32	
(55)	4.	— 2.16	— 0.52	+ 1.36	+ 1.30	+ 1.62	+ 2.23
(127)	2.	— 2.24	— 0.55	+ 1.43	+ 1.37	+ 1.79	
Means		— 1.96	— 0.42	+ 1.19	+ 1.19	+ 1.53	+ 2.23

XXVII.

		24 *Can. Ven.*	21 *Can. Ven.*	58 ♀ *Can. Ven.*	57 ♀ *Can. Ven.*	Extra Stars. 214 ♀ *Urs. maj.*	66 ♀ *Can. Ven.*	56 ♀ *Can. Ven.*
(67)	4.	— 1.10	— 0.29	+ 0.41	+ 0.99	+ 0.89	+ 0.87	+ 1.04
(125)	4.	— 0.89	— 0.28	+ 0.38	+ 0.80			
(127)	2.	— 0.74	— 0.39	+ 0.46	+ 0.68			
(128)	4.	— 0.84	— 0.35	+ 0.28	+ 0.92			+ 1.11
Means		— 0.89	— 0.33	+ 0.38	+ 0.85	+ 0.89	+ 0.87	+ 1.07

XXVIII.

		20 *Can. Ven.*	11 H *Can. Ven.*	23 *Can. Ven.*	17 *Can. Ven.*	19 *Can. Ven.*	15 *Can. Ven.*
(5)	5.	— 0.91	— 0.53	+ 0.05	+ 0.33	+ 0.39	+ 0.67
(7)	3.	— 0.63	— 0.79	+ 0.08	+ 0.42	+ 0.22	+ 0.71
(59)	1.	— 0.75	— 0.75	+ 0.07	+ 0.30	+ 0.48	+ 0.66
(60)	3.	— 0.92	— 0.84	+ 0.09	+ 0.67	+ 0.19	+ 0.82
(64)	4.	— 1.00	— 0.69	+ 0.09	+ 0.55	+ 0.34	+ 0.69
(128)	4.	— 0.87	— 0.74	+ 0.01	+ 0.43	+ 0.38	+ 0.77
Means		— 0.85	— 0.72	+ 0.06	+ 0.45	+ 0.33	+ 0.72

XXIX.

		η *Urs. maj.*	60 ♀ *Can. Ven.*	73 ♀ *Can. Ven.*	175 B *Can. Ven.*	Extra Stars. DM. 42°2403.	150B *Can. Ven.*
(64)	4.	— 4.22	+ 0.97	+ 1.39	+ 1.87	+ 2.71	
(66)	4.	— 4.29	+ 0.87	+ 1.61	+ 1.81		
(125)	4.	— 3.70	+ 0.66	+ 1.49	+ 1.57		
(130)	4.	— 3.75	+ 0.71	+ 1.56	+ 1.48		+ 0.80
Means		— 3.99	+ 0.80	+ 1.51	+ 1.58	+ 2.71	+ 0.80

XXX.

		76 ♀ *Can. Ven.*	DM. 39°2678.	75 ♀ *Can. Ven.*	72 ♀ *Can. Ven.*	59 ♀ *Can. Ven.*	62 ♀ *Can. Ven.*	78 ♀ *Can. Ven.*	Extra Star. DM. 41°2423.
(59)	1.	— 0.55	— 0.31	— 0.15		+ 0.23	+ 0.36		+ 2.07
(60)	3.	— 0.67	— 0.21	— 0.17	— 0.03	+ 0.24	+ 0.31	+ 0.55	+ 1.73
(61)	4.	— 0.61	— 0.32	— 0.16		+ 0.46	+ 0.47	+ 0.17	
(66)	4.	— 0.74	— 0.34	— 0.24	+ 0.21	+ 0.37	+ 0.32	+ 0.39	
(129)	1.	— 0.54	— 0.13	+ 0.10	+ 0.14		+ 0.43	+ 0.35 rej.	
(132)	1.	— 0.75	— 0.40	+ 0.51	— 0.25	+ 0.37	+ 0.07	+ 0.46	
(134)	1. rej.			— 0.55	+ 0.02			+ 0.77	
(136)	2.	— 0.66	— 0.14	— 0.11	+ 0.16	+ 0.16	+ 0.30	+ 0.32	
Means		— 0.65	— 0.26	— 0.10	+ 0.04	+ 0.30	+ 0.32	+ 0.44	+ 1.90

XXXI.

		ϑ *Bootae.*	λ *Bootae.*	$\varkappa$ *Bootae.*	ι *Bootae.*	13 *Bootae.*	23 ♀ *Bootae.*
(117)	1.	— 0.56	— 0.48	— 0.04	+ 0.06	+ 0.45	+ 0.55
(120)	3.	— 0.88	— 0.71	— 0.08	+ 0.24	+ 0.55	+ 0.91
(130)	4.	— 0.58	— 0.66	— 0.27	+ 0.18	+ 0.80	+ 0.54
(131)	4.	— 0.56	— 0.45	— 0.24	+ 0.19	+ 0.59	+ 0.49
(132)	1.	— 0.30	— 0.54	— 0.23	+ 0.05	+ 0.60	+ 0.41
(133)	2.	— 0.29	— 0.60	— 0.33	— 0.07	+ 0.35	+ 0.95
(134)	1.	— 0.47	— 0.49	— 0.16	+ 0.21	+ 0.51	+ 0.39
(136)	2.	— 0.57	4 — 0.29	— 0.26	— 0.14	6 + 0.73	+ 0.54
(137)	2.	— 0.53	2 — 0.50	+ 0.08	+ 0.12	6 + 0.52	+ 0.32
Means		— 0.52	— 0.52	— 0.17	+ 0.09	+ 0.57	+ 0.57

EXTRA STARS.

	22 Ϥ *Bootae.*	87 Ϥ *Can. Ven.*	DM. 51°1908.	g *Bootae.*	69 Ϥ *Bootae.*
(117)	+ 1.86	+ 1.99			
(120)	+ 1.63		+ 1.42		
(130)		+ 1.79		+ 1.10	+ 1.36
(131)		+ 1.40 rej.		+ 1.21	+ 1.51
(132)		+ 2.07			
Means	+ 1.74	+ 1.95	+ 1.42	+ 1.15	+ 1.43

XXXII.

		γ *Bootae.*	32 Ϥ *Bootae.*	34 Ϥ *Bootae.*	42 Ϥ *Bootae.*	EXTRA STAR. 50 Ϥ *Bootae.*
(8)	4.	— 2.37	+ 0.73	+ 0.74	+ 0.92	+ 0.38
(61)	4.	— 2.90		+ 1.08	+ 1.01	+ 0.66
(133)	2.	— 2.47	+ 0.89	+ 0.75	+ 0.82	
(138)	4.	— 2.86	+ 0.84	+ 1.08	+ 0.93	+ 0.86
Means		— 2.65	+ 0.82	+ 0.91	+ 0.92	+ 0.63

XXXIII.

		i *Bootae.*	39 *Bootae.*	k *Bootae.*	33 *Bootae.*	h *Bootae.*	EXTRA STAR. 112 Ϥ *Bootae.*
(137)	2.	— 0.83	+ 0.20	+ 0.14	— 0.16	+ 0.65	+ 1.04
(138)	4.	— 0.73	+ 0.13	+ 0.50	— 0.13	+ 0.25	+ 0.83
(139)	3.	— 0.85	+ 0.29	+ 0.26	— 0.07	+ 0.35	+ 1.34
(140)	3.	— 0.86	+ 0.26	+ 0.37	— 0.15	+ 0.40	+ 1.29
Means		— 0.82	+ 0.22	+ 0.32	— 0.13	+ 0.41	+ 1.12

XXXIV.

		β *Bootae.*	μ *Bootae.*	ν² *Bootae.*	ν¹ *Bootae.*	φ *Bootae.*	134 Ϥ *Bootae.*	EXTRA STAR. Comp. to μ.
(8)	4.	— 1.35	— 0.22	+ 0.23	+ 0.10	+ 0.40	+ 0.85	
(9)	3.	— 1.07	— 0.22	+ 0.20	+ 0.08	+ 0.27	+ 0.72	
(56)	2.	— 1.65	— 0.59	+ 0.50	+ 0.19	+ 0.62	+ 0.93	+ 1.90
(139)	3.	— 1.70	— 0.42	+ 0.40	+ 0.29	+ 0.58	+ 0.86	
Means		— 1.44	— 0.36	+ 0.33	+ 0.16	+ 0.47	+ 0.84	+ 1.90

XXXV.

		τ *Herculis.*	φ *Herculis.*	υ *Herculis.*	16 Ϥ *Herculis*
(9)	3.	— 0.40 rej.	— 0.43	— 0.26	2 + 1.07
(18)	4.	— 0.85	— 0.42	+ 0.01	+ 1.24
(31)	4.	— 0.65	— 0.30	0.00	+ 0.97
(140)	3.	— 1.02	— 0.47	0.00	+ 1.51
Means		— 0.84	— 0.40	— 0.06	+ 1.20

XXXVI.

		χ *Herculis.*	2 *Herculis.*	23 Ϥ *Herculis.*	4 *Herculis.*	EXTRA STARS. g *Herculis.*	19 Ϥ *Herculis.*
(20)	4.	— 0.68	— 0.35	+ 0.35	+ 0.69	— 0.70	
(23)	4.	— 0.77	— 0.08	+ 0.24	+ 0.60	— 0.64	
(31)	4.	— 0.75	+ 0.01	+ 0.21	+ 0.53	— 0.14	+ 0.52
(56)	2.	— 1.22	+ 0.21	+ 0.22	+ 0.81	— 0.65	
Means		— 0.85	— 0.05	+ 0.25	+ 0.66	— 0.53	+ 0.52

XXXVII.

		42 *Herculis.*	52 *Herculis.*	40 Ϥ *Herculis.*	46 Ϥ *Herculis.*	EXTRA STARS. 78 Ϥ *Herculis.*	Comp. of 42.
(11)	2.	— 0.74	— 0.18	+ 0.57	+ 0.35	+ 0.52	
(18)	4.	— 0.73	— 0.41	+ 0.54	+ 0.59		
(20)	4.	— 0.76	— 0.54	+ 0.60	+ 0.70	+ 0.65	
(57)	2.	— 0.73	— 0.38	+ 0.47	+ 0.64		+ 1.02
Means		— 0.74	— 0.38	+ 0.54	+ 0.57	+ 0.58	+ 1.02

XXXVIII.

		η *Herculis.*	σ *Herculis.*	100 Ϥ *Herculis.*	121 Ϥ *Herculis.*	98 Ϥ *Herculis.*	EXTRA STAR. g *Herculis.*
(23)	4.	— 1.48	— 0.92	+ 0.11	+ 0.60	+ 1.57	
(57)	2.	3 — 1.49	3 — 0.87	+ 0.09	3 + 0.79	+ 1.47	
(63)	4.	— 1.89	— 0.83	+ 0.18	+ 0.79	+ 1.73	— 0.13
(70)	3.	— 1.85	— 0.56	+ 0.16	+ 0.81	+ 1.42	
Means		— 1.68	— 0.79	+ 0.13	+ 0.75	+ 1.55	— 0.13

XXXIX.

		ι *Herculis.*	γ *Herculis.*	74 *Herculis.*	x *Herculis.*	DM. 49°2604.
(11)	2.	— 1.22	— 0.05	+ 0.12	+ 0.41	+ 0.73
(12)	1.	— 1.12 rej.	+ 0.16	— 0.07	+ 0.57	+ 0.54
(13)	6.	— 1.48	+ 0.01	+ 0.12	+ 0.32	+ 1.03
(70)	3.	— 1.76	+ 0.12	+ 0.28	+ 0.63	+ 0.71
(71)	3.	— 1.56	+ 0.13	+ 0.08	+ 0.48	+ 0.86
(72)	4.	— 1.60	+ 0.12	+ 0.06	+ 0.47	+ 0.93
Means		— 1.53	+ 0.08	+ 0.10	+ 0.48	+ 0.73

EXTRA STARS.

	120 Ϥ *Herculis.*	95 Ϥ *Draconis.*	118 Ϥ *Herculis.*	DM. 44°2695.	DM. 45°2504.	99 Ϥ *Herculis.*	95 Ϥ *Herculis.*
(11)	+ 1.17	+ 1.22	+ 1.04	+ 1.41	+ 1.41	1 + 1.79 rej.	1 + 0.41 rej.
(12)	+ 1.48 rej.		+ 1.33				
(13)	+ 1.00	+ 1.36				+ 1.29	+ 0.86
(71)	+ 1.08		+ 1.11				
Means	+ 1.08	+ 1.29	+ 1.16	+ 1.41	+ 1.41	+ 1.29	+ 0.86

XL.

		9 *Herculis.*	f *Herculis.*	135 Ş *Herculis.*	157 Ş *Herculis.*	156 Ş *Herculis.*	Extra Stars. DM. 41°2882.	110 Ş *Herculis?*	186 Ş *Herculis.*	DM. 40°3225.
(27)	4.	— 2.19	— 0.28	+ 0.54	+ 0.79	+ 1.15	2 + 1.88	2 + 3.41		
(63)	4.	— 2.07	— 0.37	+ 0.30	+ 1.04	+ 1.12			+ 0.53	+ 1.24
(68)	4.	— 2.42	— 0.36	+ 0.36	+ 1.16	+ 1.25				
(71)	3.	— 2.09	— 0.23	+ 0.39	+ 0.86	+ 1.08			+ 0.46	+ 1.35
Means		— 2.19	— 0.31	+ 0.40	+ 0.96	+ 1.15	+ 1.88	+ 3.41	+ 0.49	+ 1.29

XLI.

		169 Ş *Herculis.*	164 Ş *Herculis.*	179 Ş *Herculis.*	z *Herculis.*	Extra Stars. 182 Ş *Herculis.*	DM. 47°2541.	DM. 45°2626.	DM. 45°2627.	DM. 45°2629.
(13)	6.	— 0.27	— 0.10	+ 0.02	+ 0.34	— 1.06	+ 0.31	+ 0.36	— 0.68	+ 0.12
(19)	4.	— 0.33	+ 0.01	+ 0.10	+ 0.21		+ 0.17	+ 0.18	— 0.94	+ 0.24
(27)	4.	— 0.07	— 0.03	— 0.12	+ 0.23	— 0.98			— 0.39	
(62)	2.	— 0.05	+ 0.12	— 0.20	+ 0.64 rej.	— 1.04	+ 0.30	+ 0.18	— 0.15	+ 0.66 rej.
(73)	3.	— 0.08	— 0.10	+ 0.13	+ 0.03	— 0.88	+ 0.20			
(76)	4.	— 0.11	— 0.03	— 0.16	+ 0.29	— 1.08			— 0.46	
Means		— 0.15	— 0.04	— 0.04	+ 0.22	— 1.01	+ 0.24	+ 0.24	— 0.52	+ 0.18

XLII.

		μ *Lyrae.*	3 Ş *Lyrae.*	2 Ş *Lyrae.*	4 Ş *Lyrae.*	Extra Stars. 196 Ş *Herculis.*	1 Ş *Lyrae.*
(25)	4.	— 0.56	— 0.15	+ 0.27	+ 0.45	— 0.53	
(68)	4.	— 0.63	— 0.13	+ 0.35	+ 0.39	— 0.98	
(73)	3.	— 0.61	— 0.02	+ 0.16	+ 0.46	— 0.77	
(74)	1.	— 1.13 rej.	+ 0.14	+ 0.67 rej.	+ 0.23	— 1.02	+ 0.43
(77)	1.	— 0.90	+ 0.37 rej.	+ 0.01 rej.	+ 0.63		
(78)	1.	— 0.95	— 0.08	+ 0.54	+ 0.46	— 0.65	
(85)	4.	— 0.55	— 0.14	+ 0.33	+ 0.37	— 0.90	+ 0.44
Means		— 0.70	— 0.06	+ 0.33	+ 0.43	— 0.81	+ 0.44

XLIII.

		143 Ş *Draconis.*	164 Ş *Draconis.*	185 Ş *Herculis.*	202 Ş *Herculis.*	Extra Stars. DM. 49°2728.	197 Ş *Herculis.*	182 Ş *Herculis.*	DM. 49°2776.
(17)	4.	— 1.31	+ 0.23	+ 0.34	+ 0.74	+ 0.71			
(19)	4.	— 1.51	+ 0.04	+ 0.37	+ 0.74	+ 0.72	+ 0.31		+ 0.43
(25)	4.	— 1.14	+ 0.09	+ 0.32	+ 0.73	+ 0.50			
(88)	1.						+ 0.37		
(92)	4.	— 1.02	+ 0.21	+ 0.23	+ 0.59		+ 0.47	— 0.11	
Means		— 1.24	+ 0.14	+ 0.31	+ 0.70	+ 0.66	+ 0.38	— 0.11	+ 0.43

XLIV.

		ε. n. *Lyrae.*	ε. s. *Lyrae.*	36 Ş *Lyrae.*	25 Ş *Lyrae.*	31 Ş *Lyrae.*
(16)	4.	— 0.99	— 0.78	+ 0.57	+ 0.75	+ 0.43
(21)	3.	— 0.64	— 0.73	— 0.10	+ 0.49	+ 0.96
(22)	3.	— 0.76	— 0.95	+ 0.10	+ 0.72	+ 0.89

(74)	1.	— 0.78	— 0.80	— 0.04	+ 1.05 rej.	+ 0.94
(85)	4.	— 0.78	— 0.78	+ 0.02	+ 0.71	+ 0.81
(92)	4.	— 0.76	— 0.88	+ 0.04	+ 0.72	+ 0.88
Means		— 0.79	— 0.83	+ 0.07	+ 0.68	+ 0.81

XLV.

		16 *Lyrae.*	171 Ş *Draconis.*	1 Ş *Cygni.*	2 Ş *Cygni.*	Extra Stars. R *Lyrae.*	DM. 49°2977.	DM. 47°2779.	ζ. n. *Lyrae.*	ζ. s. *Lyrae.*
(16)	4.	— 0.71	+ 0.09	+ 0.19	+ 0.43	— 0.93				
(17)	4.	— 0.67	+ 0.18	+ 0.35	+ 0.14	— 0.97				
(22)	3.	— 0.74	+ 0.13	+ 0.33	+ 0.29	— 1.42	+ 0.47			
(28)	4.	— 0.59	0.00	+ 0.35	+ 0.23	— 1.58		+ 0.74	— 0.54	+ 0.04
(86)	4.	— 0.80		+ 0.41	+ 0.30	— 1.89				
Means		— 0.70	+ 0.10	+ 0.33	+ 0.28	— 1.56	+ 0.47	+ 0.74	— 0.54	+ 0.04

XLVI.

		9 *Lyrae.*	η *Lyrae.*	69 Ş *Lyrae.*	43 Ş *Lyrae.*	52 Ş *Lyrae.*	17 Ş *Cygni.*	14 *Cygni.*	37 Ş *Cygni.*	Extra Stars. 49 Ş *Cygni.*	52 Ş *Cygni.*
(21)	2.			+ 0.54	+ 0.66	+ 1.11	— 0.03	+ 0.04	+ 0.33	+ 0.35	
(24)	3.			+ 0.45	+ 0.83	+ 1.07	— 0.03	+ 0.02	+ 0.32	+ 0.06	+ 1.05
(28)	4.	— 1.36	— 1.25	+ 0.37	+ 1.00	+ 0.98					
(30)	4.	— 1.53	— 1.14	+ 0.34	+ 0.93	+ 1.20	— 0.07	+ 0.02	+ 0.26		
Means		— 1.44	— 1.19	+ 0.42	+ 0.85	+ 1.09	— 0.04	+ 0.03	+ 0.30	+ 0.20	+ 1.05

XLVII.

		δ *Cygni.*	9 *Cygni.*	26 Ş *Cygni.*	20 Ş *Cygni.*	Extra Stars. DM. 44°3133.	7 Ş *Cygni.*
(24)	3.	— 1.88	+ 0.37	+ 0.72	+ 0.79		
(26)	4.	— 1.58	— 0.06	+ 0.82	+ 0.93		
(86)	4.	— 1.59	+ 0.15	+ 0.69	+ 0.74	+ 2.45	
(93)	4.	— 1.93	+ 0.21	+ 0.90	+ 0.81	+ 2.84	+ 3.13
Means		— 1.74	+ 0.17	+ 0.78	+ 0.82	+ 2.64	+ 3.13

XLVIII.

		17 Ş *Cygni.*	14 *Cygni.*	37 Ş *Cygni.*	49 Ş *Cygni.*	52 Ş *Cygni.*	27 Ş *Cygni.*	Extra Star. DM. 41°3469.
(30)	4.	— 0.46	— 0.37	— 0.13	— 0.29	+ 0.85	+ 0.40	— 0.56
(33)	2.	— 0.36	— 0.58	— 0.21	— 0.26	+ 0.96	+ 0.45	— 0.24
(93)	4.	— 0.42	— 0.38	— 0.09	— 0.30	+ 0.72	+ 0.47	
(95)	4.	— 0.45	— 0.51	— 0.03	— 0.40	+ 0.93	+ 0.45	— 0.10
Means		— 0.42	— 0.46	— 0.11	— 0.31	+ 0.86	+ 0.44	— 0.30

XLIX.

		ε *Cygni.*	39 Ş *Cygni.*	56 Ş *Cygni.*	40 Ş *Cygni.*	34 Ş *Cygni.*	DM. 47°2945.	Extra Stars. DM. 47°2937.	DM. 47°3037.	61 Ş *Cygni.*
(14)	4.	— 0.77	— 0.22	— 0.04	+ 0.21	+ 0.26	+ 0.58	+ 0.94	+ 0.79	
(26)	4.	— 0.77	+ 0.01	+ 0.04	+ 0.08	+ 0.22	+ 0.43			+ 0.33
(35)	2.	— 0.78	— 0.18	— 0.02	+ 0.27	+ 0.22	+ 0.47	+ 0.33		+ 0.41
(95)	4.	— 1.03	— 0.08	+ 0.19	+ 0.04	+ 0.32	+ 0.55	+ 0.68		+ 0.45
Means		— 0.84	— 0.12	+ 0.04	+ 0.15	+ 0.25	+ 0.51	+ 0.65	+ 0.79	+ 0.40

L.

		γ *Cyeni.*	77 ♄ *Cyeni.*	114 ♄ *Cyeni.*	87 ♄ *Cyeni.*	109 ♄ *Cyeni.*	Extra Stars. 80 ♄ *Cyeni.*	DM. 40°4136.
(29)	2.	— 2.87	+ 0.24	+ 0.47	+ 0.97	+ 1.19		
(33)	2.	— 2.81	+ 0.22	+ 0.80	+ 0.75	+ 1.05	+ 1.20	+ 1.52
(35)	2.	— 3.01	+ 0.24	+ 0.66	+ 0.80	+ 1.31	+ 0.95	+ 1.40
(39)	2.	— 2.83	+ 0.09	+ 0.89	+ 0.78	+ 1.09	+ 0.99	+ 1.45
Means		— 2.88	+ 0.20	+ 0.70	+ 0.82	+ 1.16	+ 1.05	+ 1.46

LI.

		o^1 sq. *Cyeni.*	o^2 *Cyeni.*	o^1 (pr.) *Cyeni.*	86 ♄ *Cyeni.*	76 ♄ *Cyeni.*	84 ♄ *Cyeni.*	Extra Star. DM. 45°3139.
(14)	4.	— 1.48	— 1.20	— 0.11	+ 0.40	+ 0.81	+ 1.58	+ 1.74
(32)	4.	— 1.61	— 1.26	— 0.03	+ 0.51	+ 0.88	+ 1.48	+ 1.46
(37)	4.	— 1.47	— 1.25	— 0.09	+ 0.40	+ 0.82	+ 1.60	+ 1.49
(39)	2.	— 1.31	— 1.34	— 0.12	+ 0.47	+ 0.65	+ 1.66	+ 1.23
(87)	1.	— 1.29	— 1.20	— 0.07	+ 0.38	+ 0.80	+ 1.36	+ 1.07
(99)	4.	— 1.36	— 1.41	— 0.19	+ 0.45	+ 1.09	+ 1.40	+ 1.81
(131)	4.	— 1.52	— 1.21	± 0.00	+ 0.47	+ 1.05	+ 1.22	+ 1.12
(155)	2.	— 1.51	— 1.32	— 0.20	+ 0.79 rej.	+ 1.14	+ 1.42	
Means		— 1.44	— 1.28	— 0.10	+ 0.44	+ 0.91	+ 1.47	+ 1.43

LII.

		ν *Cyeni.*	57 *Cyeni.*	56 *Cyeni.*	132 ♄ *Cyeni.*	139 ♄ *Cyeni.*	110 ♄ *Cyeni.*	Extra Star. DM. 44°3639.
(29)	2.	— 1.35	— 0.16	— 0.05	+ 0.38	+ 0.46	+ 0.71	
(34)	2.	— 1.24	— 0.25	— 0.10	+ 0.30	+ 0.50	+ 0.80	
(37)	4.	— 1.10	— 0.29	— 0.09	+ 0.29	+ 0.37	+ 0.85	+ 0.83
(41)	3.	— 1.07	— 0.23	— 0.02	+ 0.20	+ 0.33	+ 0.79	+ 0.96
Means		— 1.19	— 0.23	— 0.06	+ 0.29	+ 0.41	+ 0.79	+ 0.89

LIII.

		α *Cyeni.*	55 *Cyeni*		*Cyeni.*	51 *Cyeni.*	ω¹ *Cyeni.*	104 ♄ *Cyeni.*	Extra Stars. 123 ♄ *Cyeni.*	118 ♄ *Cyeni.*
(32)	4.	— 3.72	+ 0.			+ 0.83	+ 0.98	+ 1.07	+ 0.76	
(34)	3.	— 3.46	2 — (		2	+ 0.85	+ 0.96	+ 1.02	2 + 0.68	
(42)	1.	— 3.54	—			+ 0.71	+ 1.11	+ 1.05	+ 0.58	+ 2.25
(43)	2.	— 3.79	+ 0.			+ 0.80	+ 0.84	+ 1.21	+ 0.79	
(87)	1.	— 4.00	— 0.		0.24	+ 1.94	+ 0.99	+ 1.17	+ 0.78	
(94)	4.	— 4.00	+ 0.12		+ 0.18	+ 0.79	+ 1.00	+ 1.11	+ 1.05	
Means		— 3.75	± 0.00	+ 0.56	+ 0.27	+ 0.82	+ 0.97	+ 1.10	+ 0.77	+ 2.25

LIV.

		τ *Cyeni.*	σ *Cyeni.*	162 ♄ *Cyeni.*	A *Cyeni.*	65 *Cyeni.*	ξ *Cyeni.*
(38)	4.	— 1.14	— 0.56	+ 1.40	+ 1.49	+ 0.18	— 1.35
(41)	2.	— 1.07	— 0.53	+ 1.32	+ 1.32	+ 0.20	— 1.22
(42)	1.				+ 1.51	+ 0.26	— 0.71
(43)	2.	— 1.21	— 0.85	+ 1.42	+ 1.38	+ 0.48	— 1.22
(123)	3.	— 1.32	— 0.97		+ 1.56	+ 0.32	— 0.98
Means		— 1.18	— 0.73	+ 1.38	+ 1.45	+ 0.29	— 1.10

LV.

		f^2 *Cyeni.*	f^1 *Cyeni.*	60 *Cyeni.*	134 ♄ *Cyeni.*	164 ♄ *Cyeni.*	168 ♄ *Cyeni.*	Extra Star. 140 ♄ *Cyeni.*
(94)	4.	— 1.32	— 0.24	+ 0.02	+ 0.33	+ 0.51	+ 0.73	
(96)	3.	— 1.07	— 0.07	— 0.01	+ 0.41	+ 0.28	+ 0.45	+ 0.27
(123)	3.	— 1.11	— 0.05	+ 0.08	+ 0.46	+ 0.08	+ 0.56	— 0.02
(135)	3.	— 1.00	— 0.14	+ 0.09	+ 0.43	+ 0.37	+ 0.24	+ 0.32
Means		— 1.12	— 0.12	+ 0.04	+ 0.41	+ 0.31	+ 0.49	+ 0.19

LVI.

		75 *Cyeni.*	74 *Cyeni.*	189 ♄ *Cyeni.*	77 *Cyeni.*	76 *Cyeni.*	Extra Star. 195 ♄ *Cyeni.*
(38)	4.	— 0.43	— 0.20	— 0.32	+ 0.26	+ 0.69	
(44)	3.	— 0.51	— 0.51	— 0.03	+ 0.36	+ 0.68	
(46)	4.	— 0.54	— 0.41	— 0.03	+ 0.49	+ 0.50	+ 1.25
(135)	3.	— 0.34	— 0.31	— 0.16	+ 0.30	+ 0.51	+ 1.26
Means		— 0.45	— 0.38	— 0.13	+ 0.35	+ 0.58	+ 1.25

LVII.

		ρ *Cyeni.*	π^2 *Cyeni.*	173 ♄ *Cyeni.*	g *Cyeni.*	171 ♄ *Cyeni.*	Extra Stars. 180 ♄ *Cyeni.*	Foll. ρ *Cyeni.*
(36)	3.	— 0.82	— 0.61	+ 0.33	+ 0.33	+ 0.78		
(44)	3.	— 1.01	— 0.74	+ 0.46	+ 0.37	+ 0.90	+ 1.12	
(83)	3.	— 0.99	— 0.74	+ 0.53	+ 0.43	+ 0.79		
(96)	3.	— 0.94	— 0.74	+ 0.47	+ 0.35	+ 0.86		
(141)	1.	— 1.20	2 — 0.70	+ 0.58	+ 0.37	+ 0.91		— 0.04 rej.
(142)	4.	— 1.23	— 0.73	+ 0.51	+ 0.47	+ 0.97		+ 0.27
Means		— 1.03	— 0.71	+ 0.48	+ 0.38	+ 0.87	+ 1.12	+ 0.27

LVIII.

		11 *Lacertae.*	6 *Lacertae.*	2 ♄ *Lacertae.*	13 *Lacertae.*	6 ♄ *Lacertae.*	1 ♄ *Lacertae.*	Extra Star. DM. 45°3813.
(46)	4.	— 0.30 rej.	— 0.10	— 0.10	+ 0.02	+ 0.45	+ 0.59	
(79)	1.	— 0.99	— 0.48	— 0.53 rej.	+ 0.14	+ 0.79	+ 0.50	+ 1.65
(81)	4.	— 0.74	— 0.55			+ 0.63	+ 0.58	
(141)	1.			+ 0.12		+ 0.55	+ 0.65	
(142)	4.	— 0.79	— 0.59	0.00	+ 0.04	+ 0.67	+ 0.66	
(143)	4.	— 0.81	— 0.50	— 0.09	— 0.03	+ 0.70	+ 0.72	
Means		— 0.83	— 0.44	± 0.00	+ 0.04	+ 0.63	+ 0.62	+ 1.65

LIX.

		7 *Lacertae.*	5 *Lacertae.*	2 *Lacertae.*	4 *Lacertae.*	Extra Stars. 45 ♄ *Lacertae.*	46 ♄ *Lacertae.*
(36)	3.	— 0.45	— 0.14	+ 0.26	+ 0.33	+ 0.34	+ 0.55
(45)	4.	— 0.62	+ 0.04	+ 0.34	+ 0.25	+ 0.42	+ 0.88
(47)	4.	— 0.50	— 0.14	+ 0.30	+ 0.32	+ 0.57	+ 0.81
(81)	4.	— 0.62	— 0.06	+ 0.31	+ 0.38	+ 0.73	+ 0.99
(83)	3.	— 0.80	+ 0.03	+ 0.35	+ 0.41	+ 0.89	+ 1.22
Means		— 0.60	— 0.05	+ 0.31	+ 0.34	+ 0.59	+ 0.89

LX.

		o *Androm.*	15 *Lacertae.*	2 *Androm.*	16 *Lacertae.*	14 *Lacertae.*	40 $\mathfrak{H}$ *Lacertae.*
(40)	4.	— 1.46	— 0.25	— 0.12	+ 0.37	+ 0.70	+ 0.73
(45)	4.	— 1.47	— 0.39	+ 0.22	+ 0.52	+ 0.59	+ 0.57
(79)	1.	— 2.17	— 0.08	+ 0.04	+ 0.40	+ 0.87	+ 0.91
(143)	4.	— 1.82	— 0.48	+ 0.01	+ 0.50	+ 0.93	+ 0.83
(144)	1.	— 1.95	— 0.36	+ 0.18	+ 0.79	+ 0.86	+ 0.81
(145)	4.	— 1.93	— 0.47	0.00	+ 0.65	+ 0.94	+ 0.79
Means		— 1.80	— 0.30	+ 0.06	+ 0.54	+ 0.82	+ 0.77

LXI.

		7 *Androm.*	8 *Androm.*	4 *Androm.*	11 *Androm.*	6 *Androm.*	3 *Androm.*
(40)	4.	— 0.47	— 0.39	+ 0.19	+ 0.36	+ 0.92	
(47)	4.	— 0.48	— 0.26	+ 0.05	+ 0.38	+ 0.80	— 0.51
(76)	4.	— 0.69	— 0.18	+ 0.40	+ 0.30	+ 0.85	— 0.67
(84)	3.	— 0.56	— 0.38	+ 0.25	+ 0.36		— 0.55
(89)	2.	— 0.51	— 0.57	+ 0.07	+ 0.49	+ 1.13	— 0.61
(90)	2.	— 0.55	— 0.30	+ 0.29	+ 0.19		— 0.52
(158)	2.	— 0.54	— 0.42	+ 0.23	+ 0.47		— 0.65
Means		— 0.54	— 0.36	+ 0.21	+ 0.36	+ 0.92	— 0.58

Extra Stars.

		5 *Androm.*	DM. 47°4114.	DM. 44°4347?	DM. 44°4373.	11 $\mathfrak{H}$ *Androm.*	DM. 44°4378.	DM. 47°4107.
(47)		+ 0.50	+ 1.04					
(76)		+ 0.78						
(84)		+ 0.67	+ 1.62					
(89)		+ 0.42		+ 1.52	+ 1.36	+ 1.47		
Means		+ 0.59	+ 1.33	+ 1.52	+ 1.36	+ 1.64	+ 1.64	+ 2.40
						+ 1.55	+ 1.64	+ 2.40

LXII.

		ϰ *Androm.*	ι *Androm.*	10 *Androm.*	13 *Androm.*	39 $\mathfrak{H}$ *Androm.*	23 *Androm.*	DM. 41°4933.	9 *Androm.*	42 $\mathfrak{H}$ *Androm.*
(80)	2.	4 — 1.79	4 — 1.65	+ 0.06	+ 0.20	+ 0.57	+ 0.29	+ 0.83	+ 0.40	+ 1.09
(84)	3.	— 1.59	— 1.46	+ 0.02	+ 0.36	+ 0.77	+ 0.27		+ 0.26	+ 0.75
(98)	2.	4 — 1.65	4 — 1.59	+ 0.37	+ 0.31	+ 0.61	+ 0.06	+ 0.69	+ 0.47	+ 0.75
(144)	1.	2 — 1.63	2 — 1.60							
(145)	4.	8 — 1.63	8 — 1.55	+ 0.28	+ 0.15	+ 0.49	+ 0.24	+ 0.50	+ 0.78	+ 0.78
Means		— 1.66	— 1.57	+ 0.18	+ 0.25	+ 0.61	+ 0.22	+ 0.67	+ 0.48	+ 0.84

Extra Stars.

	DM. 41°4925.	25 $\mathfrak{H}$ *Androm.*
(80)	+ 1.09	
(84)		+ 0.45
(98)		— 0.12
Means	+ 1.09	+ 0.16

LXIII.

		ϰ *Androm.*	ϕ *Androm.*	22 *Androm.*	18 *Androm.*	Extra Stars. DM. 47°50.	34 $\mathfrak{H}$ *Androm.*	DM. 48°4112.	DM. 45°26.
(82)	2.	— 1.12	+ 0.20	+ 0.24	+ 0.67	+ 0.94			
(89)	2.	— 1.27	+ 0.19	+ 0.25	+ 0.84	+ 1.41	+ 1.48		
(90)	2.	— 1.65 rej.	+ 0.17	1 + 0.34	+ 0.71				
(98)	2.	4 — 1.11	+ 0.18	4 + 0.42	+ 0.51		+ 1.58	+ 1.72	
(146)	4.	— 1.28	+ 0.12	+ 0.39	+ 0.76				+ 2.20
Means		— 1.20	+ 0.17	+ 0.33	+ 0.70	+ 1.17	+ 1.53	+ 1.72	+ 2.20

LXIV.

		79 $\mathfrak{H}$ *Androm.*	ν *Androm.*	72 $\mathfrak{H}$ *Androm.*	88 $\mathfrak{H}$ *Androm.*	91 $\mathfrak{H}$ *Androm.*
(75)	4.	— 0.62	— 0.91	— 0.40	+ 0.90	+ 1.01
(80)	2.	— 0.41	— 0.77	— 0.28	+ 0.77	+ 0.71
(146)	4.	8 — 0.45	8 — 1.02	— 0.15	+ 0.80	+ 0.82
(147)	4.	— 0.54	— 1.03	— 0.18	+ 0.64	+ 1.10
Means		— 0.50	— 0.93	— 0.25	+ 0.78	+ 0.91

LXV.

		φ *Androm.*	o *Cassiep.*	ξ *Cassiep.*	π *Cassiep.*	51 $\mathfrak{H}$ *Cassiep.*	Extra Stars. 44 $\mathfrak{H}$ *Cassiep.*	μ *Cassiep.*	ζ *Cassiep.*
(82)	2.	— 0.71	— 0.39	— 0.02	+ 0.20	+ 0.92	+ 0.50		
(91)	4.	— 0.88	— 0.18	— 0.03	+ 0.14	+ 0.97			
(97)	2.	— 0.98	— 0.14	0.00	+ 0.12	+ 0.98		+ 0.10	
(147)	4.	8 — 0.65	— 0.19	— 0.23	+ 0.04	+ 1.03			— 1.51
Means		— 0.80	— 0.22	— 0.07	+ 0.12	+ 0.97	+ 0.50	+ 0.10	— 1.51

LXVI.

		41 *Androm.*	44 *Androm.*	39 *Androm.*	97 $\mathfrak{H}$ *Androm.*	112 $\mathfrak{H}$ *Androm.*	Extra Stars. DM. 42°293.	DM. 42°283.
(75)	4.	— 0.70	— 0.02	+ 0.22	+ 0.29	+ 0.21	2 + 1.15	1 + 0.99
(91)	4.	— 0.85	— 0.16	+ 0.46	+ 0.12	+ 0.43		
(100)	1.	— 0.92 rej.	— 0.23 rej.	+ 0.67 rej.	+ 0.14 rej.			
(148)	4.	— 0.78	— 0.06	+ 0.47	0.00	+ 0.36		
(149)	4.	— 0.74	— 0.06	+ 0.30	+ 0.15	+ 0.34		
Means		— 0.80	— 0.11	+ 0.42	+ 0.14	+ 0.27	+ 1.15	+ 0.99

LXVII.

		υ *Persei.*	φ *Persei.*	ξ *Androm.*	ω *Androm.*	A *Androm.*
(97)	2.	— 1.31	— 0.32	+ 0.27	+ 0.64	+ 0.70
(103)	3.	— 1.02	0.00	+ 0.08	+ 0.24	+ 0.71
(148)	4.	8 — 1.27	12 — 0.47	+ 0.29	+ 0.44	+ 0.99
(152)	4.	12 — 1.29	12 — 0.37	8 + 0.25	8 + 0.48	+ 0.91
Means		— 1.22	— 0.29	+ 0.22	+ 0.45	+ 0.83

LXVIII.

		γ *Androm.*	υ *Androm.*	122 ♑ *Androm.*	χ *Androm.*	55 *Androm.*	120 ♑ *Androm.*
(149)	4.	8 — 3.13	12 — 0.59	8 + 0.54	8 + 0.60	+ 1.11	+ 1.46
(150)	4.	— 3.06	— 0.42	+ 0.59	+ 0.53	+ 1.00	+ 1.33
(152)	4.	— 3.11	— 0.58	+ 0.53	+ 0.53	+ 1.11	+ 1.53
(153)	4.	— 2.98	— 0.47	+ 0.55	+ 0.56	+ 0.96	+ 1.37
Means		— 3.07	— 0.51	+ 0.55	+ 0.55	+ 1.04	+ 1.42

LXIX.

		65 *Androm.*	64 *Androm.*	6 *Persei.*	2 *Persei.*	63 *Androm.*	3 ♑ *Persei.*	3 *Persei.*
(103)	3.	— 0.79	— 0.22	— 0.22	+ 0.11	+ 0.10 rej.	+ 0.22	+ 0.29
(106)	4.	— 0.97	— 0.30	— 0.23	+ 0.36	+ 0.12	+ 0.67	+ 0.32
(150)	4.	— 1.18	— 0.56	— 0.10	+ 0.44	+ 0.22	+ 0.84	+ 0.31
(154)	4.	— 0.97	— 0.49	— 0.17	+ 0.38	+ 0.16	+ 0.76	+ 0.31
Means		— 0.97	— 0.39	— 0.18	+ 0.32	+ 0.15	+ 0.76	+ 0.31

LXX.

		b *Androm.*	12 *Persei.*	c *Androm.*	14 *Persei.*	137 ♑ *Androm.*
(153)	4.	— 0.58	— 0.43	+ 0.11	+ 0.29	+ 0.60
(154)	4.	— 0.65	— 0.50	+ 0.18	+ 0.29	+ 0.69
(156)	2.	— 0.76	— 0.34	+ 0.07	+ 0.33	
(159)	4.	— 0.63	— 0.49	+ 0.19	+ 0.16	+ 0.77
Means		— 0.65	— 0.44	+ 0.14	+ 0.27	+ 0.69

Catalogue of the Stars observed by the author, with the magnitudes of various authorities.

The places are given for 1855.

No.	Name.	RA.	Dec.	H.	A.	♑.	DM.	h.	☾.	Z.	P.
1	22 Andromedae	0h 2m8	45°16′	4.6	5.2	5.2	5.1	—	—	—	5.20
2	DM. 45°26	5.2	45 18	—	—	—	6.9	—	—	—	6.81:
3	23 Andromedae	6.0	40 14	5.7	5.8	5.8	5.5	—	—	—	5.98
4	DM. 47°50	9.5	47 8	—	—	—	5.8	—	—	—	5.93:
5	72 ♑ Andromedae	20.5	43 37	—	5.8	5.4	5.4	—	—	—	5.44
6	79 ♑ Andromedae	28.9	43 41	—	5.8	5.8	5.3	—	—	—	5.23
7	ζ Cassiepeae	28.9	53 6	4.4	4.1	4.1	4.2	—	3.63	3.78	3.80:
8	44 ♑ Cassiepeae	31.2	48 33	—	—	5.8	5.8	—	—	—	5.54:
9	88 ♑ Andromedae	33.2	44 3	—	5.8	5.4	5.4	—	—	—	6.32
10	ξ Cassiepeae	34.0	49 43	4.9	5.8	4.6	5.0	—	—	—	5.04
11	π Cassiepeae	35.5	46 15	4.7	5.8	4.9	4.4	—	—	—	5.20
12	51 ♑ Cassiepeae	36.4	47 4	—	5.8	5.8	5.6	—	—	—	5.94
13	o Cassiepeae	36.7	47 29	4.5	4.9	4.9	4.7	—	—	—	4.91
14	ν Andromedae	41.9	40 18	4.8	4.4	4.4	4.6	—	—	—	4.86
15	91 ♑ Andromedae	42.2	44 12	—	5.8	6.3	6.7	—	—	—	6.44
16	97 ♑ Andromedae	51.9	43 55	—	5.8	6.3	6.1	—	—	—	6.00
17	39 Andromedae	54.8	40 33	5.8	5.8	6.3	5.8	—	—	—	6.24
18	μ Cassiepeae	58.7	54 12	5.9	5.8	5.2	5.5	—	—	—	5.19:

No.	Name.	RA.	Dec.	H.	A.	♑.	DM.	h.	☾.	Z.	P.
19	41 Andromedae	0h 59m7	43°10′	5.6	4.9	4.9	5.1	—	—	—	5.19
20	φ Andromedae	1 1.1	46 28	4.7	4.4	4.4	4.3	—	—	—	4.42
21	44 Andromedae	2.1	41 18	5.6	5.8	5.8	6.1	—	—	4.39	5.79
22	DM. 42°288	13.8	42 50	—	—	—	6.4	—	—	—	6.73:
23	ξ Andromedae	13.8	44 45	5.6	4.9	4.9	5.1	—	—	4.94	4.83
24	DM. 42°293	15.9	42 23	—	—	—	6.4	—	—	—	6.87:
25	112 ♑ Andromedae	17.8	42 41	—	5.8	5.8	5.8	—	—	—	6.11
26	ω Andromedae	19.0	44 39	5.3	4.9	4.9	4.9	—	—	4.93	5.03
27	A Andromedae	21.4	46 16	5.8	5.8	5.8	5.6	—	—	—	5.36
28	υ Andromedae	28.4	40 41	4.1	4.4	4.1	4.0	—	—	—	4.15
29	υ Persei	29.1	47 53	3.3	3.8	3.8	3.8	—	4.00	4.01	3.59
30	χ Andromedae	30.7	43 37	5.3	5.2	5.4	4.9	—	—	—	5.06
31	120 ♑ Andromedae	32.0	42 34	—	5.8	5.8	5.8	—	—	—	5.81
32	122 ♑ Andromedae	33.0	41 53	—	5.8	5.4	5.2	—	—	—	5.06
33	φ Persei	34.6	49 57	4.6	4.1	4.1	4.2	—	—	3.56	4.39
34	3 ♑ Persei	40.3	47 11	—	5.8	5.8	6.1	—	—	—	6.22
35	2 Persei	43.0	50 5	6.1	5.8	5.8	6.0	—	—	—	5.84
36	55 Andromedae	44.6	40 2	5.3	5.8	5.8	5.8	—	—	—	5.48
37	3 Persei	49.4	48 30	5.9	5.8	5.8	6.1	—	—	—	5.83
38	γ Andromedae	55.0	41 38	2.3	2.6	2.6	2.0	2.17	2.31	—	1.94
39	6 Persei	2 4.0	50 23	5.7	5.8	5.8	6.0	—	—	—	5.41
40	δ Andromedae	4.2	43 33	5.3	5.2	5.2	5.1	—	—	—	4.77
41	c Andromedae	9.9	46 42	5.7	5.2	5.2	5.1	—	—	—	5.46
42	63 Andromedae	11.4	49 29	5.9	5.8	5.8	6.1	—	—	—	5.70
43	137 ♑ Andromedae	13.9	40 44	—	5.8	5.8	6.1	—	—	—	5.94
44	64 Andromedae	14.8	49 21	5.9	5.8	5.8	5.6	—	—	—	5.23
45	65 Andromedae	16.0	49 37	5.3	4.9	5.4	4.7	—	—	—	4.73
46	12 Persei	33.1	39 34	5.2	4.9	5.2	4.7	—	—	—	4.96
47	θ Persei	34.3	48 37	4.6	4.1	4.1	4.2	—	—	—	4.13
48	14 Persei	34.7	43 41	5.2	5.8	5.8	5.4	—	—	—	5.57
49	τ Persei	44.0	52 9	4.5	4.1	4.1	4.2	—	—	—	3.97
50	47 ♑ Persei	50.6	51 47	—	4.9	5.4	5.5	—	—	—	5.16
51	γ Persei	54.3	52 56	—	3.2	3.1	3.4	3.14	3.09	3.00	2.92
52	ι Persei	58.6	49 3	—	4.1	4.1	4.3	—	—	—	4.28
53	β Persei	58.7	40 24	—	Var.	Var.	Var.	2.32	2.08	—	2.30
54	x Persei	59.7	44 18	3.9	4.4	4.4	4.4	—	—	—	3.79
55	ω Persei	3 2.0	39 4	5.1	4.9	4.9	5.1	—	—	—	4.64
56	63 ♑ Persei	5.9	50 23	—	4.9	4.9	5.8	—	—	—	5.13
57	30 Persei	8.1	43 29	5.9	5.8	5.8	5.8	—	—	—	5.59
58	29 Persei	8.3	49 41	5.6	4.9	4.9	5.5	—	—	—	5.30
59	31 Persei	8.8	49 35	5.6			5.5	—	—	—	5.29
60	l Persei	11.8	42 48	—	—	6.3	6.1	—	—	—	5.05
61	73 ♑ Persei	13.0	48 41	—	—	6.3	6.1	—	—	—	5.42:
62	α Persei	14.0	49 21	2.3	2.1	2.1	2.0	1.68	1.88	1.77	1.90
63	76 ♑ Persei	17.8	48 33	—	5.8	5.8	5.8	—	—	—	5.22
64	77 ♑ Persei	18.5	49 21	—	—	6.3	6.4	—	—	—	5.93:
65	34 Persei	19.0	49 1	5.0	4.9	5.2	5.2	—	—	—	4.95
66	σ Persei	20.4	47 28	4.6	4.9	4.6	4.7	—	—	—	4.44

No.	Name.	RA.	Dec.	M.	A.	Ph.	DM.	h.	S.	Z.	P.
67	DM. 47°844	3h20m4	47°36′	—	—	—	6.9	—	—	—	6.11
68	DM. 47°847	21.9	47 31	—	—	—	6.4	—	—	—	5.68
69	36 Persei	22.4	45 34	5.6	5.8	5.8	5.8	—	—	—	5.33
70	84 Ph Persei	22.7	44 21	—	5.8	5.8	6.4	—	—	—	6.50
71	ψ Persei	26.2	47 43	4.2	4.9	4.9	5.2	—	—	—	4.41
72	δ Persei	32.6	47 20	3.8	3.2	3.5	3.7	3.16	2.99	3.06	3.27
73	92 Ph Persei	34.6	45 39	—	5.8	6.3	6.4	—	—	—	6.15
74	ν Persei	35.4	42 8	3.7	4.1	4.1	4.2	—	—	4.39	3.84
75	96 Ph Persei	35.5	45 14	—	5.8	5.8	6.1	—	—	—	5.76
76	97 Ph Persei	39.2	43 30	—	—	5.8	6.1	—	—	—	6.03
77	99 Ph Persei	40.0	44 30	—	5.8	5.8	5.6	—	—	—	5.81
78	102 Ph Persei	43.2	48 13	—	—	6.3	5.8	—	—	—	5.76:
79	104 Ph Persei	45.5	47 27	—	5.3	5.4	5.8	—	—	—	5.55
80	A Persei	45.9	50 16	5.4	5.2	5.2	5.5	—	—	—	5.40
81	ε Persei	48.2	39 35	3.1	3.5	3.5	3.4	3.05	2.86	2.98	3.06
82	λ Persei	55.8	49 57	4.7	4.4	4.4	4.2	—	—	—	4.52
83	*c* Persei	58.1	47 20	4.5	4.1	4.1	4.4	—	—	—	4.27
84	μ Persei	4 4.3	48 2	4.7	4.4	3.8	4.2	—	4.29	—	4.35
85	*f* Persei	5.0	40 6	5.0	4.9	4.9	4.7	—	—	—	4.71
86	*b* Persei	7.4	49 56	—	4.9	4.9	4.7	—	—	—	4.82
87	120 Ph Persei	9.2	50 34	—	5.8	6.3	6.0	—	—	—	5.61
88	121 Ph Persei	10.1	49 54	—	—	5.8	7.7	—	—	—	5.76:
89	*d* Persei	11.1	46 9	5.0	4.9	4.9	5.0	—	—	—	5.06
90	*m* Persei	23.2	42 45	6.9	5.8	5.8	6.4	—	—	—	6.24
91	*e* Persei	26.7	40 58	4.5	4.9	4.9	4.9	—	—	4.38	4.21
92	133 Ph Persei	30.6	48 1	—	5.8	5.8	5.8	—	—	—	5.68
93	135 Ph Persei	32.4	49 42	—	5.8	5.8	5.4	—	—	—	5.92
94	59 Persei	32.6	43 5	6.9	5.8	5.8	5.5	—	—	—	5.42
95	4 Ph Aurigae	40.3	48 29	—	5.8	5.8	5.6	—	—	—	5.67
96	6 Ph Aurigae	42.6	42 20	—	5.8	5.8	6.1	—	—	—	5.80
97	8 Ph Aurigae	44.5	43 49	—	5.8	5.8	6.1	—	—	—	6.18
98	ε Aurigae	51.6	43 36	3.1	3.5	Var.	Var.	—	3.21	2.96	3.69:
99	ζ Aurigae	52.4	40 52	4.1	4.1	4.1	3.8	—	—	3.81	3.58
100	η Aurigae	56.4	41 2	3.6	3.8	3.8	3.5	—	—	3.40	3.21
101	22 Ph Aurigae	59.9	46 47	—	—	5.8	5.8	—	—	—	5.54
102	α Aurigae	5 6.0	45 51	0.1	0.3	0.2	0.0	0.2:	0.33	—.20	0.09:
103	λ Aurigae	8.9	39 59	4.8	4.9	4.9	4.7	—	—	—	4.72
104	34 Ph Aurigae	10.1	40 56	—	5.8	5.8	5.6	—	—	—	5.59
105	ρ Aurigae	11.6	41 39	5.6	5.3	5.8	5.8	—	—	—	5.38
106	37 Ph Aurigae	12.6	40 53	—	5.8	5.8	5.8	—	—	—	5.68
107	ο Aurigae	34.7	49 46	5.2	5.4	5.8	5.8	—	—	—	5.56
108	β Aurigae	48.9	44 56	1.6	2.1	2.1	2.0	2.15	1.84	1.89	1.96
109	π Aurigae	49.2	45 55	5.0	4.9	4.9	4.7	—	—	—	4.36
110	36 Aurigae	50.0	47 52	5.8	5.8	5.8	5.8	—	—	—	5.69
111	85 Ph Aurigae	51.6	49 52	—	5.8	6.3	5.8	—	—	—	6.02
112	DM. 43°1421	52.4	43 22	—	—	—	7.2	—	—	—	6.24:
113	38 Aurigae	52.9	42 55	6.2	5.8	5.8	6.2	—	—	—	6.07
114	39 Aurigae	54.6	42 59	6.2	—	6.3	6.7	—	—	—	5.90

No.	Name.	RA.	Dec.	M.	A.	Ph.	DM.	h.	S.	Z.	P.
115	41 Aurigae	6h 0m5	48°45′	5.8	5.8	5.8	5.8	—	—	—	5.88
116	42 Aurigae	6.8	46 27	6.5	5.8	5.8	6.4	—	—	—	6.37
117	43 Aurigae	7.5	46 23	6.5			6.4	—	—	—	6.17
118	ψ^1 Aurigae	13.7	49 21	5.3	4.9	5.2	5.8	—	—	—	4.74
119	ψ^3 Aurigae	28.7	40 1	5.9	5.8	5.8	6.0	—	—	—	5.42
120	ψ^2 Aurigae	29.0	42 37	5.3	4.9	5.2	4.7	—	—	—	4.85
121	ψ^4 Aurigae	32.5	44 39	4.8	4.9	5.2	5.4	—	—	—	4.90
122	ψ^5 Aurigae	36.3	43 43	5.7	5.8	5.4	5.5	—	—	—	5.16
123	ψ^6 Aurigae	36.6	48 55	5.7	5.8	5.8	5.4	—	—	—	5.20
124	ψ^7 Aurigae	40.5	41 57	5.3	4.9	4.9	5.1	—	—	—	4.96
125	130 Ph Aurigae	44.6	46 0	—	—	6.3	6.7	—	—	—	6.02
126	ψ^9 Aurigae	45.8	46 28	—	5.8	5.8	6.1	—	—	—	5.80
127	ψ^{10} Aurigae	47.0	45 19	—	4.9	5.2	5.8	—	—	—	4.98
128	16 Ph Lyncis	7 5.1	47 31	—	5.8	5.8	5.5	—	—	—	5.51
129	18 Ph Lyncis	7.5	49 42	—	5.8	6.3	5.2	—	—	—	5.01
130	64 Aurigae	8.0	41 7	5.7	5.8	5.8	5.8	—	—	—	5.88
131	66 Aurigae	14.1	40 57	5.6	5.8	5.8	5.6	—	—	—	5.26
132	21 Lyncis	15.8	49 30	6.1	4.9	4.9	4.4	—	—	—	4.65
133	23 Ph Lyncis	18.1	48 29	—	5.8	6.3	5.8	—	—	—	5.73
134	22 Lyncis	19.0	49 57	5.7	5.8	5.8	5.8	—	—	—	5.28:
135	26 Ph Lyncis	26.0	46 30	—	5.8	5.8	5.8	—	—	—	5.53
136	27 Ph Lyncis	27.1	49 5	—	—	6.3	6.2	—	—	—	5.79:
137	32 Ph Lyncis	30.5	48 28	—	5.8	6.3	5.6	—	—	—	5.67
138	25 Lyncis	43.9	47 45	6.0	—	—	6.0	—	—	—	6.11
139	26 Lyncis	44.2	47 56	5.2	5.8	5.8	5.4	—	—	—	5.46
140	31 Lyncis	8 12.9	43 39	4.5	4.9	4.9	4.9	—	—	—	4.26
141	34 Lyncis	31.1	46 21	5.4	5.8	5.8	5.5	—	—	—	5.33
142	35 Lyncis	42.2	44 17	5.2	5.8	5.8	5.4	—	—	—	5.71
143	75 Ph Lyncis	45.0	44 8	—	—	6.3	7.7	—	—	—	5.76:
144	76 Ph Lyncis	47.0	46 12	—	5.8	5.8	6.4	—	—	—	5.64
145	ι Ursae majoris	49.3	48 36	—	3.2	3.1	3.3	—	3.13	3.10	3.15
146	10 Ursae majoris	51.2	42 21	4.6	4.1	4.1	4.2	—	—	3.94	4.02
147	x Ursae majoris	53.7	47 44	3.7	3.5	3.5	3.7	—	3.44	3.66	3.56
148	25 Ph Ursae majoris	55.4	49 6	—	—	5.8	6.4	—	—	—	5.43
149	28 Ph Ursae majoris	57.3	39 2	—	4.9	4.6	4.6	—	—	—	4.64
150	36 Lyncis	9 4.3	43 49	5.2	4.9	4.9	4.9	—	—	—	5.45
151	38 Ph Ursae majoris	7.8	47 26	—	5.8	6.3	6.4	—	—	—	5.94
152	38 Lyncis	9.8	37 26	3.9	4.1	4.1	4.2	—	3.87	3.89	3.83
153	46 Ph Ursae majoris	19.2	46 14	5.4	5.8	5.4	5.4	—	—	—	5.39
154	DM. 45°1728	21.5	45 19	—	—	—	7.4	—	—	—	6.80:
155	θ Ursae majoris	23.1	52 21	3.2	3.2	3.1	3.1	—	3.24	3.22	3.28
156	26 Ursae majoris	24.9	52 42	—	4.9	4.9	4.9	—	—	—	4.77
157	85 Ph Lyncis	26.0	40 16	—	4.9	5.2	4.6	—	—	—	4.82
158	42 Lyncis	29.3	40 54	5.2	5.8	5.8	5.4	—	—	—	5.34
159	43 Lyncis	33.0	40 25	5.7	5.8	5.8	5.6	—	—	—	5.57
160	65 Ph Ursae majoris	39.2	46 42	5.5	4.9	5.4	5.7	—	—	—	5.00
161	31 Ursae majoris	46.2	50 30	—	4.9	5.4	5.4	—	—	—	5.37
162	19 Leonis minoris	48.8	41 45	—	4.9	5.2	5.1	—	—	—	5.23

No.	Name.	RA.	Dec.	H.	A.	♄.	DM.	h.	☉.	Z.	P.
163	λ Ursae majoris .	10h 8m3	43°40′	—	3.5.	3.5	3.7	—	3.22	3.51	3.66
164	DM. 43°2007 . .	9.8	43 47	—	—	—	6.9	—	—	—	6.64:
165	84♄ Ursae majoris	10.4	49 7	—	—	6.3	5.8	—	—	—	6.11:
166	25 Leonis minoris	12.4	42 34	—	—	—	6.6	—	—	—	6.77:
167	DM. 41°2076 . .	13.6	41 57	—	—	—	6.4	—	—	—	5.85
168	μ Ursae majoris .	13.7	42 14	—	3.2	3.1	3.3	3.1:	3.33	2.91	3.10
169*	89♄ Ursae majoris	18.9	42 20	—	—	6.3	6.9	—	—	—	5.76:
170	DM. 49°1960 . .	19.1	49 42	—	—	—	6.5	—	—	—	6.88
171	90♄ Ursae majoris	19.1	49 33	—	5.8	5.8	6.1	—	—	—	6.54
172	DM. 41°2093 . .	19.2	41 57	—	—	—	7.4	—	—	—	5.73:
173	31 Leonis minoris	19.5	37 26	—	4.4	4.4	4.0	—	—	—	4.34
174	DM. 41°2097 . .	21.6	41 13	—	—	—	7.4	—	—	—	6.60:
175	32 Leonis minoris	21.7	39 40	—	5.8	5.8	6.1	—	—	—	5.94
176	95♄ Ursae majoris	24.8	41 9	—	4.9	4.9	5.1	—	—	—	4.89
177	35 Leonis minoris	28.0	37 5	—	5.8	5.8	6.1	—	—	7.04	5.68:
178	38 Leonis minoris	30.8	38 39	—	5.8	5.4	5.5	—	—	6.39	5.98
179	105♄ Ursae majoris	35.0	46 58	—	4.9	4.9	5.1	—	—	—	5.36
180	DM. 46°1659 . .	36.8	46 19	—	—	—	6.9	—	—	—	7.52:
181	ω Ursae majoris .	45.6	43 58	—	4.9	4.9	5.1	—	—	—	4.89:
182	DM. 42°2162 . .	48.0	42 47	—	—	—	6.1	—	—	—	6.02:
183	47 Ursae majoris .	51.3	41 12	—	4.9	4.9	4.7	—	—	—	5.10
184	121♄ Ursae majoris	51.9	46 17	—	5.8	5.8	5.8	—	—	—	5.56
185	122♄ Ursae majoris	52.1	43 41	—	5.8	5.8	6.1	—	—	—	6.08
186	DM. 43°2069 . .	52.5	43 30	—	—	—	6.9	—	—	—	6.63:
187	49 Ursae majoris .	52.7	39 59	—	4.9	4.9	4.7	—	—	—	5.14
188	51 Ursae majoris .	56.5	39 1	—	5.8	5.4	6.1	—	—	—	6.10:
189	130♄ Ursae majoris	11 1.5	43 59	—	—	6.3	6.4	—	—	—	5.89:
190	ψ Ursae majoris .	1.5	45 18	—	3.2	3.5	3.7	3.16	3.27	2.97	3.02
191	136♄ Ursae majoris	8.5	50 16	—	5.8	5.8	5.8	—	—	—	5.56:
192	56 Ursae majoris .	14.9	44 17	—	5.8	5.8	5.2	—	—	—	5.03
193	DM. 40°2432 . .	20.8	40 6	—	—	—	8.0	—	—	—	7.19:
194	57 Ursae majoris .	21.3	40 8	—	4.9	4.9	4.9	—	—	—	5.40
195	58 Ursae majoris .	22.7	43 59	—	5.8	5.8	6.1	—	—	—	5.98
196	158♄ Ursae majoris	30.0	51 26	—	5.8	6.3	5.8	—	—	—	6.35:
197	59 Ursae majoris .	30.6	44 25	—	5.8	5.8	5.8	—	—	—	5.56
198	60 Ursae majoris .	30.7	47 38	—	—	5.8	6.0	—	—	—	6.32:
199	χ Ursae majoris .	38.4	48 34	—	4.1	3.8	4.0	—	—	3.71	3.74
200	γ Ursae majorrs .	46.2	54 30	—	2.6	2.6	2.7	2.42	2.34	2.45	2.59
201	65 Ursae majoris .	47.5	47 18	—	—	5.8	7.2	—	—	—	6.93:
202*	DM. 47°1914 . .	47.6	47 17	—			7.4	—	—	—	—
203	DM. 41°2251 . .	49.4	41 10	—	—	—	8.4	—	—	—	6.58:
204*	DM. 41°2252 . .	49.6	41 5	—	—	—	6.9	—	—	—	6.97:
205	179♄ Ursae majoris	49.8	41 10	—	5.8	5.8	6.4	—	—	—	6.56:
206*	DM. 40°2485 . .	51.1	41 0	—	—	—	7.8	—	—	—	6.89:
207	67 Ursae majoris .	54.8	43 51	—	4.9	5.2	5.0	—	—	—	5.18
208	DM. 43°2182 . .	55.1	43 55	—	—	—	6.6	—	—	—	6.60:

169, 206. Some uncertainty about the identification.
202. Not observed separately: mag. of 201 + 202 = 6.39:
204. Perhaps 179 ♄ includes both this star and 205.

No.	Name.	RA.	Dec.	H.	A.	♄.	DM.	h.	☉.	Z.	P.
209	DM. 49°2110 . .	11h 58m8	49°58′	—	—	—	6.9	—	—	—	6.82:
210	2 Canum venat. .	12 8.9	41 28	—	5.8	5.2	6.4	—	—	—	5.55
211	3 Canum venat. .	12.7	49 47	—	5.8	5.2	5.5	—	—	—	5.60
212	DM. 49°2132 . .	15.5	49 9	—	—	—	7.2	—	—	—	6.87:
213	4 Canum venat .	16.7	43 21	—	5.8	5.8	6.1	—	—	—	5.96
214	DM. 43°2221 . .	17.9	43 40	—	—	—	6.9	—	—	—	6.54:
215	6 Canum venat. .	18.7	39 50	—	5.2	5.2	5.3	—	—	5.28	5.02
216	11♄ Canum venat.	20.4	42 10	—	5.8	5.8	6.4	—	—	—	6.38
217	8 Canum venat. .	26.9	42 8	—	4.4	4.4	4.4	—	—	—	4.54
218	9 Canum venat. .	31.8	41 40	—	5.8	5.8	6.9	—	—	—	6.23:
219	21♄ Canum venat.	35.4	46 40	—	—	6.3	6.7	—	—	—	6.80:
220	23♄ Canum venat.	37.6	44 54	—	5.8	5.8	6.0	—	—	—	6.22
221	10 Canum venat.	38.1	40 4	—	5.8	5.4	6.0	—	—	—	5.93
222	26♄ Canum venat.	38.3	46 13	—	5.2	5.2	5.4	—	—	—	5.13
223	11 Canum venat.	42.0	49 16	—	5.8	5.8	5.8	—	—	—	6.25
224	29♄ Canum venat.	43.0	49 33	—	—	6.3	6.4	—	—	—	6.42
225	30♄ Canum venat.	43.3	38 19	—	5.8	5.2	5.8	—	—	—	5.93
226	32♄ Canum venat.	48.3	47 59	—	5.8	5.8	5.6	—	—	—	5.66
227	12 Canum venat.	49.2	39 6	—	3.2	2.8	3.1	3.01	2.69	2.97	3.22
228	36♄ Canum venat.	50.5	46 57	—	5.8	5.8	6.4	—	—	—	6.02
229*	40♄ Canum venat.	57.3	43 47	—	—	5.8	5.8	—	—	—	6.82:
230	42♄ Canum venat.	59.4	46 3	—	5.8	5.8	5.8	—	—	—	5.58
231	15 Canum venat.	13 3.0	39 19	—	4.9	4.9	6.1	—	—	—	6.31
232	17 Canum venat.	3.4	39 16	—			6.1	—	—	—	6.08
233	49♄ Canum venat.	7.1	40 56	—	4.9	4.9	4.9	—	—	5.21	5.08
234	19 Canum venat.	9.1	41 37	—	5.8	5.2	5.8	—	—	—	5.98
235	20 Canum venat.	11.1	41 20	—	4.6	4.6	4.3	—	—	4.94	4.96
236	21 Canum venat.	12.1	50 27	—	4.9	4.9	4.9	—	—	—	5.33
237	23 Canum venat.	13.8	40 56	—	5.3	5.4	5.4	—	—	—	5.74
238	56♄ Canum venat.	14.5	44 45	—	—	6.3	6.4	—	—	—	6.54:
239	57♄ Canum venat.	15.7	44 40	—	—	5.8	6.4	—	—	—	6.36
240	58♄ Canum venat.	20.1	46 47	—	5.8	5.8	5.8	—	—	—	5.95
241	59♄ Canum venat.	22.1	41 29	—	5.8	5.8	6.1	—	—	—	6.58
242	DM. 42°2403 . .	22.8	42 59	—	—	—	7.1	—	—	—	7.78:
243	214♄ Ursae majoris	22.8	51 29	—	5.8	5.8	6.6	—	—	—	6.39:
244	60♄ Canum venat.	25.0	42 51	—	5.8	5.8	6.1	—	—	—	6.13
245	62♄ Canum venat.	28.0	39 33	—	5.8	6.3	6.1	—	—	—	6.60
246	24 Canum venat.	28.5	49 46	—	4.9	4.9	4.7	—	—	—	4.86
247	65♄ Canum venat.	29.1	44 56	—	5.8	5.8	6.4	—	—	—	6.88
248	66♄ Canum venat.	30.7	50 14	—	—	6.3	6.4	—	—	—	6.37:
249	72♄ Canum venat.	36.3	42 24	—	5.8	6.3	5.8	—	—	—	6.37
250	73♄ Canum venat.	37.4	46 15	—	5.8	6.3	6.4	—	—	—	6.74
251	DM. 41°2423 . .	39.9	41 46	—	—	—	7.7	—	—	—	7.96:
252	DM. 39°2678 . .	40.0	39 14	—	—	—	5.8	—	—	—	6.11
253	75♄ Canum venat.	40.1	41 49	—	5.3	5.4	5.5	—	—	—	6.24
254	76♄ Canum venat.	40.8	39 18	—	5.2	5.2	5.4	—	—	—	5.76
255	η Ursae majoris .	41.8	50 3	—	2.1	2.1	2.0	1.82	1.69	1.89	2.01

229. Some uncertainty about the identification.

No.	Name.	RA.	Dec.	M.	A.	Ş.	DM.	h.	⊜.	Z.	P.
256	78 Ş Canum venat.	13h 42m 0	42° 46′	—	5.8	6.3	6.4	—	—	—	6.71
257	87 Ş Canum venat.	56.4	46 27	—	5.8	5.8	5.8	—	—	—	6.60:
258	22 Ş Bootae	57.6	51 41	—	—	6.3	5.8	—	—	—	6.41:
259	23 Ş Bootae	14 2.1	44 32	—	5.3	5.4	5.4	—	—	—	5.41
260	13 Bootae	2.9	50 9	5.2	5.8	5.8	5.4	—	—	—	5.41
261	DM. 43°2391	5.8	43 2	—	—	—	6.8	—	—	—	6.36
262	κ Bootae	8.3	52 27	4.3	4.4	4.4	4.4	—	—	4.57	4.77
263	32 Ş Bootae	8.5	42 13	—	5.8	5.8	6.0	—	—	—	6.43
264	34 Ş Bootae	10.5	40 25	—	5.8	5.8	6.1	—	—	—	6.44
265	λ Bootae	10.9	46 46	4.1	4.1	4.1	4.0	4.20	—	4.41	4.47
266	ι Bootae	11.1	52 2	4.3	4.4	4.4	4.4	—	—	4.88	4.99
267	DM. 51°1908	12.2	51 59	—	—	—	6.7	—	—	—	6.14:
268	42 Ş Bootae	13.9	39 29	—	5.8	5.8	5.8	—	—	—	6.18:
269	θ Bootae	20.3	52 31	4.1	3.8	4.1	4.0	—	—	4.15	4.47
270	*g* Bootae	23.6	50 30	5.7	5.8	5.8	5.6	—	—	—	5.91:
271	γ Bootae	26.2	38 57	3.0	2.8	3.5	2.8	3.24	3.18	3.23	3.36
272	69 Ş Bootae	29.6	50 1	—	5.8	5.8	5.5	—	—	—	6.15
273	33 Bootae	33.5	45 1	5.1	5.8	5.2	5.2	—	—	—	5.64
274	*h* Bootae	44.2	46 43	5.7	5.8	5.4	5.8	—	—	—	6.10
275	39 Bootae	44.8	49 20	5.1	5.8	5.8	5.6	—	—	—	5.94
276	112 Ş Bootae	55.7	47 51	—	—	5.8	6.1	—	—	—	6.71:
277	β Bootae	56.5	40 58	3.5	3.2	3.1	3.1	3.7:	3.60	3.60	3.68
278	*i* Bootae	59.0	48 13	4.8	4.9	4.6	4.4	—	—	—	5.03
279	*k* Bootae	15 0.6	48 43	5.1	4.9	5.2	5.4	—	—	—	6.02
280	134 Ş Bootae	17.3	40 6	—	5.8	5.8	5.4	—	—	—	5.65
281	μ Bootae	19.0	37 53	4.3	3.8	4.1	4.0	—	4.21	4.45	4.61
282	DM. 37°2637	19.0	37 51	—	—	—	6.9	—	—	—	6.56:
283	ν Bootae (pr.)	25.7	41 20	4.7	4.1	4.1	4.4	—	—	—	5.06
284	ν Bootae (sq.)	26.6	41 25	4.5			4.7	—	—	—	5.20
285	φ Bootae	32.6	40 50	4.9	4.9	4.9	4.9	—	—	—	5.33
286	1 Ş Herculis	33.6	47 17	—	5.8	5.4	5.4	—	—	—	5.97
287	χ Herculis	47.7	42 51	4.2	4.4	4.6	4.3	—	—	4.81	4.73
288	2 Herculis	49.8	43 34	4.9	5.8	5.4	5.4	—	—	—	5.42
289	4 Herculis	50.7	42 58	4.8	5.8	5.8	5.8	—	—	—	6.02
290	υ Herculis	58.3	46 26	4.2	4.4	4.6	4.2	—	—	4.88	4.88
291	φ Herculis	16 4.2	45 20	3.9	4.1	4.1	3.8	—	—	4.53	4.59
292	19 Ş Herculis	7.0	42 45	—	—	6.3	6.1	—	—	—	5.90:
293	23 Ş Herculis	15.0	40 2	—	5.8	5.8	5.4	—	—	—	5.68
294	τ Herculis	15.4	46 42	3.6	3.5	3.5	3.7	—	3.79	4.15	4.21
295	*g* Herculis	23.9	42 12	4.6	5.2	Var.	4.7	—	—	—	5.00:
296	40 Ş Herculis	27.4	45 56	—	5.8	5.8	5.5	—	—	—	5.96
297	σ Herculis	29.4	42 45	4.0	4.1	4.4	4.0	—	—	4.41	4.43
298	46 Ş Herculis	32.0	46 55	—	5.8	6.3	5.8	—	—	—	5.98
299	DM. 49°2530	34.6	49 9	—	—	—	7.2	—	—	—	6.37:
300	42 Herculis	34.8	49 13	4.5	5.2	4.9	4.9	—	—	—	4.86
301	η Herculis	37.9	39 11	3.2	3.2	3.5	3.1	—	3.71	3.66	3.65
302	52 Herculis	45.0	46 14	4.3	4.4	4.9	4.9	—	—	—	5.16
303	78 Ş Herculis	49.3	47 39	—	—	6.3	6.1	—	—	—	5.99:
304	95 Ş Herculis	1.0	49 1	—	—	6.3	5.6	—	—	—	6.27:

No.	Name.	RA.	Dec.	M.	A.	Ş.	DM.	h.	⊜.	Z.	P.
305	98 Ş Herculis	17h 3m 1	40° 42′	—	5.8	5.8	6.1	—	—	—	6.44
306	99 Ş Herculis	3.5	48 35	—	—	5.8	6.4	—	—	—	6.64:
307	95 Ş Draconis	4.7	51 2	—	—	6.3	6.4	—	—	—	6.64:
308	100 Ş Herculis	4.8	40 58	—	4.9	4.9	5.1	—	—	—	5.22
309	DM. 45°2504	7.5	45 31	—	—	—	6.1	—	—	—	6.74
310	DM. 49°2604	8.0	49 56	—	—	—	6.1	—	—	—	6.14
311	110 Ş Herculis	12.7	43 17	—	—	neb.	neb.	—	—	—	8.47:
312	112 Ş Herculis	13.1	49 52	—	5.8	5.8	7.1	—	—	—	not. obs.
313	118 Ş Herculis	15.7	45 26	—	—	6.3	6.4	—	—	—	6.53:
314	DM. 44°2695	16.2	44 21	—	—	—	6.4	—	—	—	6.74:
315	74 Herculis	16.3	46 22	5.9	5.3	5.4	5.4	—	—	—	5.61
316	120 Ş Herculis	16.6	48 20	—	—	6.3	6.4	—	—	—	6.46:
317	121 Ş Herculis	17.0	40 7	—	4.9	5.2	5.1	—	—	—	5.75
318	*x* Herculis	22.9	48 24	5.7	5.8	5.4	5.8	—	—	—	5.94
319	135 Ş Herculis	28.5	41 21	—	5.8	5.8	5.4	—	—	—	5.87
320	*y* Herculis	32.9	48 41	5.7	5.8	5.4	5.8	—	—	—	5.60
321	ι Herculis	35.4	46 5	3.8	3.5	3.5	4.0	3.55	3.98	4.28	4.21
322	DM. 41°2882	36.7	41 44	—	—	—	6.4	—	—	—	7.14:
323	156 Ş Herculis	41.1	38 57	—	5.8	6.3	5.8	—	—	—	6.52
324	157 Ş Herculis	41.2	39 23	—	—	6.3	5.8	—	—	—	6.36
325	164 Ş Herculis	43.2	47 41	—	5.8	5.4	6.1	—	—	—	6.62
326	DM. 47°2541	44.1	47 38	—	—	—	6.7	—	—	—	6.85:
327	*z* Herculis	46.3	48 27	5.9	5.8	5.8	6.4	—	—	—	6.84
328	DM. 40°3225	46.5	40 6	—	—	—	6.6	—	—	—	6.64:
329	168 Ş Herculis	47.4	40 1	—	—	6.3	6.1	—	—	—	5.94:
330	169 Ş Herculis	48.0	46 42	—	5.8	5.8	6.2	—	—	—	6.53
331	*f* Herculis	48.6	40 2	5.1	4.9	4.9	4.9	—	—	—	5.26
332	θ Herculis	51.3	37 16	3.8	4.1	4.1	3.7	—	4.23	3.96	3.64
333	DM. 45°2626	52.5	45 1	—	—	—	6.6	—	—	—	6.86:
334	DM. 45°2627	52.7	45 23	—	—	—	6.1	—	—	—	6.21:
335	DM. 45°2629	53.0	45 53	—	—	—	6.2	—	—	—	6.82:
336	179 Ş Herculis	54.7	45 28	—	5.8	5.8	6.2	—	—	—	6.62
337	182 Ş Herculis	55.8	45 31	—	5.8	6.3	5.8	—	—	—	5.80:
338	185 Ş Herculis	59.4	48 29	—	5.8	6.3	6.1	—	—	—	6.35
339	DM. 49°2728	18 2.5	49 28	—	—	—	7.2	—	—	—	6.66:
340	196 Ş Herculis	3.2	43 27	—	4.9	5.2	5.1	—	—	—	5.11:
341	197 Ş Herculis	3.5	49 41	—	5.8	6.3	6.4	—	—	—	6.41:
342	202 Ş Herculis	7.8	48 16	—	5.8	6.3	6.4	—	—	—	6.68
343	1 Ş Lyrae	8.1	41 7	—	—	5.8	5.6	—	—	—	6.19:
344	2 Ş Lyrae	8.3	38 46	—	5.8	5.8	5.8	—	—	—	6.09
345	3 Ş Lyrae	11.1	42 8	—	5.3	5.2	5.4	—	—	—	5.75
346	4 Ş Lyrae	12.5	40 53	—	5.8	5.8	5.8	—	—	—	6.17
347	DM. 49°2776	17.5	49 39	—	—	—	6.4	—	—	—	6.46:
348	143 Ş Draconis	17.8	49 3	—	4.9	4.9	5.0	—	—	—	5.01
349	μ Lyrae	19.5	39 26	5.5	5.2	4.9	5.0	—	—	—	5.20
350	ε Lyrae (bor.)	39.5	39 31	4.2	4.1	4.4	4.3	—	—	—	4.86
351	ε Lyrae (aust.)	39.6	39 28	4.2		4.6	4.5	—	—	—	4.82
352	ζ Lyrae (bor.)	39.8	37 28	4.1	4.4	4.4	4.4	—	—	4.32	5.28:
353	ζ Lyrae (aust.)	39.8	37 27	4.2			5.4	—	—		5.78:

No.	Name.	RA.	Dec.	H.	A.	Ṗ.	DM.	h.	S.	Z.	P.
354	25 Ṗ Lyrae	18h41m6	41°17′	—	5.8	5.8	5.8	—	—	—	6.13
355	164 Ṗ Draconis	44.4	48 36	—	5.8	5.8	5.8	—	—	—	6.20
356	31 Ṗ Lyrae	47.4	41 13	—	5.8	5.8	6.4	—	—	—	6.25
357	36 Ṗ Lyrae	50.2	41 25	—	5.8	5.8	5.4	—	—	—	5.62
358	R Lyrae	50.9	43 45	4.8	4.6	Var.	Var.	—	—	—	4.40:
359	171 Ṗ Draconis	51.0	48 41	—	5.8	5.8	5.4	—	—	—	5.83
360	43 Ṗ Lyrae	54.0	40 29	—	5.8	5.8	6.4	—	—	—	6.22
361	16 Lyrae	57.4	46 44	5.3	4.9	5.2	5.4	—	—	—	5.14
362	52 Ṗ Lyrae	19 1.6	41 12	—	5.8	5.8	6.1	—	—	—	6.42
363	DM. 47°2779	8.2	47 8	—	—	—	6.4	—	—	—	6.38:
364	1 Ṗ Cygni	8.3	49 35	—	5.8	5.8	6.1	—	—	—	6.02
365	η Lyrae	8.8	38 54	4.6	4.4	4.4	4.4	—	—	4.64	4.46
366	θ Lyrae	11.4	37 53	4.7	4.4	4.4	4.5	—	—	4.54	4.25
367	2 Ṗ Cygni	12.7	46 44	—	5.8	5.8	6.0	—	—	—	5.98
368	DM. 49°2977	14.8	49 18	—	—	—	6.1	—	—	—	6.15:
369	69 Ṗ Lyrae	19.4	43 6	—	5.8	5.4	5.4	—	—	—	5.85
370	DM. 44°3133	21.5	44 39	—	—	—	6.1	—	—	—	6.82:
371	7 Ṗ Cygni	21.6	44 43	—	5.8	6.3	6.7	—	—	—	7.24:
372	17 Ṗ Cygni	30.0	42 5	—	5.8	5.4	5.6	—	—	—	5.43
373	20 Ṗ Cygni	32.1	44 23	—	5.8	5.4	5.2	—	—	—	5.26
374	θ Cygni	32.6	49.53	5.0	4.6	4.6	4.7	—	—	5.00	4.70
375	14 Cygni	34.7	42 29	5.2	5.3	5.4	5.8	—	—	—	5.44
376	26 Ṗ Cygni	36.4	45 11	—	5.4	5.4	5.4	—	—	—	5.23
377	27 Ṗ Cygni	37.0	39 55	—	5.8	5.8	6.1	—	—	—	6.25
378	DM. 41°3469	38.9	41 26	—	—	—	5.8	—	—	—	5.61:
379	δ Cygni	40.5	44 46	2.9	3.2	2.8	3.1	3.03	2.87	2.94	3.05
380	34 Ṗ Cygni	43.2	47 34	—	—	5.8	5.6	—	—	—	6.10
381	37 Ṗ Cygni	45.7	40 14	—	5.8	5.8	5.4	—	—	—	5.78
382	DM. 47°2937	47.0	47 2	—	—	—	6.1	—	—	—	6.44:
383	39 Ṗ Cygni	47.6	46 40	—	5.8	5.8	5.6	—	—	—	5.78
384	40 Ṗ Cygni	47.9	47 35	—	5.8	5.8	5.6	—	—	—	6.01
385	DM. 47°2945	48.8	48 27	—	—	—	6.7	—	—	—	6.32
386	49 Ṗ Cygni	52.2	39 59	—	5.3	5.4	5.6	—	—	—	5.61
387	52 Ṗ Cygni	53.1	41 52	—	5.8	5.8	6.4	—	—	—	6.57
388	56 Ṗ Cygni	54.8	45 23	—	5.8	5.8	5.6	—	—	—	5.92
389	*e* Cygni	57.3	49 42	5.3	5.3	5.2	5.0	—	—	—	5.16
390	61 Ṗ Cygni	20 0.2	47 49	—	5.8	5.8	6.1	—	—	—	6.23:
391	DM. 47°3037	6.2	47 48	—	—	—	6.2	—	—	—	6.56:
392	o_1 Cygni (pr.)	8.8	46 23	5.5	4.1	4.4	4.9	—	—	—	5.15
393	o_1 Cygni (sq.)	9.1	46 18	4.5			4.0	—	—	4.20	4.00
394	o_2 Cygni	11.0	47 16	4.8	4.4	4.6	4.9	—	—	4.35	4.13
395	76 Ṗ Cygni	11.3	45 8	—	5.8	5.8	6.1	—	—	—	6.02
396	77 Ṗ Cygni	11.8	39 55	—	5.8	5.4	5.3	—	—	—	5.29
397	80 Ṗ Cygni	13.0	40 17	—	—	6.3	6.1	—	—	—	6.02
398	DM. 45°3139	14.2	45 52	—	—	—	6.5	—	—	—	6.46:
399	84 Ṗ Cygni	15.3	46 23	—	5.8	5.8	6.4	—	—	—	6.45
400	DM. 40°4136	16.9	40 40	—	—	—	6.4	—	—	—	6.38
401	γ Cygni	17.0	39 47	2.2	2.6	2.6	2.8	2.33	2.42	2.09	2.64
402	86 Ṗ Cygni	17.4	45 19	—	5.8	5.8	5.6	—	—	—	5.66

No.	Name.	RA.	Dec.	H.	A.	Ṗ.	DM.	h.	S.	Z.	P.
403	87 Ṗ Cygni	20h17m6	40°34′	—	5.8	5.8	5.8	—	—	—	5.83
404	ω_1 Cygni	22.6	48 54	5.7	—	5.8	5.8	—	—	—	5.88
405	ω_2 Cygni (pr.)	25.6	48 28	5.2	4.9	4.9	4.7	—	—	—	5.28
406	ω_2 Cygni (sq.)	26.8	48 44	5.5	5.3	5.4	5.7	—	—	—	5.53
407	104 Ṗ Cygni	29.2	46 12	—	5.8	5.8	5.8	—	—	—	5.99
408	109 Ṗ Cygni	34.3	40 4	—	5.8	5.8	5.8	—	—	—	6.12
409	110 Ṗ Cygni	35.0	42 56	—	5.8	5.8	5.8	—	—	—	5.90
410	α Cygni	36.5	44 46	1.6	1.7	1.5	1.5	1.49	1.28	1.46	1.82
411	114 Ṗ Cygni	36.7	41 12	—	5.8	5.4	5.2	—	—	—	5.72
412	51 Cygni	37.7	49 49	5.3	5.3	5.2	5.5	—	—	—	5.76
413	118 Ṗ Cygni	39.8	46 46	—	5.8	5.8	6.4	—	—	—	6.99:
414	123 Ṗ Cygni	43.1	47 18	—	—	6.3	5.4	—	—	—	5.71:
415	55 Cygni	44.0	45 35	5.1	5.3	5.4	5.4	—	—	—	5.06
416	56 Cygni	45.0	43 31	5.1	5.2	5.4	5.4	—	—	—	5.17
417	57 Cygni	48.1	43 50	4.8	5.2	5.2	5.3	—	—	—	5.03
418	132 Ṗ Cygni	48.3	44 38	—	5.8	5.8	6.1	—	—	—	5.47
419	134 Ṗ Cygni	50.9	46 52	—	5.8	6.3	5.8	—	—	—	5.89
420	DM. 44°3639	51.5	44 23	—	—	—	6.6	—	—	—	5.99:
421	ν Cygni	51.8	40 37	4.7	4.1	4.1	4.0	—	—	—	4.21
422	139 Ṗ Cygni	53.1	43 54	—	5.8	5.4	5.8	—	—	—	5.58
423	140 Ṗ Cygni	53.9	49 54	—	5.8	5.4	5.4	—	—	—	5.70:
424	f_1 Cygni	54.9	46 57	5.2	5.2	5.2	5.2	—	—	—	5.43
425	60 Cygni	56.1	45 35	5.6	5.8	5.2	5.5	—	—	—	5.57
426	ξ Cygni	59.7	43 21	4.2	4.1	4.1	4.0	—	—	—	4.12
427	f_2 Cygni	21 1.6	47 4	5.1	5.2	5.2	4.5	—	—	—	4.57
428	τ Cygni	9.0	37 25	4.0	4.1	4.1	4.0	—	—	3.82	4.04
429	σ Cygni	11.7	38 47	4.7	4.4	4.4	4.4	—	—	4.48	4.43
430	*A* Cygni	11.9	42 5	—	5.8	5.8	6.2	—	—	—	6.31
431	68 Cygni	13.1	43 20	5.7	4.9	4.9	5.2	—	—	—	5.31
432	162 Ṗ Cygni	13.3	40 26	—	—	6.3	6.4	—	—	—	6.25
433	164 Ṗ Cygni	14.5	48 54	—	5.2	5.4	5.3	—	—	—	5.81
434	168 Ṗ Cygni	17.0	48 46	—	5.3	5.8	5.4	—	—	—	5.96
435	171 Ṗ Cygni	20.0	46 5	—	5.8	5.8	5.8	—	—	—	5.88
436	173 Ṗ Cygni	21.7	48 12	—	4.9	5.2	5.4	—	—	—	5.53
437	*g* Cygni	24.1	45 54	5.1	4.9	4.9	5.3	—	—	—	5.46
438	180 Ṗ Cygni	27.8	49 18	—	—	5.8	5.8	—	—	—	6.09:
439	ρ Cygni	28.5	44 57	4.0	4.4	4.1	4.2	—	—	4.12	4.24
440	Following ρ Cygni	—	—	—	—	—	—	—	—	—	5.35:
441	74 Cygni	31.1	39 46	5.0	4.9	4.9	4.9	—	—	—	5.26
442	75 Cygni	34.5	42 37	5.0	5.3	5.4	5.1	—	—	—	5.20
443	76 Cygni	35.8	40 9	5.9	—	—	6.1	—	—	—	6.09
444	77 Cygni	36.6	40 25	5.6	—	—	5.7	—	—	—	5.89
445	189 Ṗ Cygni	37.3	40 30	—	5.2	5.4	5.0	—	—	—	5 47
446	π_2 Cygni	41.4	48 38	4.6	4.4	4.4	4.4	—	—	—	4.52
447	195 Ṗ Cygni	43.8	40 28	—	5.8	5.8	6.1	—	—	—	6.67:
448	1 Ṗ Lacertae	57.1	43 57	—	5.8	5.4	5.8	—	—	—	5.88
449	2 Ṗ Lacertae	22 0.2	44 19	—	5.3	5.4	4.6	—	—	—	5.35
450	DM. 45°3813	2.8	45 2	—	—	—	6.1	—	—	—	6.77:

No.	Name.	RA.	Dec.	H.	A.	Ф.	DM.	h.	G.	Z.	P.
451	6 Ф Lacertae	22h 7m9	44°43′	—	5.8	5.8	5.8	—	—	—	5.89
452	2 Lacertae	15.0	45 48	4.7	4.6	4.6	4.9	—	—	—	4.91
453	4 Lacertae	18.6	48 45	4.9	4.9	4.9	4.9	—	—	—	4.95
454	5 Lacertae	23.5	46 58	4.9	4.9	4.9	4.7	—	—	—	4.60
455	6 Lacertae	24.2	42 23	4.9	4.9	4.9	4.9	—	—	—	4.97
456	7 Lacertae	25.3	49 32	4.4	4.1	4.1	4.0	—	—	—	4.13
457	11 Lacertae	34.2	43 31	5.1	4.9	4.9	4.7	—	—	—	4.63
458	13 Lacertae	37.7	41 4	5.3	5.8	5.8	5.1	—	—	—	5.38
459	14 Lacertae	43.8	41 11	5.7	5.8	5.8	6.0	—	—	—	6.11
460	15 Lacertae	45.5	42 31	5.3	5.8	5.4	4.9	—	—	—	5.15
461	40 Ф Lacertae	47.2	43 59	—	5.8	6.3	5.8	—	—	—	6.07
462	16 Lacertae	49.8	40 51	5.7	5.8	5.4	6.1	—	—	—	5.87
463	45 Ф Lacertae	50.1	48 58	—	5.8	5.4	4.5	—	—	—	5.16:
464	46 Ф Lacertae	50.7	47 55	—	5.8	5.2	5.2	—	—	—	5.42:
465	o Andromedae	55.3	41 33	3.7	3.8	3.8	3.8	—	—	3.72	3.86
466	2 Andromedae	56.0	41 58	5.6	5.8	5.8	6.1	—	—	—	5.46
467	3 Andromedae	57.7	49 16	5.1	5.2	4.9	4.7	—	—	—	4.73
468	4 Andromedae	23 1.0	45 36	5.9	5.8	5.8	5.4	—	—	—	5.42
469	5 Andromedae	1.2	48 31	4.6	4.9	4.9	4.9	—	—	—	5.74:
470	6 Andromedae	3.8	42 47	6.1	—	6.3	5.8	—	—	—	6.02
471*	DM. 44°4347	4.2	44 44	—	—	—	7.2	—	—	—	6.54:
472	7 Andromedae	6.0	48 37	4.6	4.9	4.9	4.9	—	—	—	4.77
473	11 Ф Andromedae	10.5	44 24	—	5.8	5.8	6.1	—	—	—	6.57:
474	8 Andromedae	11.1	48 13	5.1	5.2	5.2	4.7	—	—	—	4.92
475	DM. 44°4373	11.1	44 42	—	—	—	6.4	—	—	—	6.41:
476	9 Andromedae	11.5	40 59	6.4	5.8	5.8	6.1	—	—	—	6.21
477	DM. 47°4107	12.2	47 42	—	—	—	7.4	—	—	—	7.30:
478	DM. 44°4378	12.2	44 21	—	—	—	6.7	—	—	—	6.65:
479	11 Andromedae	12.7	47 50	5.4	5.8	5.8	5.8	—	—	—	5.55
480	DM. 47°4114	12.9	47 36	—	—	—	6.4	—	—	—	6.38:
481	10 Andromedae	13.0	41 16	6.4	—	5.8	5.7	—	—	—	5.95
482	13 Andromedae	20.2	42 7	5.8	5.8	5.8	6.1	—	—	—	6.01
483	25 Ф Andromedae	30.5	43 37	—	—	6.3	6.1	—	—	—	5.94:
484	λ Andromedae	30.5	45 41	4.1	4.1	4.1	3.7	—	—	3.85	3.89
485	DM. 48°4112	30.9	48 12	—	—	—	7.3	—	—	—	6.40:
486	ι Andromedae	31.1	42 28	4.6	4.1	4.1	4.2	—	—	4.41	4.44
487	18 Andromedae	32.1	49 40	5.1	5.3	5.4	5.5	—	—	—	5.53
488	x Andromedae	33.3	43 31	4.6	4.1	4.4	4.6	—	—	4.28	4.36
489	ψ Andromedae	38.9	45 37	5.1	4.9	5.4	5.1	—	—	—	5.07
490	34 Ф Andromedae	40.4	46 1	—	—	6.3	6.4	—	—	—	6.24
491	39 Ф Andromedae	49.7	41 51	—	5.8	5.8	6.1	—	—	—	6.32
492	42 Ф Andromedae	54.3	41 33	—	5.8	5.8	6.4	—	—	—	6.52
493	DM. 41°4925	55.7	41 56	—	—	—	6.7	—	—	—	6.73:
494	DM. 41°4933	57.2	41 18	—	—	—	5.8	—	—	—	6.37

471. Some uncertainty about the identification.

A number of rejections of observations have been made in taking the differences of the magnitudes of stars from the means of their groups, which, having been made after the calculations of the differences of the means of the groups, do not affect these differences. The following table will give some idea of what effect these rejections would have had.

Groups.	Number of set.	Obs. before rejection.	Obs. after rejection.	Calc.	No. of rounds.	Remarks.
IX—XI	(105)	+ 0.22	+ 0.25	+ 0.20	2	
XI—XII	(186)	+ 0.72	+ 0.63	+ 0.80	2	Clouds.
XVIII—XIX	(122)	+ 0.20	+ 0.18	+ 0.26	3	Posture very uncomfortable.
XVIII—XX	(48)	− 0.02	+ 0.01	+ 0.18	1	Hazy, windy, altitude low. Comp. not used.
XXIII—XXIV	(15)	− 1.80	− 1.89	− 1.90	3	Only night when stars were observed on their own color.
XLII—XLIV	(74)	+ 0.37	+ 0.32	+ 0.30	1	Strong wind; lamp blown out.
LVI—LXVIII	(46)	+ 0.16	+ 0.27	+ 0.28	4	Great changes in brightness of lamp.
LVIII—LX	(79)	− 0.31	− 0.20	− 0.07	1	Bad seeing, lamp blown. Comp. not used.
LXI—LXIII	(90)	+ 0.60	+ 0.45	+ 0.37	2	Very windy. Effect of pinhole correction. Comp. not used.
LXVII—LXIX	(103)	− 0.97	− 1.07	− 1.08	3	3 Ф *Persei* rej. Possibly variable. Obs. good.
I—XXI	(177)	− 1.49	− 1.61	− 1.61	2	Presence of clouds suspected.
XXIII—XLI	(62)	− 3.16	− 3.02	− 3.11	2	*z Herculis* rej. Suspected variable.
XI—XXXI	(117)	+ 0.52	+ 0.40	+ 0.71	1	Wrong star observed.
LI—I	(155)	+ 1.34	+ 1.29	+ 1.33	2	Obs. of rejected star are very discordant.

The following stars were observed with more than one group and we have therefore different independent results for their magnitudes.

Name of star.	Group.	No. of nights.	Mag.	Group.	No. of nights.	Mag.	Adopted mag.
42 Aurigae	XIII	4	6.41	Extra XII	1	6.20:	6.37
43 Aurigae	XIII	4	6.17	Extra XII	1	6.20:	6.18
ψ_1 Aurigae	XIII	4	4.76	Extra XII	1	4.66:	4.74
35 Lyncis	XVI	4	5.18	Extra XVII	2	5.15:	5.17
105 Ф Ursae majoris	XXI	7	5.36	Extra XX	1 rej.	4.95:	5.36
121 Ф Ursae majoris	XXI	7	5.53	Extra XX	1 rej.	5.09:	5.53
χ Ursae majoris	XXIII	7	3.74	Extra XX	1 rej.	3.10:	3.74
59 Ф Canum venat.	XXX	6	6.58	Extra XXIX	1 rej.	6.13:	6.58
g Herculis	Extra XXXVI	4	5.00:	Extra XXXVIII	1	4.99:	5.00:
182 Ф Herculis	Extra XLI	5	5.76:	Extra XLIII	1	5.98:	5.80:
17 Ф Cygni	XLVIII	4	5.42	XLVI	3	5.44	5.43
14 Cygni	XLVIII	4	5.38	XLVI	3	5.52	5.44
37 Ф Cygni	XLVIII	4	5.75	XLVI	3	5.81	5.78
49 Ф Cygni	XLVIII	4	5.60	Extra XLVI	2	5.66:	5.61
52 Ф Cygni	XLVIII	4	6.61	Extra XLVI	1	6.39:	6.57

The following remarks relate to known or suspected variables.

10. ξ *Cassiepeae*. Argelander makes this star 5.8, and Heis, 4.6. Wm. Herschel makes it 0.4 fainter than o; Argelander, 0.9 fainter; Heis, 0.3 brighter; the DM., 0.3 fainter; and I, 0.1 fainter. 1876, Apr. 8, it seems about equal to o.

11. π *Cassiepeae*. Argelander makes this star 5.8, and the DM., 4.4. Wm. Herschel makes it 0.2 fainter than o; Argelander, 0.9 fainter; Heis, equal; the DM., 0.3 fainter; and I, 0.3 fainter. 1876, Apr. 8, I find it but little than o.

53. β *Persei*. This variable was always observed near its maximum.

88. 121 Ḫ *Persei*. The absence of this star from the Uranometria, and the great difference between the magnitude assigned to it by Heis and me (5.8) and that of the *Durchmusterung* (7.7) certainly creates a suspicion of variability.

98. ε *Aurigae*. An irregular variable.

129. 18 Ḫ *Lyncis*. Heis makes this star 6.3, and I, 5.0. Argelander makes it equal to 26 *Lyncis*; Heis makes it 0.5 fainter; the DM., 0.2 brighter; and I, 0.4 brighter. Argelander makes it equal to 16 Ḫ *Lyncis*, Heis makes it 0.5 fainter; the DM., 0.3 brighter; and I, 0.5 brighter. 1876, April 8, it seems not to have changed since my measures.

190. ψ *Ursae majoris*. Ptolemy made this star 4.5 and 1.2 fainter than μ, which it now equals. 1876, April 8, I find $\mu > \psi > \theta$. The star is orange.

201. 65 *Ursae majoris*. I have rejected this star from the mean of its group, not because I think it variable but because its near companion makes accordant measures very difficult to obtain.

218. 9 *Canum venaticorum*. The magnitudes of Argelander and Heis differ very much from that of the DM., and my measures are discordant. But I have excluded the star from the group, not so much on suspicion of variability, as because I do not feel sure that I always observed the same star, as there are three near together.

247. 65 Ḫ *Canum venaticorum*. Argelander and Heis make this 5.8; I, 6.9. They make it equal to 66 Ḫ; the DM. makes it 0.3 fainter; and I, 0.7 fainter. Argelander and the DM. make it equal to 73 Ḫ; Heis makes it 0.5 brighter; and I, 0.1 fainter.

255. η *Ursae majoris*. Two measures in June, 1873, make this star 0.5 brighter than in June, 1874. But unfortunately the values of the cap constants are so uncertain that nothing can be concluded.

268. 42 Ḫ *Bootae*. The configuration of stars in this neighborhood is perplexing, and possibly two different stars are confounded.

269. θ *Bootae*. The disagreement of different observers in reference to the relative magnitudes of θ and λ *Bootae* is noticeable. On 1875, July 27, at Munich, I found θ much the brighter.

276. 112 Ḫ *Bootae*. Although this faint star was measured every night, yet 73 Ḫ which is marked as 6.0 in the DM. entirely escaped observation. This was owing partly to its omission from the Uranometria and partly to its being near the junction of three groups.

279. k *Bootae*. Argelander makes this star 4.9; I, 6.0. Wm. Herschel makes it equal to 39; the DM., 0.2 brighter; Heis, 0.6 brighter; Argelander 0.9 brighter; and I, 0.1 fainter. 1875, July 27, at Munich, I inclined to think it brighter than 39. 1876, April 7, at Berlin, I found it a little brighter than 39, and made its magnitude by comparison with 39, *h*, and 134 Ḫ, to be 5.75.

295. g *Herculis*. An irregular variable.

312. 112 Ḫ *Herculis*. This I believe is the only star in the Uranometria within the assigned limits of declination which I never succeeded in observing. I have little doubt that the identification of the Argelander and Heis is wrong and that they really observed DM. 49° 2604.

327. z *Herculis*. I am confident that this star is variable. It is now of the 7th magnitude and very considerably fainter than 120 Ḫ or 148 Ḫ, which Heis and the DM. call 6.3 but which are, in

fact, hardly brighter than 6.6. Yet the star was seen by Ptolemy who makes it 5.6; in which he is supported by Sûfi and Ulugh. Tycho and Hevelius call it a nebula. Wm. Herschel makes it 5.9; Argelander and Heis, 5.8; the DM., 6.4; and I, 6.9. Wm. Herschel makes *z* 0.2 fainter than *y*; Argelander makes them equal; Heis makes *z* 0.4 fainter; the DM., 0.6 fainter; and I, 1.3 fainter. 1876, April 6, I find that my magnitudes of stars in this vicinity continue to represent them well.

By mistake, z *Herculis* has been retained in the group and 182 Ḫ, a star which there is no reason to suspect, has been excluded.

334. DM. 45° 2627. I believe this star to be variable. I found it the brightest of the group except 182 Ḫ *Herculis*, and yet neither the Uranometria now Heis has it. My measures show some discrepancies, even rejecting that of set (19), which is affected by a large color correction. The star remarkably ruddy.

340. 196 Ḫ *Herculis*. This star was rejected from the mean of the group on an inadequate suspicion of variability.

358. R *Lyrae*. A known variable.

378. DM. 41° 3469. Although I make this star 5.6, and the DM. marks it 5.8. It is omitted by both Argelander and Heis, and its variability is highly probable. When I first observed it, it seemed brighter than 14 *Cygni*, but I think its light afterwards diminished.

398. DM. 45° 3139. This star was omitted from the group on a suspicion of variability which is not without foundation.

413, 414. 118 Ḫ and 123 Ḫ *Cygni*. The Uranometria only contains the former star as 5.8; Heis calls the former 5.8 and the latter 6.3; the DM. calls the former 6.4 and the latter 5.4; and I call the former 7.0 and the latter 5.7. I only succeeded in observing Argelander's star, once; and never failed to observe the other, which seemed to me brighter than 51 *Cygni*.

440. *Star following* ρ *Cygni*. This star was perhaps DM. + 45° 3637. 1875, Aug. 2, I could find no star which could possibly be identified with the star observed. The star was red.

449. 2 Ḫ *Lacertae*. I have some slight suspicion of this star, founded on the discrepancies in my measures, and the fact that the DM. makes it 0.8 brighter than Heis. The star is reddish.

463. 45 Ḫ *Lacertae*. This is a reddish star and there is a difference of 1.3 between the magnitudes of the Uranometria and the Durchmusterung. I have rejected not only this star but also 46 Ḫ from the mean of the group, but my suspicion of the latter depends only on the discrepancies of my own measures.

469. 5 *Andromedae*. The magnitude of Sir Wm. Herschel differs by 1.1 from mine.

CHAPTER IV.

COMPARISONS OF THE DIFFERENT OBSERVERS.

Having reduced the magnitudes of the different observers to the same scale, I have made some statistical comparisons between them. I am sorry to say that at the time when this work was done, I only knew the magnitudes of Ptolemy, Ulugh, and Tycho, in whole numbers without the thirds, and I had not seen the magnitudes of Sûfi. I will first give the tables of the comparisons which have been made, and will then endeavor to make some inferences from them. Where the unreduced magnitudes are not used, the scale is that which I have finally adopted, which is explained in Chapter II.

Comparisons of the catalogues of Ptolemy, Ulugh Beg and Tycho Brahe, each with each.

Ptolemy with Ulugh Beg.

	Mags.	U. 1	2	3	4	5	6	neb.	omits.	
	1	15	—	—	—	—	—	—	—	15
	2	—	31	11	1	—	—	—	—	43
	3	—	2	169	17	1	—	—	10	199
	4	—	1	12	380	58	4	—	16	471
II.	5	—	—	2	6	168	22	—	16	214
	6	—	—	—	—	4	44	—	2	50
	ἀμ.	—	—	—	2	3	2	—	2	9
	νεφ.	—	—	—	—	—	—	4	—	4
	omitted.	—	1	3	14	17	6	—	—	
		15	35	197	420	251	78	4		

Tycho Brahe with Ulugh Beg.

	Mag.	U. 1	2	3	4	5	6	neb.	omits.	
	1	10	2	—	—	—	—	—	—	12
	2	2	23	16	—	—	—	—	—	41
	3	—	1	110	40	2	—	—	7	160
T.	4	—	—	36	180	43	6	1	60	326
	5	—	—	1	81	68	8	—	53	211
	6	—	—	—	13	33	35	—	83	164
	neb.	—	—	—	—	—	4	1	—	5
	omit.	3	9	35	106	105	25	2	—	
		15	35	198	420	251	78	4		

Tycho Brahe with Ptolemy.

	Mag.	II. 1	2	3	4	5	6	νεφ.	ἀμ.	omit.	
	1	10	2	—	—	—	—	—	—	—	12
	2	2	26	12	1	—	—	—	—	—	41
	3	—	2	111	33	4	—	—	—	10	160
T.	4	—	—	36	195	25	3	1	7	59	326
	5	—	—	3	92	59	3	—	—	54	211
	6	—	—	1	20	32	29	—	—	82	164
	neb.	—	—	—	—	1	3	1	—	—	5
	omit.	3	13	36	130	93	12	2	2	—	
		15	43	199	471	214	50	4	9		

Comparisons for Northern Stars alone.

Ptolemy with Ulugh Beg.

	Mags.	U. 1	2	3	4	5	6	neb.	omitted.
	1	8	—	—	—	—	—	—	—
	2	—	20	3	—	—	—	—	—
	3	—	2	96	6	1	—	—	1
	4	—	—	8	209	28	3	—	6
II.	5	—	—	1	4	91	11	—	6
	6	—	—	—	—	2	30	—	10
	ἀμ.	—	—	—	2	3	2	—	2
	νεφ.	—	—	—	—	—	—	3	—
	omitted.	—	—	—	12	11	3	—	—

Tycho Brahe with Ulugh Beg.

	Mags.	U. 1	2	3	4	5	6	neb.	omitted.
	1	6	—	—	—	—	—	—	—
	2	2	20	11	—	—	—	—	—
	3	—	1	72	25	1	—	—	5
T.	4	—	—	24	137	39	5	1	44
	5	—	—	1	47	44	8	—	41
	6	—	—	—	9	23	26	—	72
	neb.	—	—	—	—	—	1	1	—
	omitted.	—	1	3	22	40	12	1	—

Tycho Brahe with Ptolemy.

	Mags.	II. 1	2	3	4	5	6	ἀμ.	νεφ.	omitted.
	1	6	—	—	—	—	—	—	—	—
	2	2	21	9	1	—	—	—	—	—
	3	—	2	70	23	2	—	—	—	7
	4	—	—	21	146	24	3	7	1	44
	5	—	—	1	60	38	3	—	—	41
	6	—	—	1	15	19	23	—	—	71
	neb.	—	—	—	—	—	1	—	1	—
	omitted.	—	—	4	21	43	6	2	1	—

Details of the Ptolemy and Ulugh Beg Comparison.

Ptolemy and Ulugh both make 1st mag. in

Constellation.	Total No.	No. northern.
Auriga	1	1
Taurus	1	1
Canis minor	1	1
Leo	2	2
Bootes	1	1
Lyra	1	1
Eridanus	1	—
Orion	2	1
Canis major	1	—
Argo Navis	1	—
Virgo	1	—
Piscis Austrinus	1	—
Centaurus	1	—
	15	8

Ptolemy's 2nd as made by Ulugh.

Constellation.	No. II.		Ulugh makes 2nd		3rd		4th	
Ursa minor	2	2	1	1	1	1	—	—
Perseus	2	2	2	2	—	—	—	—
Ursa major	6	6	4	4	2	2	—	—
Andromeda	1	1	1	1	—	—	—	—
Pegasus	3	3	3	3	—	—	—	—
Auriga	1	1	1	1	—	—	—	—
Gemini	2	2	2	2	—	—	—	—
Leo	2	2	2	2	—	—	—	—
Corona Bor.	1	1	1	1	—	—	—	—
Cycnus	1	1	1	1	—	—	—	—
Orion	4	1	4	1	—	—	—	—
Columba	2	—	—	—	2	—	—	—
Hydra	1	—	1	—	—	—	—	—
Aquila et Ant.	1	1	1	1	—	—	—	—
Libra	2	—	—	—	2	—	—	—
Lupus	1	—	1	—	—	—	—	—
Argo Navis	7	—	3	—	3	—	1	—
Centaurus	4	—	3	—	1	—	—	—
	43	23	31	20	11	3	1	—

Ptolemy's 3rd mag.

Constellation.	Total No.	No. northern.	Ulugh makes: 2		3		4		5		omits.	
Ursa minor	1	1	—	—	1	1	—	—	—	—	—	—
Draco	8	8	—	—	8	8	—	—	—	—	—	—
Cepheus	1	1	—	—	1	1	—	—	—	—	—	—
Cassiepea	4	4	—	—	4	4	—	—	—	—	—	—
Perseus	5	5	—	—	5	5	—	—	—	—	—	—
Ursa major	8	8	—	—	8	8	—	—	—	—	—	—
Canes Ven.	1	1	—	—	1	1	—	—	—	—	—	—
Andromeda	4	4	1	1	2	2	1	1	—	—	—	—
Pegasus	4	4	—	—	4	4	—	—	—	—	—	—
Pisces	2	2	—	—	2	2	—	—	—	—	—	—
Triangulum	3	3	—	—	3	3	—	—	—	—	—	—
Aries	3	3	—	—	3	3	—	—	—	—	—	—
Auriga	2	2	1	1	1	1	—	—	—	—	—	—
Taurus	6	6	—	—	6	6	—	—	—	—	—	—
Gemini	4	4	—	—	3	3	1	1	—	—	—	—
Leo	6	6	—	—	6	6	—	—	—	—	—	—
Bootes	4	4	—	—	3	3	1	1	—	—	—	—
Hercules	5	5	—	—	4	4	—	—	1	1	—	—
Lyra	2	2	—	—	2	2	—	—	—	—	—	—
Cycnus	5	5	—	—	5	5	—	—	—	—	—	—
Delphinus	5	5	—	—	4	4	1	1	—	—	—	—
Cetus	9	2	—	—	8	2	1	—	—	—	—	—
Eridanus	5	—	—	—	3	—	2	—	—	—	—	—
Orion	8	4	—	—	7	3	1	1	—	—	—	—
Lepus	2	—	—	—	2	—	—	—	—	—	—	—
Canis major	5	—	—	—	5	—	—	—	—	—	—	—
Argo Navis	10	—	—	—	5	—	5	—	—	—	—	—
Hydra	4	—	—	—	3	—	1	—	—	—	—	—
Virgo	5	5	—	—	5	5	—	—	—	—	—	—
Crater	1	—	—	—	1	—	—	—	—	—	—	—
Corvus	5	—	—	—	5	—	—	—	—	—	—	—
Centaurus	2	—	—	—	2	—	—	—	—	—	—	—
Serpens	5	5	—	—	5	5	—	—	—	—	—	—
Ophiuchus	5	1	—	—	5	1	—	—	—	—	—	—
Aquila et Antinous	8	5	—	—	6	4	1	—	—	—	1	1
Libra	1	—	—	—	1	—	—	—	—	—	—	—
Scorpius	13	—	—	—	8	—	—	—	—	—	5	—
Sagittarius	6	—	—	—	6	—	—	—	—	—	—	—
Capricornus	4	—	—	—	4	—	—	—	—	—	—	—
Aquarius	9	1	—	—	6	—	2	1	—	—	1	—
Piscis Austrinus	3	—	—	—	—	—	—	—	—	—	3	—
Centaurus	4	—	—	—	4	—	—	—	—	—	—	—
Lepus	2	—	—	—	2	—	—	—	—	—	—	—
	199	106	2	2	169	96	17	6	1	1	10	1

Ptolemy's 4th mag.

Constellation.	Total No.	No. northern.	Ulugh makes: 2		3		4		5		6		omits.	
Ursa minor	5	5	—	—	1	1	3	3	1	1	—	—	—	—
Draco	16	16	—	—	1	1	8	8	7	7	—	—	—	—
Cepheus	8	8	—	—	—	—	7	7	1	1	—	—	—	—
Cassiepea	6	6	—	—	—	—	6	6	—	—	—	—	—	—
Perseus	16	16	—	—	—	—	15	15	1	1	—	—	—	—
Lynx	3	3	—	—	—	—	2	2	—	—	—	—	1	1
Ursa major	7	7	—	—	1	1	4	4	2	2	—	—	—	—
Andromeda	15	15	—	—	—	—	14	14	1	1	—	—	—	—
Pegasus	9	9	—	—	—	—	7	7	1	1	—	—	1	1
Pisces	26	20	—	—	—	—	25	19	1	1	—	—	—	—
Triangulum	1	1	—	—	—	—	—	—	1	1	—	—	—	—
Aries	4	4	—	—	—	—	4	4	—	—	—	—	—	—
Auriga	8	8	—	—	1	1	5	5	2	2	—	—	—	—
Taurus	12	12	—	—	—	—	10	10	1	1	—	—	1	1
Gemini	11	11	—	—	2	2	9	9	—	—	—	—	—	—
Canis minor	1	1	—	—	—	—	1	1	—	—	—	—	—	—
Cancer	10	10	—	—	—	—	8	8	—	—	—	—	2	2
Leo	9	9	—	—	—	—	8	8	—	—	—	—	1	1
Bootes	8	8	—	—	—	—	8	8	—	—	—	—	—	—
Corona Borealis	5	5	—	—	—	—	5	5	—	—	—	—	—	—
Hercules	18	18	—	—	1	1	14	14	3	3	—	—	—	—
Lyra	7	7	—	—	—	—	6	6	1	1	—	—	—	—
Cycnus	11	11	—	—	—	—	10	10	1	1	—	—	—	—
Sagitta	1	1	—	—	—	—	1	1	—	—	—	—	—	—
Delphinus	2	2	—	—	—	—	—	—	—	—	2	2	—	—
Cetus	12	5	—	—	—	—	12	5	—	—	—	—	—	—
Eridanus	25	—	—	—	—	—	24	—	1	—	—	—	—	—
Orion	15	10	—	—	—	—	13	8	2	2	—	—	—	—
Monoceros	2	—	—	—	—	—	—	—	—	—	—	—	2	—
Lepus	6	—	—	—	—	—	6	—	—	—	—	—	—	—
Columba	3	—	—	—	—	—	3	—	—	—	—	—	—	—
Canis major	10	—	—	—	—	—	8	—	2	—	—	—	—	—
Argo Navis	19	—	—	—	—	—	15	—	3	—	—	—	1	—
Hydra	16	8	—	—	2	—	11	7	1	1	—	—	2	—
Virgo	6	—	—	—	—	—	5	—	1	—	—	—	—	—
Crater	10	—	—	—	—	—	9	—	1	—	—	—	—	—
Corvus	1	—	—	—	—	—	1	—	—	—	—	—	—	—
Centaurus	1	—	—	—	—	—	1	—	—	—	—	—	—	—
Serpens	12	6	—	—	—	—	11	5	1	1	—	—	—	—
Ophiuchus	17	10	—	—	1	1	13	9	3	—	—	—	—	—
Aquila et Antinous	2	2	—	—	—	—	1	1	—	—	1	1	—	—
Libra	9	—	—	—	—	—	8	—	1	—	—	—	—	—
Lupus	7	—	—	—	—	—	—	—	4	—	1	—	2	—
Scorpius	3	—	—	—	1	—	2	—	—	—	—	—	—	—
Sagittarius	9	—	—	—	—	—	9	—	—	—	—	—	—	—

Constellation.	Total No.	No. northern.	2		3		4		5		6		omits.	
Capricornus	9	—	—	—	—	—	8	—	1	—	—	—	—	—
Aquarius	18	—	—	—	—	—	15	—	1	—	—	—	2	—
Piscis Austrinus	9	—	—	—	—	—	5	—	3	—	—	—	1	—
Centaurus	15	—	1	—	—	—	10	—	4	—	—	—	—	—
Lupus	6	—	—	—	1	—	4	—	1	—	—	—	—	—
Ara	9	—	—	—	—	—	5	—	4	—	—	—	—	—
Corona Australis	1	—	—	—	—	—	1	—	—	—	—	—	—	—
	471	264	1	—	12	8	380	209	58	28	4	3	16	6

Ptolemy's 5th mag.

Constellation.	Total No.	No. northern.	3		4		5		6		omits.	
Draco	5	5	—	—	—	—	5	5	—	—	—	—
Cepheus	4	4	—	—	—	—	3	3	1	1	—	—
Cassiepea	1	1	—	—	—	—	1	1	—	—	—	—
Perseus	3	3	—	—	—	—	3	3	—	—	—	—
Camelopardalus	1	1	—	—	—	—	1	1	—	—	—	—
Ursa major	5	5	—	—	—	—	5	5	—	—	—	—
Canes venatici	1	1	—	—	—	—	1	1	—	—	—	—
Andromeda	4	4	—	—	—	—	4	4	—	—	—	—
Pegasus	3	3	—	—	—	—	3	3	—	—	—	—
Pisces	3	3	—	—	—	—	3	3	—	—	—	—
Aries	9	9	—	—	—	—	9	9	—	—	—	—
Auriga	2	2	—	—	—	—	—	—	2	2	—	—
Taurus	23	23	—	—	2	2	17	17	1	1	3	3
Gemini	6	6	—	—	—	—	5	5	—	—	1	1
Cancer	3	3	—	—	—	—	3	3	—	—	—	—
Leo	7	7	—	—	—	—	5	5	1	1	1	1
Leo minor	1	1	—	—	—	—	1	1	—	—	—	—
Bootes	9	9	—	—	—	—	9	9	—	—	—	—
Corona Borealis	1	1	—	—	1	1	—	—	—	—	—	—
Hercules	2	2	—	—	1	1	1	1	—	—	—	—
Cycnus	2	2	—	—	—	—	1	1	1	1	—	—
Sagitta	3	3	—	—	—	—	3	3	—	—	—	—
Cetus	4	—	1	—	—	—	—	—	—	—	3	—
Eridanus	2	—	—	—	—	—	2	—	—	—	—	—
Orion	3	3	—	—	—	—	2	2	—	—	1	1
Lepus	4	—	—	—	—	—	4	—	—	—	—	—
Canis major	6	—	—	—	—	—	5	—	—	—	1	—
Argo Navis	7	—	—	—	—	—	7	—	—	—	—	—
Hydra	1	1	—	—	—	—	—	—	1	1	—	—
Virgo	15	7	1	1	1	—	11	6	1	—	1	—
Corvus	1	—	—	—	—	—	1	—	—	—	—	—
Centaurus	3	—	—	—	—	—	3	—	—	—	—	—
Ophiuchus	7	—	—	—	—	—	7	—	—	—	—	—
Aquila et Antinous	4	3	—	—	—	—	1	—	3	3	—	—

Ulugh makes

Constellation.	Total No.	No. northern.	3		4		5		6		omits.	
Libra	4	—	—	—	—	—	3	—	—	—	1	—
Lupus	1	—	—	—	—	—	—	—	—	—	1	—
Scorpius	1	—	—	—	—	—	1	—	—	—	—	—
Sagittarius	9	—	—	—	—	—	8	—	1	—	—	—
Capricornus	11	—	—	—	—	—	5	—	4	—	2	—
Aquarius	12	1	—	—	—	—	9	—	2	1	1	—
Piscis Austrinus	2	—	—	—	—	—	1	—	1	—	—	—
Centaurus	5	—	—	—	—	—	5	—	—	—	—	—
Lupus	6	—	—	—	1	—	5	—	—	—	—	—
Ara	2	—	—	—	—	—	1	—	1	—	—	—
Corona Australis	6	—	—	—	—	—	4	—	2	—	—	—
	214	113	2	1	6	4	168	91	22	11	16	6

Ptolemy's 6th mag.

Constellation.	Total No.	No. northern.	5		6		omits.	
Draco	2	2	—	—	2	2	—	—
Cassiepea	2	2	—	—	2	2	—	—
Pisces	7	7	1	1	6	6	—	—
Aries	1	1	—	—	1	1	—	—
Auriga	1	1	—	—	—	—	1	1
Taurus	1	1	—	—	1	1	—	—
Leo	5	5	1	1	4	4	—	—
Corona Borealis	1	1	—	—	1	1	—	—
Hercules	3	3	—	—	3	3	—	—
Sagitta	1	1	—	—	1	1	—	—
Delphinus	3	3	—	—	3	3	—	—
Orion	5	5	—	—	5	5	—	—
Canis major	1	—	1	—	—	—	—	—
Hydra	1	—	—	—	1	—	—	—
Virgo	4	1	—	—	3	1	1	—
Libra	1	—	—	—	1	—	—	—
Sagittarius	1	—	—	—	1	—	—	—
Capricornus	6	—	1	—	5	—	—	—
Aquarius	1	—	—	—	1	—	—	—
Argo Navis	1	—	—	—	1	—	—	—
Corona Australis	2	—	—	—	2	—	—	—
	50	33	4	2	44	30	2	1

Ptolemy's ἀμαυροί

Constellation.	Total No.	No. northern.
Perseus	1	1
Lynx	1	1

Ulugh makes

4		5		6		omits.	
—	—	1	1	—	—	—	—
—	—	—	—	1	1	—	—

Constellation.	Total No.	No. northern.	4		5		6		omits.	
Equuleus	4	4	1	1	2	2	1	1	—	—
Leo minor	1	1	1	1	—	—	—	—	—	—
Coma Berenices	2	2	—	—	—	—	—	—	2	2
	9	9	2	2	3	3	2	2	2	2

Ptolemy's νεφ

Ulugh makes nebulae

Constellation.	Total No.	No. northern.		
Perseus	1	1	1	1
Cancer	1	1	1	1
Orion	1	1	1	1
Scorpius	1	—	1	—
	4	3	4	3

Details of the Tycho Brahe and Ulugh Beg Comparison.

Tycho's 1st mags

Ulugh makes

Constellation.	Total No.	No. northern.	1st		2nd	
Auriga	1	1	1	1	—	—
Taurus	1	1	1	1	—	—
Leo	2	2	2	2	—	—
Bootes	1	1	1	1	—	—
Lyra	1	1	1	1	—	—
Orion	1	—	1	—	—	—
Canis major	1	—	1	—	—	—
Hydra	1	—	—	—	1	—
Virgo	1	—	1	—	—	—
Scorpius	1	—	—	—	1	—
Piscis Austrinus	1	—	1	—	—	—
	12	6	10	6	2	—

Tycho's 2nd mags

Ulugh makes

Constellation.	Total No.	No. northern.	1		2		3	
Ursa minor	2	2	—	—	1	1	1	1
Draco	1	1	—	—	—	—	1	1
Perseus	1	1	—	—	1	1	—	—
Ursa major	7	7	—	—	4	4	3	3
Canes venatici	1	1	—	—	—	—	1	1
Andromeda	3	3	—	—	2	2	1	1
Pegasus	3	3	—	—	3	3	—	—
Auriga	1	1	—	—	1	1	—	—
Taurus	1	1	—	—	—	—	1	1
Gemini	3	3	—	—	2	2	1	1
Canis minor	1	1	1	1	—	—	—	—
Leo	2	2	—	—	2	2	—	—
Corona Borealis	1	1	—	—	1	1	—	—

Constellation.	Total No.	No. northern.	1		2		3	
Cycnus	1	1	—	—	1	1	—	—
Cetus	2	1	—	—	—	—	2	1
Orion	5	2	1	1	4	1	—	—
Canis major	1	—	—	—	—	—	1	—
Serpens	1	1	—	—	—	—	1	1
Aquila et Antinous	1	1	—	—	1	1	—	—
Libra	2	—	—	—	—	—	2	—
Scorpius	1	—	—	—	—	—	1	—
	41	33	2	2	23	20	16	11

Tycho's 3rd mags

Ulugh makes

Constellation.	Total No.	No. northern.	2		3		4		5		omits.	
Ursa minor	1	1	—	—	1	1	—	—	—	—	—	—
Draco	11	11	—	—	7	7	3	3	—	—	1	1
Cepheus	2	2	—	—	—	—	2	2	—	—	—	—
Cassiepea	5	5	—	—	4	4	1	1	—	—	—	—
Perseus	5	5	1	1	4	4	—	—	—	—	—	—
Lynx	1	1	—	—	—	—	1	1	—	—	—	—
Ursa major	3	3	—	—	3	3	—	—	—	—	—	—
Andromeda	1	1	—	—	1	1	—	—	—	—	—	—
Pegasus	3	3	—	—	3	3	—	—	—	—	—	—
Pisces	1	1	—	—	1	1	—	—	—	—	—	—
Aries	2	2	—	—	1	1	1	1	—	—	—	—
Taurus	5	5	—	—	4	4	1	1	—	—	—	—
Gemini	4	4	—	—	2	2	2	2	—	—	—	—
Canis minor	1	1	—	—	—	—	1	1	—	—	—	—
Cancer	2	2	—	—	—	—	2	2	—	—	—	—
Leo	5	5	—	—	5	5	—	—	—	—	—	—
Leo minor	3	3	—	—	—	—	—	—	—	—	3	3
Coma Berenices	1	1	—	—	—	—	—	—	1	1	—	—
Bootes	6	6	—	—	3	3	3	3	—	—	—	—
Hercules	9	9	—	—	5	5	4	4	—	—	—	—
Lyra	2	2	—	—	2	2	—	—	—	—	—	—
Cycnus	6	6	—	—	5	5	—	—	—	—	1	1
Delphinus	5	5	—	—	4	4	1	1	—	—	—	—
Cetus	7	1	—	—	6	1	1	—	—	—	—	—
Eridanus	7	—	—	—	3	—	3	—	—	—	1	—
Orion	4	—	—	—	4	—	—	—	—	—	—	—
Lepus	2	—	—	—	2	—	—	—	—	—	—	—
Canis major	5	—	—	—	4	—	1	—	—	—	—	—
Argo Navis	3	—	—	—	1	—	2	—	—	—	—	—
Hydra	3	—	—	—	1	—	2	—	—	—	—	—
Virgo	5	5	—	—	5	5	—	—	—	—	—	—
Corvus	3	—	—	—	3	—	—	—	—	—	—	—
Serpens	9	6	—	—	4	4	5	2	—	—	—	—

Constellation.	Total No.	No. northern.	2		3		4		5		omits.	
Ophiuchus	7	3	—	—	6	3	1	—	—	—	—	—
Aquila et Antinous	10	5	—	—	6	4	2	1	1	—	1	—
Libra	1	—	—	—	—	—	1	—	—	—	—	—
Scorpius	2	—	—	—	2	—	—	—	—	—	—	—
Capricornus	4	—	—	—	4	—	—	—	—	—	—	—
Aquarius	4	—	—	—	4	—	—	—	—	—	—	—
	160	104	1	1	110	72	40	25	2	1	7	5

Tycho's 4th mags

Ulugh makes

Constellation.	Total No.	No. northern.	3		4		5		6		neb.		omits.	
Ursa minor	3	3	—	—	3	3	—	—	—	—	—	—	—	—
Draco	13	13	1	1	5	5	6	6	1	1	—	—	—	—
Cepheus	7	7	—	—	5	5	2	2	—	—	—	—	—	—
Cassiepea	5	5	—	—	5	5	—	—	—	—	—	—	—	—
Perseus	11	11	—	—	9	9	1	1	—	—	—	—	1	1
Lynx	1	1	—	—	1	1	—	—	—	—	—	—	—	—
Ursa major	15	15	5	5	4	4	3	3	—	—	—	—	3	3
Andromeda	10	10	—	—	9	9	—	—	—	—	—	—	1	1
Equuleus	4	4	—	—	1	1	2	2	1	1	—	—	—	—
Pegasus	9	9	1	1	5	5	—	—	—	—	—	—	3	3
Pisces	5	5	1	1	4	4	—	—	—	—	—	—	—	—
Triangulum	3	3	3	3	—	—	—	—	—	—	—	—	—	—
Aries	5	5	2	2	1	1	2	2	—	—	—	—	—	—
Auriga	7	7	2	2	5	5	—	—	—	—	—	—	—	—
Taurus	13	13	2	2	6	6	5	5	—	—	—	—	—	—
Gemini	8	8	2	2	6	6	—	—	—	—	—	—	—	—
Cancer	3	3	—	—	2	2	—	—	—	—	—	—	1	1
Leo	11	11	1	1	6	6	2	2	1	1	—	—	1	1
Leo minor	4	4	—	—	1	1	—	—	—	—	—	—	3	3
Coma Berenices	12	12	—	—	—	—	2	2	—	—	—	—	10	10
Bootes	13	13	—	—	5	5	4	4	—	—	—	—	4	4
Corona Borealis	5	5	—	—	4	4	—	—	—	—	—	—	1	1
Hercules	16	16	—	—	11	11	3	3	—	—	—	—	2	2
Lyra	3	3	—	—	1	1	—	—	—	—	—	—	2	2
Cycnus	13	13	—	—	10	10	1	1	—	—	—	—	2	2
Argo Navis	2	—	—	—	—	—	—	—	—	—	—	—	2	—
Vulpecula cum Ansere	3	3	—	—	—	—	—	—	—	—	—	—	3	3
Sagitta	3	3	—	—	1	1	2	2	—	—	—	—	—	—
Delphinus	1	1	—	—	—	—	—	—	1	1	—	—	—	—
Cetus	10	5	1	—	9	5	—	—	—	—	—	—	—	—
Eridanus	8	—	—	—	6	—	1	—	—	—	—	—	1	—
Orion	16	13	3	2	6	4	2	2	—	—	1	1	4	3
Monoceros	8	4	—	—	—	—	—	—	—	—	—	—	8	4
Lepus	6	—	—	—	5	—	—	—	—	—	—	—	1	—
Canis major	2	—	—	—	2	—	—	—	—	—	—	—	—	—

Constellation.	Total No.	No. northern.	3		4		5		6		neb.		omits.	
Hydra	10	6	2	—	5	5	1	1	—	—	—	—	2	—
Virgo	7	1	1	1	6	—	—	—	—	—	—	—	—	—
Crater	8	—	—	—	7	—	1	—	—	—	—	—	—	—
Corvus	2	—	2	—	—	—	—	—	—	—	—	—	—	—
Serpens	6	6	—	—	5	5	1	1	—	—	—	—	—	—
Ophiuchus	10	8	—	—	9	8	1	—	—	—	—	—	—	—
Aquila et Antinous	2	1	—	—	—	—	—	—	1	1	—	—	1	—
Libra	4	—	—	—	—	—	1	—	1	—	—	—	2	—
Scorpius	4	—	3	—	1	—	—	—	—	—	—	—	—	—
Sagittarius	7	—	2	—	4	—	—	—	—	—	—	—	1	—
Capricornus	1	—	—	—	1	—	—	—	—	—	—	—	—	—
Aquarius	7	—	2	—	4	—	—	—	—	—	—	—	1	—
	326	250	36	24	180	137	43	39	6	5	1	1	60	44

Tycho's 5th mags

Ulugh makes

Constellation.	Total No.	No. northern.	3		4		5		6		omits.	
Ursa minor	1	1	—	—	—	—	1	1	—	—	—	—
Draco	7	7	—	—	—	—	6	6	—	—	1	1
Cepheus	2	2	—	—	—	—	1	1	—	—	—	—
Cassiepea	1	—	—	—	—	—	—	—	—	—	1	1
Perseus	9	6	6	6	2	2	—	—	—	—	1	1
Ursa major	7	7	—	—	4	4	—	—	—	—	3	3
Canes Venatici	1	1	—	—	1	1	—	—	—	—	—	—
Andromeda	8	8	—	—	4	4	3	3	—	—	1	1
Pegasus	3	3	—	—	1	1	2	2	—	—	—	—
Pisces	17	15	—	—	14	12	3	3	—	—	—	—
Triangulum	1	1	—	—	—	—	1	1	—	—	—	—
Aries	4	4	—	—	1	1	3	3	—	—	—	—
Auriga	8	8	—	—	—	—	1	1	2	2	5	5
Taurus	17	17	—	—	5	5	3	3	1	1	8	8
Gemini	4	4	—	—	2	2	—	—	—	—	2	2
Canis minor	1	1	—	—	—	—	—	—	—	—	1	1
Cancer	6	6	—	—	4	4	1	1	—	—	1	1
Leo	4	4	—	—	1	1	1	1	1	1	1	1
Bootes	4	4	—	—	—	—	4	4	—	—	—	—
Corona Borealis	2	2	—	—	1	1	—	—	—	—	1	1
Hercules	1	1	—	—	—	—	—	—	—	—	1	1
Lyra	5	5	—	—	4	4	—	—	—	—	1	1
Cycnus	1	1	—	—	—	—	—	—	1	1	—	—
Sagitta	1	1	—	—	—	—	1	1	—	—	—	—
Delphinus	1	1	—	—	—	—	—	—	1	1	—	—
Cetus	2	1	—	—	—	—	—	—	—	—	2	1
Eridanus	4	—	—	—	2	—	1	—	—	—	1	—
Orion	16	12	—	—	4	2	2	2	1	1	9	7
Monoceros	3	2	—	—	—	—	—	—	—	—	3	2

Constellation.	Total No.	No. northern.	3		4		5		6		omits.	
Lepus	4	—	—	—	1	—	3	—	—	—	—	—
Canis major	4	—	—	—	1	—	2	—	—	—	1	—
Hydra	7	3	—	—	6	2	—	—	—	—	1	1
Virgo	8	6	—	—	1	—	4	4	—	—	3	2
Crater	1	—	—	—	—	—	—	—	—	—	1	—
Corvus	2	—	—	—	1	—	1	—	—	—	—	—
Centaurus	4	—	—	—	1	—	3	—	—	—	—	—
Serpens	1	1	—	—	1	1	—	—	—	—	—	—
Ophiuchus	2	—	—	—	—	—	2	—	—	—	—	—
Aquila et Antinous	2	1	—	—	—	—	—	—	1	1	1	—
Libra	2	—	—	—	—	—	2	—	—	—	—	—
Scorpius	2	—	—	—	1	—	1	—	—	—	—	—
Sagittarius	2	—	—	—	2	—	—	—	—	—	—	—
Capricornus	7	—	—	—	4	—	3	—	—	—	—	—
Aquarius	22	1	—	—	13	1	6	—	—	—	3	—
	211	141	1	1	81	47	68	44	8	8	53	41

Tycho's 6th mags

Ulugh makes

Constellation.	No.	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	omits.	Northern alone.
Draco	3	3	—	—	—	—	—	—	3	3
Cassiepea	19	19	—	—	1	1	2	2	16	16
Perseus	2	2	—	—	1	1	—	—	1	1
Camelopardalus	10	10	—	—	—	—	—	—	10	10
Lynx	2	2	—	—	—	—	—	—	2	2
Pegasus	5	5	1	1	2	2	—	—	2	2
Pisces	13	13	3	3	2	2	5	5	3	3
Aries	10	10	—	—	3	3	1	1	6	6
Auriga	4	4	—	—	1	1	—	—	3	3
Taurus	11	11	1	1	6	6	1	1	3	3
Gemini	6	6	—	—	2	2	—	—	4	4
Canis minor	2	2	—	—	—	—	—	—	2	2
Cancer	4	4	1	1	2	2	—	—	1	1
Leo	6	6	—	—	—	—	2	2	4	4
Bootes	3	3	—	—	1	1	—	—	2	2
Corona Borealis	2	2	1	1	—	—	1	1	—	—
Hercules	2	2	—	—	—	—	2	2	—	—
Lyra	2	2	1	1	1	1	—	—	—	—
Cycnus	1	1	—	—	—	—	—	—	1	1
Sagitta	1	1	—	—	—	—	1	1	—	—
Delphinus	3	3	—	—	—	—	3	3	—	—
Orion	10	7	1	1	—	—	4	4	5	2
Lepus	1	—	—	—	1	—	—	—	—	—
Hydra	3	1	—	—	—	—	1	1	2	—
Sextans	1	1	—	—	—	—	—	—	1	1

Constellation.	No.	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	omits.	Northern alone.
Virgo	10	7	—	—	3	1	2	1	5	5
Aquila et Antinous	2	2	—	—	—	—	1	1	1	1
Libra	1	—	—	—	—	—	—	—	1	—
Sagittarius	5	—	1	—	2	—	2	—	—	—
Capricornus	13	—	3	—	2	—	6	—	2	—
Aquarius	7	1	—	—	3	—	1	1	3	—
	164	130	13	9	33	23	35	26	83	72

Tycho's nebulae

Ulugh makes

Constellation.	No.	Northern alone.	6	Northern alone.	Nebulae.	Northern alone.
Cancer	1	1	—	—	1	1
Hercules	1	1	1	1	—	—
Capricornus	3	—	3	—	—	—
	5	2	4	1	1	1

Details of the Tycho Brahe and Ptolemy Comparison.

Tycho's 1st mags

Ptolemy makes

Constellation.	No.	Northern alone.	1	Northern alone.	2	Northern alone.
Auriga	1	1	1	1	—	—
Taurus	1	1	1	1	—	—
Leo	2	2	2	2	—	—
Bootes	1	1	1	1	—	—
Lyra	1	1	1	1	—	—
Orion	1	—	1	—	—	—
Canis major	1	—	1	—	—	—
Hydra	1	—	—	—	1	—
Virgo	1	—	1	—	—	—
Scorpius	1	—	—	—	1	—
Piscis Austrinus	1	—	1	—	—	—
	12	6	10	6	2	—

Tycho's 2nd mags

Ptolemy makes

Constellation.	No.	Northern alone.	1	Northern alone.	2	Northern alone.	3	Northern alone.	4	Northern alone.
Ursa minor	2	2	—	—	1	1	1	1	—	—
Draco	1	1	—	—	—	—	1	1	—	—
Perseus	1	1	—	—	1	1	—	—	—	—
Ursa major	7	7	—	—	6	6	1	1	—	—
Canes venatici	1	1	—	—	—	—	1	1	—	—
Andromeda	3	3	—	—	1	1	2	2	—	—
Pegasus	3	3	—	—	3	3	—	—	—	—
Auriga	1	1	—	—	1	1	—	—	—	—
Taurus	2	2	—	—	—	—	1	1	1	1

Constellation.	No.	Northern alone.	1	Northern alone.	2	Northern alone.	3	Northern alone.	4	Northern alone.
Gemini	2	2	—	—	2	2	—	—	—	—
Cauis minor	1	1	1	1	—	—	—	—	—	—
Leo	2	2	—	—	2	2	—	—	—	—
Corona Borealis	1	1	—	—	1	1	—	—	—	—
Cycnus	1	1	—	—	1	1	—	—	—	—
Cetus	2	1	—	—	—	—	2	1	—	—
Orion	5	2	1	1	4	1	—	—	—	—
Canis major	1	—	—	—	—	—	1	—	—	—
Serpens	1	1	—	—	—	—	1	1	—	—
Aquila et Antinous	1	1	—	—	1	1	—	—	—	—
Libra	2	—	—	—	2	—	—	—	—	—
Scorpius	1	—	—	—	—	—	1	—	—	—
	41	33	2	2	26	21	12	9	1	1

Tycho's 3rd mags

Ptolemy makes

Constellation.	No.	Northern alone.	2	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	Omits.	Northern alone.
Ursa minor	1	1	1	1	—	—	—	—	—	—	—	—
Draco	11	11	—	—	7	7	3	3	—	—	1	1
Cepheus	2	2	—	—	—	—	2	2	—	—	—	—
Cassiepea	5	5	—	—	4	4	1	1	—	—	—	—
Perseus	5	5	1	1	4	4	—	—	—	—	—	—
Lynx	1	1	—	—	—	—	1	1	—	—	—	—
Ursa major	3	3	—	—	3	3	—	—	—	—	—	—
Andromeda	1	1	—	—	1	1	—	—	—	—	—	—
Pegasus	3	3	—	—	3	3	—	—	—	—	—	—
Pisces	1	1	—	—	1	1	—	—	—	—	—	—
Triangulum	2	2	—	—	1	1	1	1	—	—	—	—
Auriga	1	1	—	—	—	—	—	—	—	—	1	1
Taurus	5	5	—	—	4	4	1	1	—	—	—	—
Gemini	3	3	—	—	3	3	—	—	—	—	—	—
Canis minor	1	1	—	—	—	—	1	1	—	—	—	—
Cancer	2	2	—	—	—	—	2	2	—	—	—	—
Leo	5	5	—	—	5	5	—	—	—	—	—	—
Leo minor	3	3	—	—	—	—	—	—	—	—	3	3
Coma Berenices	1	1	—	—	—	—	—	—	—	—	1	1
Bootes	6	6	—	—	4	4	2	2	—	—	—	—
Hercules	9	9	—	—	4	4	4	4	1	1	—	—
Lyra	2	2	—	—	2	2	—	—	—	—	—	—
Cycnus	6	6	—	—	5	5	—	—	—	—	1	1
Delphinus	5	5	—	—	5	5	—	—	—	—	—	—
Cetus	7	1	—	—	5	1	1	—	1	—	—	—
Eridanus	7	—	—	—	5	—	1	—	—	—	1	—
Orion	4	—	—	—	4	—	—	—	—	—	—	—
Lepus	2	—	—	—	2	—	—	—	—	—	—	—
Canis major	5	—	—	—	4	—	1	—	—	—	—	—

Constellation.	No.	Northern alone.	2	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	Omits.	Northern alone.
Argo Navis	3	—	—	—	2	—	1	—	—	—	—	—
Hydra	3	—	—	—	—	—	2	—	—	—	1	—
Virgo	5	5	—	—	4	4	—	—	1	1	—	—
Corvus	3	—	—	—	3	—	—	—	—	—	—	—
Serpens	9	6	—	—	4	4	5	2	—	—	—	—
Ophiuchus	7	3	—	—	5	1	2	2	—	—	—	—
Aquila et Antinous	10	5	—	—	7	4	1	1	1	—	1	—
Libra	1	—	—	—	—	—	1	—	—	—	—	—
Scorpius	2	—	—	—	2	—	—	—	—	—	—	—
Sagittarius	4	—	—	—	4	—	—	—	—	—	—	—
Aquarius	4	—	—	—	4	—	—	—	—	—	—	—
	160	104	2	2	111	70	33	23	4	2	10	7

Tycho's 4th mags

Ptolemy makes

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	dμ.	Northern alone.	veg.	Northern alone.	Omits.	Northern alone.
Ursa minor	3	3	—	—	3	3	—	—	—	—	—	—	—	—	—	—
Draco	13	13	—	—	9	9	3	3	1	1	—	—	—	—	—	—
Cepheus	7	7	—	—	6	6	1	1	—	—	—	—	—	—	—	—
Cassiepea	5	5	—	—	5	5	—	—	—	—	—	—	—	—	—	—
Perseus	11	11	—	—	9	9	—	—	—	—	1	1	—	—	1	1
Lynx	1	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—
Ursa major	15	15	4	4	4	4	3	3	—	—	—	—	—	—	4	4
Andromeda	10	10	—	—	9	9	—	—	—	—	—	—	—	—	1	1
Equuleus	4	4	—	—	—	—	—	—	—	—	4	4	—	—	—	—
Pegasus	13	13	1	1	9	9	—	—	—	—	—	—	—	—	3	3
Pisces	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—
Triangulum	4	4	3	3	1	1	—	—	—	—	—	—	—	—	—	—
Aries	10	10	2	2	6	6	2	2	—	—	—	—	—	—	—	—
Auriga	6	6	1	1	5	5	—	—	—	—	—	—	—	—	—	—
Taurus	15	15	2	2	7	7	6	6	—	—	—	—	—	—	—	—
Gemini	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—
Cancer	3	3	—	—	3	3	—	—	—	—	—	—	—	—	—	—
Leo	11	11	1	1	7	7	2	2	1	1	—	—	—	—	—	—
Leo minor	4	4	—	—	—	—	—	—	—	—	1	1	—	—	3	3
Coma Berenices	12	12	—	—	—	—	—	—	—	—	1	1	—	—	11	11
Bootes	13	13	—	—	5	5	4	4	—	—	—	—	—	—	4	4
Corona Borealis	5	5	—	—	4	4	—	—	—	—	—	—	—	—	1	1
Hercules	16	16	—	—	14	14	—	—	—	—	—	—	—	—	2	2
Lyra	3	3	—	—	1	1	—	—	—	—	—	—	—	—	2	2
Cycnus	13	13	—	—	11	11	—	—	—	—	—	—	—	—	2	2
Sagitta	6	6	—	—	1	1	2	2	—	—	—	—	—	—	3	3
Delphinus	1	1	—	—	—	—	—	—	1	1	—	—	—	—	—	—
Cetus	10	5	2	—	8	5	—	—	—	—	—	—	—	—	—	—
Eridanus	8	—	—	—	7	—	—	—	—	—	—	—	—	—	1	—

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	dit.	Northern alone.	vep.	Northern alone.	Omits.	Northern alone.
Orion	16	13	4	4	7	5	—	—	—	—	—	—	1	1	4	3
Monoceros	8	4	—	—	1	—	—	—	—	—	—	—	—	—	7	4
Lepus	6	—	—	—	5	—	—	—	—	—	—	—	—	—	1	—
Canis major	2	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—
Argo Navis	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—
Hydra	10	5	2	1	7	4	—	—	—	—	—	—	—	—	1	—
Virgo	7	1	2	—	5	1	—	—	—	—	—	—	—	—	1	—
Crater	8	—	—	—	8	—	—	—	—	—	—	—	—	—	—	—
Corvus	2	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—
Serpens	6	2	—	—	6	2	—	—	—	—	—	—	—	—	—	—
Ophiuchus	10	9	—	—	10	9	—	—	—	—	—	—	—	—	—	—
Aquila et Antinous	2	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—
Libra	4	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—
Lupus	1	—	—	—	1	—	—	—	—	—	—	—	—	—	3	—
Scorpius	3	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—
Sagittarius	7	—	2	—	4	—	—	—	—	—	—	—	—	—	—	—
Capricornus	1	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—
Aquarius	7	—	3	—	3	—	—	—	—	—	—	—	—	—	—	—
	326	246	36	21	195	146	25	24	3	3	7	7	1	1	59	44

Tycho's 5th mags

Ptolemy makes

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	Omits.	Northern alone.
Ursa minor	1	1	—	—	1	1	—	—	—	—	—	—
Draco	7	7	—	—	4	4	2	2	—	—	1	1
Cepheus	2	2	1	1	—	—	1	1	—	—	—	—
Cassiepea	1	1	—	—	—	—	—	—	—	—	1	1
Perseus	9	9	—	—	7	7	1	1	—	—	1	1
Ursa major	7	7	—	—	2	2	2	2	—	—	3	3
Andromeda	8	8	—	—	5	5	2	2	—	—	1	1
Canes Venatici	1	1	—	—	—	—	1	1	—	—	—	—
Pegasus	3	3	—	—	2	2	1	1	—	—	—	—
Pisces	17	17	—	—	15	15	2	2	—	—	—	—
Triangulum	4	4	—	—	1	1	3	3	—	—	—	—
Aries	6	6	—	—	1	1	—	—	—	—	5	5
Auriga	11	11	—	—	1	1	2	2	1	1	7	7
Taurus	9	9	—	—	4	4	5	5	—	—	—	—
Gemini	4	4	—	—	2	2	—	—	—	—	2	2
Canis minor	1	1	—	—	—	—	—	—	—	—	1	1
Cancer	6	6	—	—	4	4	1	1	—	—	1	1
Leo	4	4	—	—	1	1	1	1	1	1	1	1
Bootes	4	4	—	—	—	—	4	4	—	—	—	—
Corona Borealis	2	2	—	—	—	—	1	1	—	—	1	1
Hercules	1	1	—	—	—	—	—	—	—	—	1	1
Lyra	5	5	—	—	4	4	—	—	—	—	1	1

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	Omits.	Northern alone.
Cygnus	1	1	—	—	—	—	1	1	—	—	—	—
Sagitta	1	1	—	—	—	—	1	1	—	—	—	—
Delphinus	1	1	—	—	1	1	—	—	—	—	—	—
Cetus	2	1	—	—	—	—	—	—	—	—	2	1
Eridanus	4	—	—	—	2	—	1	—	—	—	1	—
Orion	16	12	—	—	4	2	1	1	1	1	10	8
Monoceros	4	2	—	—	1	—	—	—	—	—	3	2
Lepus	4	—	—	—	1	—	3	—	—	—	—	—
Canis major	3	—	—	—	—	—	2	—	—	—	1	—
Hydra	7	3	—	—	5	2	—	—	—	—	2	1
Virgo	8	7	—	—	—	—	5	5	—	—	3	2
Crater	1	—	—	—	—	—	—	—	—	—	1	—
Corvus	2	—	—	—	1	—	1	—	—	—	—	—
Centaurus	4	—	—	—	1	—	3	—	—	—	—	—
Serpens	1	1	—	—	1	1	—	—	—	—	—	—
Ophiuchus	2	—	—	—	1	—	—	—	—	—	1	—
Aquila et Antinous	2	1	—	—	—	—	1	1	—	—	1	—
Libra	2	—	—	—	1	—	1	—	—	—	—	—
Lupus	1	—	—	—	—	—	1	—	—	—	—	—
Scorpius	1	—	—	—	1	—	—	—	—	—	—	—
Sagittarius	2	—	—	—	2	—	—	—	—	—	—	—
Capricornus	7	—	—	—	4	—	3	—	—	—	—	—
Aquarius	22	—	2	—	12	—	6	—	—	—	2	—
	211	143	3	1	92	60	59	38	3	3	54	41

Tycho's 6th mags

Ptolemy makes

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	Omits.	Northern alone.
Draco	3	3	—	—	—	—	—	—	—	—	3	3
Cassiepea	19	19	—	—	—	—	1	1	2	2	16	16
Perseus	2	2	—	—	—	—	1	1	—	—	1	1
Camelopardalus	10	10	—	—	—	—	—	—	—	—	10	10
Lynx	2	2	—	—	—	—	—	—	—	—	2	2
Pegasus	5	5	—	—	2	2	2	2	—	—	1	1
Pisces	13	13	—	—	3	3	1	1	6	6	3	3
Aries	10	10	—	—	—	—	3	3	1	1	6	6
Auriga	4	4	—	—	1	1	—	—	—	—	3	3
Taurus	11	11	—	—	2	2	4	4	1	1	4	4
Gemini	6	6	—	—	—	—	2	2	—	—	4	4
Canis minor	2	2	—	—	—	—	—	—	—	—	2	2
Cancer	4	4	—	—	—	—	2	2	—	—	2	2
Leo	6	6	—	—	—	—	—	—	2	2	4	4
Bootes	3	3	—	—	—	—	1	1	—	—	2	2
Corona Borealis	2	2	—	—	1	1	—	—	1	1	—	—
Hercules	2	2	—	—	—	—	—	—	2	2	—	—
Lyra	2	2	—	—	2	2	—	—	—	—	—	—

Constellation.	No.	Northern alone.	3	Northern alone.	4	Northern alone.	5	Northern alone.	6	Northern alone.	Omits.	Northern alone.
Cycnus	1	1	—	—	—	—	—	—	—	—	1	1
Sagitta	1	1	—	—	—	—	—	—	1	1	—	—
Delphinus	3	3	—	—	1	1	—	—	2	2	—	—
Orion	10	7	—	—	1	1	—	—	4	4	5	2
Lepus	1	—	—	—	—	—	1	—	—	—	—	—
Hydra	3	1	—	—	—	—	1	—	—	—	2	1
Virgo	11	7	—	—	1	1	2	1	2	1	6	4
Aquila et Antinous	2	2	1	1	1	1	—	—	—	—	—	—
Libra	1	—	—	—	—	—	—	—	—	—	1	—
Sagittarius	5	—	—	—	1	—	3	—	1	—	—	—
Capricornus	13	—	—	—	3	—	4	—	4	—	2	—
Aquarius	7	1	—	—	1	—	4	1	—	—	2	—
	164	129	1	1	20	15	32	19	29	23	82	71

Tycho's Nebulae

Constellation.	No.	Northern alone.
Cancer	1	1
Hercules	1	1
Capricornus	3	—
	5	2

Ptolemy makes

5	Northern alone.	6	Northern alone.	νεφ.	Northern alone.
—	—	—	—	1	1
—	—	1	1	—	—
1	—	2	—	—	—
1	—	3	1	1	1

Comparison of Ptolemy and the Uranometria.

Northern Constellations.

Enter Argelander's unreduced magnitude at the top and Ptolemy's at the side. The table shows the numbers of stars.

	1	1.2	2.1	2	2.3	3.2	3	3.4	4.3	4	4.5	5.4	5	5.6	6.5	6	Var.	Cl.
1	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	—	1	3	15	4	1	1	—	—	—	—	—	—	—	—	—	1	—
3	—	—	—	5	9	8	27	28	9	12	4	1	1	—	1	—	4	—
4	—	—	—	—	1	1	11	18	27	52	41	37	41	8	3	8	1	—
5	—	—	—	—	—	—	—	1	1	4	10	10	39	12	9	23	—	—
6	—	—	—	—	—	—	—	—	—	—	1	1	5	3	6	16	—	—
ἀμ.	—	—	—	—	—	—	—	—	1	—	3	3	—	1	1	—	—	—
νεφ.	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
Absent	—	—	—	—	—	1	1	2	16	26	31	276	83	54	101	3	8	

In what follows, I shall employ my finally adopted scale of magnitudes, for which $\log \rho = 0.4402$.

Σ will denote the algebraical sum; S the sum without regard to signs. Quantities marked with a star have been obtained from the observations; the others by calculation.

Comparison of Ptolemy and Hevelius.

$m_{\Pi} - m_{H}$.

For Red Stars of Schmidt's Catalogue.

	Number.	Magnitude.	Square.
*$\Sigma\Delta \div n$ (Southern Stars only)	34	— .059	.0035

Comparison of Ptolemy and Sir Wm. Herschel.

$m_{\Pi} - m_{H}$.

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	286½	141.3	—
* " " − "	298½	166.7	—
$\Sigma\Delta$	585	— 25.4	—
$S\Delta$	585	308.0	—
*$\Sigma\Delta^2$	585	—	263.14
$\frac{\Sigma\Delta}{n}$	—	— .0434	.0019
$1.2533 \frac{S\Delta}{n}$	—	.6598	.4353
$\sqrt{\frac{\Sigma\Delta^2}{n}}$	—	.6710	.4502

Northern Stars only.

*Sum of + differences II.'s 2nd mag.	7	2.5	—
* " " − " " " 3rd "	11	4.9	—
* " " + " " " "	30	15.6	—
* " " − " " " 4th "	61	42.2	—
* " " + " " " "	96	53.6	—
* " " − " " " 5th "	100	53.1	—
* " " + " " " "	52	22.9	—
* " " − " " " 6th "	37	19.0	—
* " " + " " " "	20	12.4	—
* " " − " " " "	4	0.9	—
Sum of + differences all mags	205	107.0	—
" " − " " "	213	120.1	—
$\Sigma\Delta$	418	— 13.1	—
$S\Delta$	418	227.1	—
$\frac{\Sigma\Delta}{n}$	—	— .0313	.0010
$1.2533 \frac{S\Delta}{n}$	—	.6809	.4646

Red Stars of Schmidt's Catalogue.

*$\frac{\Sigma\Delta}{n}$	64	+ .233	.0543

Comparison of Ptolemy and Sir John Herschel.

$m_{\Pi} - m_{h}$.

	Numbers.	Magnitudes.	Squares.
*Sum of — differences	153	86.83	—
* „ „ + „	176	114.16	—
$\Sigma\Delta$	329	+ 25.33	—
$S\Delta$	329	200.99	—
*$\Sigma\Delta^2$	329	—	188.2510
$\frac{\Sigma\Delta}{n}$	—	+ .0769	.0059
$1.2533\frac{S\Delta}{n}$	—	.7656	.5861
$\sqrt{\frac{\Sigma\Delta^2}{n}}$	—	.7564	.5722

Stars to 15° S. only.

*Sum of + differences Π.'s 2nd mag.	11	4.24	—
* „ „ — „ „ „ „	17	8.25	—
* „ „ + „ „ 3rd „	38	18.54	—
* „ „ — „ „ „ „	55	33.89	—
* „ „ + „ „ 4th „	39½	31.74	—
* „ „ — „ „ „ „	18½	4.76	—
* „ „ + „ „ 5th „	6	5.61	—
* „ „ — „ „ „ „	0	.00	—
Sum of + differences all mags	94½	60.13	—
„ „ — „ „ „ „	90½	46.90	—
$\Sigma\Delta$	185	+ 13.23	—
$S\Delta$	185	107.03	—
$\frac{\Sigma\Delta}{n}$	—	+ .0715	.0051
$1.2533\frac{S\Delta}{n}$	—	.7250	.5256

Northern Stars only.

*Sum of + differences Π.'s 2nd mag.	9	3.81	—
* „ „ — „ „ „ „	17	7.38	—
* „ „ + „ „ 3rd „	34	16.66	—
* „ „ — „ „ „ „	40	21.05	—
* „ „ + „ „ 4th „	29	24.25	—
* „ „ — „ „ „ „	7	1.11	—
* „ „ + „ „ 5th „	2	3.40	—
* „ „ — „ „ „ „	0	0	—
$\Sigma\Delta$	138	+ 18.58	—
$S\Delta$	138	77.66	—
$\frac{\Sigma\Delta}{n}$	—	+ .1346	.0181
$1.2533\frac{S\Delta}{n}$	—	.7053	.4974

Red Stars of Schmidt's Catalogue.

	Numbers.	Magnitudes.	Squares.
*$\frac{\Sigma\Delta}{n}$	43	+ .402	.1616

Comparison of Ptolemy and the Durchmusterung.

$m_{\Pi} - m_{DM.}$

	Numbers.	Magnitudes.	Squares.
*Sum of + differences Π.'s 2nd mag.	4	2.30	—
* „ „ — „ „ „ „	19	5.34	—
* „ „ + „ „ 3rd „	39	14.24	—
* „ „ — „ „ „ „	65	40.94	—
* „ „ + „ „ 4th „	122	58.71	—
* „ „ — „ „ „ „	127	64.02	—
* „ „ + „ „ 5th „	66	38.15	—
* „ „ — „ „ „ „	44	18.47	—
* „ „ + „ „ 6th „	17	10.84	—
* „ „ — „ „ „ „	15	2.53	—
„ „ + „ all mags	248	124.24	—
„ „ — „ „ „	270	131.30	—
$\Sigma\Delta$	518	— 7.06	—
$S\Delta$	518	255.54	—
$\frac{\Sigma\Delta}{n}$	—	— .0136	.0002
$1.2533\frac{S\Delta}{n}$	—	.6185	.3825

Schmidt's Red Stars.

$\frac{\Sigma\Delta}{n}$	57	+ .224	.0502

Comparison of Ptolemy and Seidel.

$m_{\Pi} - m_{\odot}$.

	Numbers.	Magnitudes.	Squares.
*Sum of + differences Π.'s 2nd mag.	12	4.51	—
* „ „ — „ „ „ „	18	8.67	—
* „ „ + „ „ 3rd „	48	24.54	—
* „ „ — „ „ „ „	55	23.80	—
* „ „ + „ „ 4th „	52	39.86	—
* „ „ — „ „ „ „	4	2.42	—
* „ „ + „ „ 5th „	2	3.62	—
* „ „ — „ „ „ „	0	0	—
$\Sigma\Delta$	133	— 3.42	—
$S\Delta$	133	61.52	—
$\frac{\Sigma\Delta}{n}$	—	.0257	.0007
$1.2533\frac{S\Delta}{n}$	—	.5797	.3361

Schmidt's Red Stars.

	Numbers.	Magnitudes.	Squares.
$\frac{\Sigma\Delta}{n}$	36	+ .247	.0610

Comparison of Hevelius and Sir Wm. Herschel.

$m_H - m_H$.

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	440½	155.8	—
* „ „ — „	538½	247.8	—
$\Sigma\Delta$	979	− 92.0	—
$S\Delta$	979	403.6	—
$\frac{\Sigma\Delta}{n}$	—	− .0940	.0088
$1.2533\frac{S\Delta}{n}$	—	.5168	.2671

Schmidt's Red Stars.

$\frac{\Sigma\Delta}{n}$	79	+ .032	—
$\frac{\Sigma\Delta}{n}$ (Southern Stars only)	27	− .097	—

Comparison of Hevelius and Sir John Herschel.

$m_H - m_h$.

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	129½	55.86	—
* „ „ — „	123½	60.03	—
$\Sigma\Delta$	253	− 4.17	—
$S\Delta$	253	115.89	—
$\frac{\Sigma\Delta}{n}$	—	− .0165	.0003
$1.2533\frac{S\Delta}{n}$	—	.5740	.3295

Schmidt's Red Stars.

$\frac{\Sigma\Delta}{n}$	46	+ .194	—
$\frac{\Sigma\Delta}{n}$ (Southern Stars only)	20	+ .194	—

Comparison of Hevelius and the Durchmusterung.

$m_H - m_{DM.}$

	Numbers.	Magnitudes.	Squares.
*Sum of + differences H.'s 2nd mag.	20	4.82	—
* „ „ — „ „ „ „	8	5.04	—

	Numbers.	Magnitudes.	Squares.
*Sum of + differences H.'s 3rd mag.	38	11.13	—
* „ „ — „ „ „ „	59	31.22	—
* „ „ + „ „ 4th „	85	25.29	—
* „ „ — „ „ „ „	128	63.37	—
* „ „ + „ „ 5th „	177	48.75	—
* „ „ — „ „ „ „	162	81.48	—
* „ „ + „ „ 6th „	198	80.58	—
* „ „ — „ „ „ „	154	55.59	—
* „ „ + „ „ fractional mags	15	3.40	—
* „ „ — „ „ „ „	17	7.90	—
„ „ + „ „ all „	533	173.97	—
„ „ — „ „ „ „	528	244.60	—
$\Sigma\Delta$	1061	− 70.63	—
$S\Delta$	1061	418.57	—
$\frac{\Sigma\Delta}{n}$	—	− .0666	.0044
$1.2533\frac{\Sigma\Delta}{n}$	—	.4906	.2407

Schmidt's Red Stars.

$\frac{\Sigma\Delta}{n}$	70	+ .147	—

Comparison of Hevelius and Seidel.

$m_H - m_S$.

Schmidt's Red Stars.

	Numbers.	Magnitudes.	Squares.
$\frac{\Sigma\Delta}{n}$	35	+ .026	—

Comparison of the two Herschels.

$m_H - m_h$.

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	109	34.95	—
* „ „ — „	93	30.72	—
$\Sigma\Delta$	202	+ 4.23	—
$S\Delta$	202	65.67	—
$\frac{\Sigma\Delta}{n}$	—	+ .0209	.0004
$1.2533\frac{S\Delta}{n}$	—	.4074	.1660
$*\sqrt{\frac{\Sigma\Delta^2}{n}}$	—	.4520	.2043

Northern Stars only.

*Sum of + differences	52½	13.39	—
* „ „ — „	58½	15.15	—

	Numbers.	Magnitudes.	Squares.
$\Sigma\Delta$	111	− 1.76	—
$S\Delta$	111	28.54	—
$\frac{\Sigma\Delta}{n}$	—	− .0158	.0002
$1.2533\frac{S\Delta}{n}$	—	.3222	.1038

Southern Stars only.

*Sum of + differences	$44\frac{1}{2}$	17.52	—
*" " − "	$50\frac{1}{2}$	19.58	—
$\Sigma\Delta$	95	− 2.06	—
$S\Delta$	95	37.10	—
$\frac{\Sigma\Delta}{n}$	—	− .0217	.0005
$1.2533\frac{S\Delta}{n}$	—	.4895	.2396

Schmidt's Red Stars.

$\frac{\Sigma\Delta}{n}$	34	+ .018	—

Comparison of Sir Wm. Herschel and the Durchmusterung.

$m_H - m_{DM.}$

	Numbers.	Magnitudes.	Squares.
*Mean discrepancy from 75 groups	432	0.440	.1936

Schmidt's Red Stars.

$*\frac{\Sigma\Delta}{n}$	60	+ 0.004	—

Comparison of Sir Wm. Herschel and Seidel.

$m_H - m_{\odot}$

	Numbers.	Magnitudes.	Squares.
$*1.2533\frac{S\Delta}{n}$	167	0.397	.1576
$*\sqrt{\frac{\Sigma\Delta^2}{n}}$	167	.418	.1747

Schmidt's Red Stars.

$*\frac{\Sigma\Delta}{n}$	31	− .188	—

Comparison of Sir John Herschel and the Durchmusterung.

$m_h - m_{DM.}$

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	$76\frac{1}{2}$	22.69	—
*" " − "	$65\frac{1}{2}$	15.33	—

	Numbers.	Magnitudes.	Squares.
$\Sigma\Delta$	142	+ 7.36	—
$S\Delta$	142	38.02	—
$\frac{\Sigma\Delta}{n}$	—	+ .0518	.0027
$1.2533\frac{S\Delta}{n}$	—	.3380	.1142

Schmidt's Red Stars.

$*\frac{\Sigma\Delta}{n}$	31	− .032	—

Comparison of Sir John Herschel and Seidel.

$m_h - m_{\odot}$

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	80	19.75	—
*" " − "	73	17.40	—
$\Sigma\Delta$	153	+ 2.35	—
$S\Delta$	153	37.15	—
$\frac{\Sigma\Delta}{n}$	—	+ .0154	.0002
$1.2533\frac{S\Delta}{n}$	—	.3045	.0927

Schmidt's Red Stars.

$*\frac{\Sigma\Delta}{n}$	33	− .181	—

Comparison of the Durchmusterung and Seidel.

$m_{DM.} - m_{\odot}$

	Numbers.	Magnitudes.	Squares.
*Sum of + differences	80	17.87	—
*" " − "	90	21.25	—
$\Sigma\Delta$	170	− 3.38	—
$S\Delta$	170	39.12	—
$\frac{\Sigma\Delta}{n}$	—	− .0199	.0004
$1.2533\frac{S\Delta}{n}$	—	.2884	.0832
$*\Sigma\Delta^2$	—	—	14.8761
$\frac{\Sigma\Delta^2}{n}$	—	—	.0880

Schmidt's Red Stars.

$*\frac{\Sigma\Delta}{n}$	35	− .139	—

Comparison of the Durchmusterung and Zöllner.

$m_{DM.} - m_Z$.

	Numbers.	Magnitudes.	Squares.
Sum of + differences	104	28.52	—
" " − "	115	35.58	—
$\Sigma\Delta$	219	− 7.06	—
$S\Delta$	219	64.10	—
$\frac{\Sigma\Delta}{n}$	—	− .0322	.0010
$1.2533\frac{S\Delta}{n}$	—	.374	.1399
Schmidt's Red Stars.			
$\frac{\Sigma\Delta}{n}$	25	+ .127	—

Comparison of Seidel and Zöllner.

$m_{\mathfrak{S}} - m_Z$.

91 Stars in 16 Groups, half determined by $\mathfrak{S}$; 42 + differences, and 49 − differences.

	Numbers.	Magnitudes.	Squares.
$S\Delta$	—	18.82	—
$1.2533\frac{S\Delta}{n}$	—	.272	.0740
Schmidt's Red Stars.			
$\frac{\Sigma\Delta}{n}$	15	+ .268	—

Comparison of the unreduced magnitudes of Argelander and Heis, for Northern Stars only.

$\mathfrak{H}$.

A. Mags.	1	1.2	2.1	2	2.3	3.2	3	3.4	4.3	4	4.5	5.4	5	5.6	6.5	6	6.7	Var.	Sums.
1	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	6
1.2	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
2.1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
2	—	—	—	18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	18
2.3	—	—	—	1	12	—	—	—	—	—	—	—	—	—	—	—	—	—	14
3.2	—	—	—	—	1	7	1	1	—	—	—	—	—	—	—	—	—	1	10
3	—	—	—	—	—	3	29	4	—	—	—	—	—	—	—	—	—	—	36
3.4	—	—	—	—	—	—	1	44	1	1	—	—	—	—	—	—	—	—	49
4.3	—	—	—	—	—	—	—	3	24	9	1	—	—	—	—	—	—	2	37
4	—	—	—	—	—	—	—	—	5	74	11	3	—	—	—	—	—	—	96
4.5	—	—	—	—	—	—	—	—	1	7	64	10	1	—	—	—	—	3	83
5.4	—	—	—	—	—	—	—	—	—	—	2	74	12	1	—	—	—	—	90
5	—	—	—	—	—	—	—	—	—	—	5	17	241	71	17	3	1	1	356
5.6	—	—	—	—	—	—	—	—	—	—	—	—	8	72	20	—	—	1	101
6.5	—	—	—	—	—	—	—	—	—	—	—	—	4	17	112	35	2	1	171
6	—	—	—	—	—	—	—	—	—	—	—	1	5	40	205	884	123	1	1259
Var.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10	10
Omitted	—	—	—	—	—	—	—	—	—	—	—	—	3	3	17	152	1410	10	1595
Sums	5.	3	2	19	13	10	31	52	31	91	83	105	274	204	371	1074	1536	32	3936

Probable Errors of the Different Observers.

There are two methods of calculating the probable errors of the different observers, from a comparison of their magnitudes. The first of these is due to Argelander; and depends upon the comparison of three observers with each other. Let ε_1, ε_2, and ε_3, be the mean errors of the three observers, and η_{12}, η_{13}, and η_{23}, the mean discrepancies of the three pairs of observers. Then, according to the principles of the method of least squares,

$$\varepsilon_1^2 + \varepsilon_2^2 = \eta_{12}^2,$$
$$\varepsilon_1^2 + \varepsilon_3^2 = \eta_{13}^2,$$
$$\varepsilon_2^2 + \varepsilon_3^2 = \eta_{23}^2.$$

Consequently, if we write σ for $\frac{1}{2}(\eta_{12}^2 + \eta_{13}^2 + \eta_{23}^2)$, we have

$$\varepsilon_1^2 = \sigma - \eta_{23}^2,$$
$$\varepsilon_2^2 = \sigma - \eta_{13}^2,$$
$$\varepsilon_3^2 = \sigma - \eta_{12}^2.$$

The other method of calculating the probable errors is applicable only in special cases. It depends upon a principle stated on page 12, according to which if two observers have had their scales of magnitudes reduced to the scale of equable distribution, by the method of Chapter II, there will be a discrepancy in the mean magnitude of any large number of stars, according to the two observers, which will be equal to .608 of the difference of the squares of their mean errors; so that if θ denote this discrepancy, we have the two equations

$$\varepsilon_1^2 + \varepsilon_2^2 = \eta_{12}^2,$$
$$\text{and } \varepsilon_1^2 - \varepsilon_2^2 = \frac{\theta}{.608},$$

whence $\varepsilon_1^2 = \frac{1}{2}\left(\eta_{12}^2 + \frac{\theta}{.608}\right)$, and $\varepsilon_2^2 = \frac{1}{2}\left(\eta_{12}^2 - \frac{\theta}{.608}\right)$.

There are several circumstances which detract from the accuracy of these methods, and sometimes render them useless. In the first place, there is a certain part of every observers error which is due simply to the fact that the smallest divisions of a magnitude which he employs have finite values; and when two different observers divide magnitudes differently, discrepancies between them will arise from this source, which are entirely independent of any real difference between their estimates of the magnitudes of the stars. Two observers might agree entirely as to the relative brightness of all the stars, and yet, simply from making the divisions between their different classes at different points, apparent discrepancies would be introduced, which would not disappear under any possible system of reducing their magnitudes. Another im-

perfection in these methods of calculating the probable errors arises from the different susceptibility of the eyes of different observers to light of different colors, whether it be due to some organic cause, or to differences in atmosphere or methods of observation. All methods of calculating probable errors *à posteriori*, depend upon the principles of least squares, and consequently suppose that no one source of error preponderates over the rest, and every circumstance which can introduce large discrepancies between the magnitudes of different observers, must more or less invalidate the calculated probable errors. All those observers who make their observations upon stars reduced in brightness must, from the physiological principle referred to upon page 6, make the red stars too faint. Seidel's observations, for example, upon stars greatly out of focus, show this effect; and Ptolemy and Hevelius, who probably estimated magnitudes while observing stars through pin-holes, also make red stars too faint. On the other hand, observers who estimate magnitudes under a very clear atmosphere, or with the aid of telescopes, must make the red stars too bright. Another source of discrepancies which are more or less constant, and therefore take off from the worth of the results obtained for the probable errors, is that the observers have occupied stations differing greatly in latitude. Ptolemy and Hevelius make low stars to faint, but Heis makes the low southern stars rather too bright, as compared with Behrmann and Sir John Herschel. The variability of stars interferes also with the application of these methods, and other circumstances may occasionally injure or destroy the value of the results obtained. I have found, as a fact, that the comparisons of Sir J. Herschel with Ptolemy and with Hevelius cannot be made use of, the discrepancies in both cases being much too large.

The following table shows the values of the squares of the mean discrepancies. I have always preferred the value derived from the formula $\frac{\Sigma\Delta^2}{n} - \left(\frac{\Sigma\Delta}{n}\right)^2$, and when I had not this value I used $\left(\frac{S\Delta}{n}\right)^2 \frac{\partial}{2}$.

Observers.	*(Discrepancy)²*	*Observers.*	*(Discrepancy)²*
Π ♅	.4483	♅ DM	.1936
Π DM	.3825	♅ ♋	.1576
Π ♋	.3361	h DM	.1142
H ♅	.2671	h ♋	.0927
H DM	.2407	DM ♋	.0876
♅ h	.2039		

The following values are adopted for $\frac{\Sigma\Delta}{n}$:

Π DM	.1632
H DM	.0666.

The value for Ptolemy and the Durchmusterung has been obtained by striking out all of Π's 5th and 6th magnitude stars, and allowing only one fourth weight to his 4th magnitude; on account of his great number of omissions of faint stars.

The resulting squares of mean errors obtained from these figures are as follows:

Triad.	Π.	H.	♅.	h.	DM.	♋.
Π ♅ DM	.3186	—	.1297	—	.0639	—
Π ♅ ♋	.3134	—	.1349	—	—	.0227
Π DM ♋	.3155	—	—	—	.0670	.0206
H ♅ DM	—	.1571	.1100	—	.0836	—
♅ h DM	—	—	.1417	.0622	.0519	—
♅ h ♋	—	—	.1344	.0695	—	.0232
♅ DM ♋	—	—	.1318	—	.0618	.0258
h DM ♋	—	—	—	.0596	.0545	.0330
Pair.						
Π DM	.3254	—	—	—	.0570	—
H DM	—	.1751	—	—	.0656	—

The probable errors corresponding to these squares of mean errors are shown in the following table:

	Π.	H.	♅.	h.	DM.	♋.
Π ♅ DM	.3807	—	.2429	—	.1705	—
Π ♅ ♋	.3776	—	.2477	—	—	.1016
Π DM ♋	.3745	—	—	—	.1746	.0968
H ♅ DM	—	.2673	.2237	—	.1950	—
♅ h DM	—	—	.2539	.1682	.1537	—
♅ h ♋	—	—	.2473	.1778	—	.1027
♅ DM ♋	—	—	.2449	—	.1677	.1083
h DM ♋	—	—	—	.1647	.1575	.1225
Π DM	.3848	—	—	—	.1610	—
H DM	—	.2822	—	—	.1728	—

From these numerical results, I prefer to select the most probable values by an exercise of judgment rather than by any rigid rule. The following cannot be far wrong.

Observer.	*(Mean Error)²*	*Probable Error.*
Π	.3155	.3745
H	.1571	.2673
♅	.1327	.2457
h	.0695	.1778
DM	.0642	.1709
♋	.0231	.1025

We may now make use of these values for the Durchmusterung and for Seidel to calculate the probable errors of Zöllner and of myself. For Zöllner we have,

	ε^2.
From DM	.0757
„ ℭ	.0509

Whence we may take Z's probable error as .1690 and his mean error square as .0625. For myself, we have,

	ε^2.
From DM	.0528
„ ℭ	.0682

Whence P's probable error may be taken as .1650 and his mean error square as .0600.

It will be seen that the results both for Zöllner and for me present great discrepancies. These are undoubtedly owing to faulty reductions of the scales, and must, I think, be ultimately ascribed to the fact that the scale of equable distribution of Chapter II is not an equiphotometric scale. Seidel's observations being confined chiefly to 2nd and 3rd magnitude stars (for the first magnitude stars are not brought into the comparisons with non-photometric observers) do not extend over too great a range to be capable of being brought into agreement with a scale of equable distribution. If for my observations two scales are adopted; one for the bright stars, which are thus made to agree as nearly as possible with Seidel, and the other for the faint ones, which are brought in a similar manner into agreement with the *Durchmusterung*, the probable errors become

for the faint stars	.125
„ „ bright stars	.111.

This is therefore the only part of the error which is due to observation. Of this, a great part is of a systematic character depending on the Right Ascension, and would disappear if the distant groups were only better bound together.

It was at first my intention to compare every tenth group with a group of stars about the pole, as Prof. Winlock also urged me to do; but it was found that stars so low as the pole could not be observed. The subjoined table shows the algebraical mean of the differences between me and the Durchmusterung, for stars in successive series of ten groups, and also the same mean difference between me and the best attainable magnitude for a few stars observed either by Seidel or by Sir John Herschel or found in the small list of stars very accurately observed by Heis and given in his *De Magnitudine Numerosa Stellarum*. A constant has been added in both cases, to make the sum of the differences equal to zero.

Groups.	*No. of Stars.*	DM — P.	*No. of Stars.*	Best mag — P.
LXVI to VI	49	+m27	5	+m21
VI „ XVI	51	+ .23	2	+ .05

Groups.	*No. of Stars.*	DM — P.	*No. of Stars.*	Best mag. — P.
XVI to XXVI	49	+m10	7	+m07
XXVI „ XXXVI	46	— .30	1	— .36
XXXVI „ XLVI	43	— .19	5	— .14
XLVI „ LVI	52	— .01	4	+ .09
LVI „ LXVI	60	— .06	4	— .14

It is evident from the agreement which this table exhibit between the Durchmusterung and the entirely independent observations of Seidel and the others, that the right ascension correction is to be applied only to my observations. I find that the means of the excess of the Durchmusterung magnitudes over mine, for the four successive quadrants, are as follows:

0^h to 6^h	+m116
6 „ 12	+ .109
12 „ 18	— .273
18 „ 24	— .091

whence we find, that the correction to my magnitudes is

$$-0^m04 + 0^m24 \sin(\alpha + 1^h11^m).$$

The mean square of the discrepancy from the Durchmusterung, of the magnitudes so corrected, is .0835; and therefore *the probable error of my corrected magnitudes is*

$$\pm 0^m0937.$$

This does not apply either to the few very bright stars, or to the extra stars whose magnitudes are marked with a colon, in the Catalogue. For both these classes, my error is greater. On the other hand, this numerical value of my probable error is somewhat too large, owing to its depending on comparison with a scale of equable distribution, instead of an isophotometric scale. The formula of correction is of an artificial character, and consequently the corrections have by no means as good an effect as a reöbservation of distant groups would have. I regret that circumstances have not allowed me to make such reöbservations.

The following table shows the corrections to be applied to my magnitudes of the different stars in the Catalogue on pp. 128—138, in order to obtain my finally corrected magnitudes.

Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.
1— 4	+ 0m04	22, 23	+ 0m10	46— 48	+ 0m16	119—123	+ 0m18
5	+ 0.05	24—27	+ 0.11	49— 53	+ 0.17	124—127	+ 0.17
6— 8	+ 0.06	28—34	+ 0.12	54— 67	+ 0.18	128—131	+ 0.16
9—15	+ 0.07	35—37	+ 0.13	68— 81	+ 0.19	132—136	+ 0.15
16	+ 0.08	38—40	+ 0.14	82—107	+ 0.20	137—139	+ 0.14
17—21	+ 0.09	41—45	+ 0.15	108—118	+ 0.19	140	+ 0.11

Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.	Nos. of Catalogue.	Cor.
141	+ 0ᵐ10	195—198	— 0ᵐ08	274—278	— 0ᵐ25	428—433	— 0ᵐ14
142	+ 0.09	199	— 0.09	279—282	— 0.26	434—437	— 0.13
143—148	+ 0.08	200—206	— 0.10	283—289	— 0.27	438—444	— 0.12
149, 150	+ 0.07	207—209	— 0.11	290—328	— 0.28	445—447	— 0.11
151, 152	+ 0.06	210	— 0.12	329—346	— 0.27	448—450	— 0.09
153—157	+ 0.05	211—216	— 0.13	347—349	— 0.26	451, 452	— 0.08
158, 159	+ 0.04	217	— 0.14	350—361	— 0.25	454—456	— 0.07
160, 161	+ 0.03	218—223	— 0.15	362—368	— 0.24	457	— 0.06
162	+ 0.02	224—228	— 0.16	369—372	— 0.23	458, 459	— 0.05
163—168	0.00	229—232	— 0.17	373—380	— 0.22	460—464	— 0.04
169—176	— 0.01	233—238	— 0.18	381—388	— 0.21	465—469	— 0.03
177, 178	— 0.02	239—244	— 0.19	389—393	— 0.20	470—478	— 0.02
179, 180	— 0.03	245—254	— 0.20	394—403	— 0.19	479—482	— 0.01
181—187	— 0.04	255, 256	— 0.21	404—407	— 0.18	483—486	0.00
188—190	— 0.05	257—261	— 0.22	408—415	— 0.17	487—490	+ 0.01
191	— 0.06	262—269	— 0.23	416—424	— 0.16	491	+ 0.02
192—194	— 0.07	270—273	— 0.24	425—427	— 0.15	492—494	+ 0.03

On the General Variability of Stars.

It is obvious that there must be a certain portion of the mean square of the discrepancy between any two observers, which is due to the variability of the stars. This will be a function of t, the interval of time between their epochs, and we may denote it by φt. In consequence of this, the mean square of the discrepancy between two observers is equal to

$$\varepsilon_1^2 + \varepsilon_2^2 + \varphi t_{12}.$$

If, therefore, the square of the mean error of a certain observer, A, is calculated according to the method of Argelander, from comparison with two other observers, B and C, it will be affected by an error which is equal to

$$\tfrac{1}{2}\left(\varphi t_{AB} + \varphi t_{AC} - \varphi t_{BC}\right) = \tfrac{1}{2}\left(\varphi t_{AB} + \varphi t_{AC} + \varphi t_{BC}\right) - \varphi t_{BC}.$$

Since φt is a positive quantity which increases with t, it is obvious that the algebraical value of the error so introduced into the results for ε_A^2, ε_B^2, and ε_C^2, will be smallest for the observer who occupies an intermediate chronological position. Let us call the earliest and latest of the three observers the extremes, and the intermediate one the mean. Then we find from the table given above, that the excess of the deduced square of the mean error of Sir Wm. Herschel, over that of the Durchmusterung, when Herschel is an extreme, and the Durchmusterung is a mean, amounts to .0700; but when Herschel is a mean, and the Durchmusterung an extreme, it is only .0658. Confining our attention to the four best triads, which are Π H DM, Π H ℌ,

H h ℌ, and H DM ℌ, we find the following values of ε^2. For the DM as extreme, .0639; as mean, .0618. For Sir Wm. Herschel, as extreme, .1344 and .1318; as mean, .1349 and .1297. It appears from these numbers that the general variability of stars is a barely appreciable quantity.

With regard to the nature of the function φ, three hypotheses are possible. First, it may be supposed that the general variability of stars is such that the variation during any given interval of time, say a year, is entirely independent of the variation during the preceding year, not being subject to any law at all, except the general laws of probability. In this case, the accumulated variability, in any period of time, would be proportional to the square root of the interval, and φt would be proportional to t. In the second place, we may suppose that the variability of stars generally conforms to a certain law, and that it is, in fact, progressive; so that if a star has been increasing in brightness for a year, the probability that it will increase in brightness during the next year, is greater than if it had been diminishing. In this case, the accumulated variation, in long periods of time, will be relatively greater, as compared with the variations in short times, than if there had been no such law. Finally, we may suppose that the variability of stars has generally more or less of a cyclical character; so that if a star has been increasing in brightness it is more likely to diminish during the next year, than if it had been decreasing. In this case, the variations during long periods will accumulate less rapidly than if there were no law; and if the variability of all stars is cyclical, the mean discrepancy between two observers which accumulates on this account will approximate to a constant value. On account of the immense intervals of time which separate Ptolemy and Sûfi from modern observers, if there was anything more than an extremely minute general variability of the first or second nature, it must have produced changes in the general aspect of the heavens: but it is sufficient to compare the table given by Schjellerup of the magnitudes of Sûfi and Argelander, to convince oneself that however good an observer Sûfi may have been (and there is internal evidence to show that he is decidedly inferior, for example to Sir Wm. Herschel) the general change in this period of 900 years cannot have amounted to a quarter of a magnitude. If therefore we accept the numbers which have just been cited as evidence of the existence of a general variability, it is certain that this variability is of a cyclical character, and that the mean period is not a very long one.

CHAPTER V.

ON THE FORM OF THE GALACTIC CLUSTER.

The chief end of observations of the magnitudes of stars is to determine the form of the cluster in which our sun is situated. Sir Wm. Herschel assumed the star-density to be homogeneous throughout the cluster, and sought to determine by his gauges the shape of its exterior surface. Struve assumed the condensation of stars to be in parallel planes, and sought to ascertain the numerical law of the decrease in density with the distance from the central plane. I shall make no attempt to obtain numerical accuracy in my inferences upon this subject, but shall content myself with endeavoring to show the general forms of the surfaces of equal star-density throughout the cluster. I shall make the assumption that the proportions of stars of different intrinsic lights is the same in all parts of space. I shall at first proceed as if the intrinsic light of all stars were the same, and I shall afterwards inquire how far the conclusions so derived are affected by the assumption of the greatest possible variation in the intrinsic light of the stars.

Let L be the intrinsic light of a star;

l, its apparent light;

r, its distance;

$\mathrm{FL}.d\mathrm{L}$, the proportion of stars whose intrinsic light lies between L and $\mathrm{L}+d\mathrm{L}$ (which I assume to be the same throughout space);

$fl.dl$, the number of stars whose apparent light lies between l and $l+dl$;

φr, the number of stars in a unit of space at the distance r.

Our problem is to determine the general forms of the surfaces for which φr is constant. The given quantities are the values of $\int fl.dl$. Let us consider a portion of the heavens equal to $\frac{1}{4\partial}$ of the entire sphere. The volume of space in that direction, at the distance r, is equal to $r^2.dr$, and the number of stars in that space is equal to $\varphi r.r^2.dr$. Then the number of stars in that space whose intrinsic brightness lies between L and $\mathrm{L}+d\mathrm{L}$ is equal to $\mathrm{FL}.\varphi r.r^2.dr.d\mathrm{L}$. Now we have

$$l = \frac{\mathrm{L}}{r^2} \tag{1}$$

and therefore, substituting for L its value as given by this equation, and integrating, we have for the total number of stars in the part of the heavens just defined, whose apparent brightness lies between l and $l+dl$,

$$fl.dl = dl\int_0^\infty \mathrm{F}(lr^2).\varphi r.r^4.dr. \tag{2}$$

If φr had a constant value, b, throughout space, we should have, on substituting L for lr^2,

$$fl.dl = \left(\tfrac{1}{2}b\int_0^\infty \mathrm{FL}.\mathrm{L}^{\frac{1}{2}}.d\mathrm{L}\right)l^{-\frac{3}{2}}.dl.$$

or

$$\int fl.dl = -\left(\tfrac{1}{3}b\int_0^\infty \mathrm{FL}.\mathrm{L}^{\frac{3}{2}}.d\mathrm{L}\right)l^{-\frac{3}{2}}.$$

The numbers of stars as bright or brighter than any given apparent light, ought therefore to be inversely proportional to the three-halves power of the light. Now we have found, in Chapter II, that, denoting by $\nu(m)$ the number of stars as bright or brighter than magnitude m, in the Northern heavens, we have

$$m = -\tfrac{1}{3} + 1.89\ldots\log.\nu(m)$$

and this would give, according to the formula just obtained, 0.352 for the logarithm of the ratio of light between successive magnitudes. This value is, however, considerably smaller than has ever been found from observation. In fact, a single glance at the heavens is sufficient to show that the stars are not uniformly distributed throughout space, but that there is a great concentration in the plane of the milky way. The effect of such a concentration obviously is to make the number of stars at a given distance proportional to something between r and r^2, so that $\nu(m)$ will become proportional to something between l^{-1} and $l^{-\frac{3}{2}}$, and the logarithm of the ratio of light between successive magnitudes will be between 0.528 and 0.352, which limits do in fact include all the observed values of this quantity.

The stars are concentrated toward the plane of the milky way; but is the star-density constant within that plane, or subject only to irregular variations? Supposing L to be equal to unity for all stars, we have

$$\varphi r.r^2.dr = fl.dl,$$

and consequently

$$\varphi r = 2l^{\frac{1}{2}}.fl. \tag{3}$$

Now Argelander, in his Preface to the third volume of the Durchmusterung, has given the following numbers of stars of different magnitudes contained in a sample of the milky way,

Mags.	Numbers.
7	562
8	2068
9	18916

These numbers are undoubtedly somewhat too small for the fainter stars, because, as Argelander has himself explained, there was some tendency to omit stars where they were the thickest. Notwithstanding this, if we take the limiting magnitudes from my

table on page 26, and, using Rosén's value for the ratio of light between successive magnitudes, calculate the mean densities of the stars at the distances corresponding to the different magnitudes, on the supposition that $L = 1$, we obtain the following values:

Mags.	Densities.
7	536
8	542
9	587

If, in place of using my reduced magnitudes, the unreduced magnitudes had been used, the increase in the density would have been still more marked. This increase of the density with the radius is not accidental, but proves that the denser parts of the milky way have really an annular form.

I next propose to consider the distribution of the brighter stars. For this purpose I divide the entire sphere of the heavens into 32 equal regions, as follows. First, a circular region around that pole of the milky way which lies in Coma Berenices, and which I term the Beronicean pole. Secondly, a similar region around the opposite pole of the milky way, which I term the Magellanic pole because the Magellanic Clouds are upon that side. Between these, five zones, parallel to the milky way, and each divided perpendicularly into six equal parts. These five zones I term respectively the Beronicean Apogalactic, the Beronicean Perigalactic, the Engalactic, the Magellanic Perigalactic, and the Magellanic Apogalactic zones. The different regions of each zone are distinguished by the letters A, B, C, D, E, and F, each of which is appropriated to a line of regions in the same galactic meridian. The numbers of stars of each magnitudes within each of these regions, as shown upon Heis's maps down to 20° S. and upon Behrmann's maps south of that parallel, are shown in the following tables. The 6th magnitude includes Heis's 6.7, which seems to have the same limit as Behrmann's 6th*.

Region.	1m	2m	3m	4m	5m	6m	*Cl. Neb.*
Beronicean Pole	1	—	4	6	41	164	2
Beronicean Apogalactic:							
A Virgo region . . .	1	3	6	6	18	132	—
B Leo region	1	3	4	11	22	159	—
C Leo minor region . .	—	—	11	9	34	174	—
D Dipper region . . .	—	7	8	4	40	190	1
E Hercules region . .	—	3	11	17	34	175	2
F Libra region . . .	—	3	7	9	26	143	1

* The numbers of stars between 20° and 30° S., according to Heis and Behrmann are as follows: —

Magnitude.	1	2	3	4	5	6	6.7
H.	1	4	12	31	78	119	68
B.	1	4	9	32	82	270	—

Region.	1	2	3	4	5	6	*Cl. Neb.*
Beronicean Perigalactic:							
A Centaurus region . .	—	—	7	10	38	149	1
B Hydra region	—	1	3	9	28	148	2
C Gemini region . . .	1	1	3	9	26	202	2
D Polaris region . . .	—	1	4	13	44	196	—
E Lyra region	1	1	6	17	40	201	1
F Ophiuchus region . .	1	4	8	9	36	96	1
Engalactic:							
A Crux region	4	2	11	20	61	182	4
B Argo region	—	1	3	11	43	178	2
C Capella and Procyon reg.	2	3	8	6	44	197	2
D Cassiepea region . .	—	4	6	17	53	228	3
E Cygnus region . . .	1	2	6	24	54	224	1
F Scutum region . . .	—	4	14	10	28	112	4
Magellanic Perigalactic:							
A Octans region . . .	—	2	—	16	53	151	—
B Canis major region . .	2	6	6	20	66	243	1
C Orion region	3	4	10	28	59	266	—
D Andromeda region . .	—	4	4	26	45	221	1
E Equuleus region . .	—	3	9	15	35	191	1
F Corona Australis region	—	1	3	6	32	128	1
Magellanic Apogalactic:							
A Hydrus region . . .	1	—	6	11	35	204	1
B Middle Eridanus region	—	—	2	14	38	152	—
C North Eridanus-region .	—	1	7	12	19	109	—
D Pisces ribbons region .	—	—	3	7	8	75	—
E Aquarius region . . .	—	—	4	13	32	100	—
F Piscis Austrinus region	1	2	2	10	36	123	—
Magellanic Pole	—	2	4	5	29	132	—
Summaries:							
Polar regions (×3) . .	3	6	24	33	210	888	—
Apogalactic (mean) . . .	2	11	40½	61½	171	868	—
Perigalactic (mean) . . .	4	14	31½	89	251	1096	—
Engalactic	7	16	48	88	283	1121	—
All except Engalactic:							
Beronicean side	6	27	92	129	427	2129	—
Magellanic side	7	25	60	183	487	2095	—
Series:							
A	7	7	30	63	205	818	—
B	3	11	18	65	197	880	—
C	6	9	39	64	182	948	—
D	0	16	25	67	190	910	—
E	2	9	36	86	195	891	—
F	2	14	34	44	158	602	—

The most striking fact in reference to these numbers is that the stars are as thick in the regions of the galactic poles as they are in the apogalactic regions, and that they are very nearly as thick in the perigalactic as in the engalactic regions. It seems to me that an important inference can safely be drawn from this circumstance. Let the stars differ in intrinsic light as much as they may, the number of them must at least depend upon the depth of the concentrated stratum through which we are looking, and the milky way could not be the result of a concentration of stars in parallel planes, without producing a greater concentration in the engalactic than in the perigalactic regions, these latter having a mean elevation above the great circle of the milky way, of 22°. It seems to be an unavoidable conclusion that between the perigalactic and apogalactic regions the line of sight approaches to being tangential to the surfaces of equal condensation, and that there is therefore a locus of maximum condensation of an annular form, so that the surfaces resemble Cassinian ovals in their sections. To my mind this conclusion, based on this general consideration, is more trustworthy than anything which can be derived from a consideration of the numbers in their details. I have however made some calculations, based upon the supposition that all the stars have equal intrinsic light, adopting at the same time Rosén's photometric ratio between the different magnitudes. It is true that Rosén's ratio is probably too small for bright stars, but owing to the engalactic and perigalactic regions containing each of them parts of the heavens where the stars vary very much in thickness, it is proper to assign a small value to the ratio, because of the effect which has been pointed out above, of a concentration of the stars more or less toward planes; and if a larger ratio were taken for the apogalactic and polar regions, the general conclusions to which the assumption of Rosén's ratio leads would only be strengthened. Assuming the magnitudes of Behrmann and Heis to be the same, and adopting my reduction of Heis's scale, given on page 36, I have calculated the solid contents of the portions of space within which the stars of each magnitude would be contained, and, dividing the numbers of stars by these volumes, I get the following mean star-densities:

Densities of Stars.

	1m	2m	3m	4m	5m	6m
Galactic Poles	.52	.23	.30	.18	.39	.30
Apogalactic Zones	.34	.42	.51	.34	.32	.29
Perigalactic Zones	.69	.54	.39	.49	.46	.37
Engalactic Zone	1.21	.62	.60	.49	.52	.37

Radii Vectores.

1st Mag.	2nd	3rd	4th	5th	6th
1.27	2.80	4.06	5.81	8.15	12.9

I have next plotted these densities on a system of rectangular coördinates, taking the radii vectores for abscissas and the densities for ordinates. Instead of making a separate construction for each zone, I have plotted them all upon the same construction, merely distinguishing the different zones by differently shaped dots, and have then drawn curves of the densities for the different zones, having regard, in drawing each of them, (especially for the smaller radii vectores) not merely to the position of the points for that curve, but also to the harmony of the different curves. On the curves so drawn, I have read off the radii vectores for the different densities, and have thus obtained the data for plotting the curves of equal condensation. The result is shown upon Plate III, and it will be seen that there appears to be a concentrated ring, lying much within the milky way, and at a distance of a 2½ mag. star. This is shown not so much by the concentrated part itself, which might result from an accidental distribution of the numbers, as it is by the concave forms of the outer surfaces of equal condensation. The existence of a ring of bright stars, in the approximate plane of the milky way, has already been noticed by more than one astronomer.

In regard to our position with reference to this ring, it is evident from the total numbers of stars on the Beronicean and Magellanic sides, that we are upon the Beronicean side of the most concentrated plane of bright stars, just as we are upon the same side of the milky way. It is true that the 6th magnitude stars do not show this, but this is probably owing to Behrmann's faintest stars not being really so faint as I have assumed them to be. The comparison of the numbers of stars on the different galactic meridians seems to show that we are not far removed from the axis of the inner ring, and the irregularity of the numbers makes it difficult to decide to which side we incline. There are somewhat more stars on the meridian C, in the direction of Capella and Procyon, but nothing in reference to our position can be concluded from this.

Let us now briefly consider the manner in which our conclusions are affected by the variations in the intrinsic brightness of the stars. Reverting to equation (2),

$$fl.dl = dl\int_0^\infty \mathrm{F}(lr^2).\varphi r.r^4.dr,$$

let us suppose that φr is expressed in the form

$$\varphi r = \alpha r^{2a-5} + \beta r^{2b-5} + \gamma r^{2c-5} + \text{etc.},$$

where the number of terms is finite. Substituting in (2) this value of φr, and also making L the variable instead of r, we have

$$fl = \tfrac{1}{2} l^{-1} \int_0^{\infty} \mathrm{F}(lr^2) \left(\alpha r^{2a-1} + \beta r^{2b-1} + \text{etc.}\right) r^{-1} d(lr^2)$$

$$= \tfrac{1}{2}\alpha \int_0^{\infty} \mathrm{L}^{a-1}.\mathrm{FL}.d\mathrm{L}.l^{-a} + \tfrac{1}{2}\beta \int_0^{\infty} \mathrm{L}^{b-1}.\mathrm{FL}.d\mathrm{L}.l^{-b} + \text{etc.}$$

If therefore fl be expressed in the form

$$(4) \qquad fl = \mathrm{A}l^{-a} + \mathrm{B}l^{-b} + \mathrm{C}l^{-c} + \text{etc.},$$

where a, b, c are such values, integral or fractional, as will enable us to express fl with the requisite degree of accuracy in the fewest terms, we find

$$(5) \qquad \tfrac{1}{2}\varphi r = \frac{\mathrm{A}}{[\mathrm{L}^{a-1}]} r^{2a-5} + \frac{\mathrm{B}}{[\mathrm{L}^{b-1}]} r^{2b-5} + \frac{\mathrm{C}}{[\mathrm{L}^{c-1}]} r^{2c-5} + \text{etc.},$$

where $[\mathrm{L}^x]$ denotes the mean value of L^x.

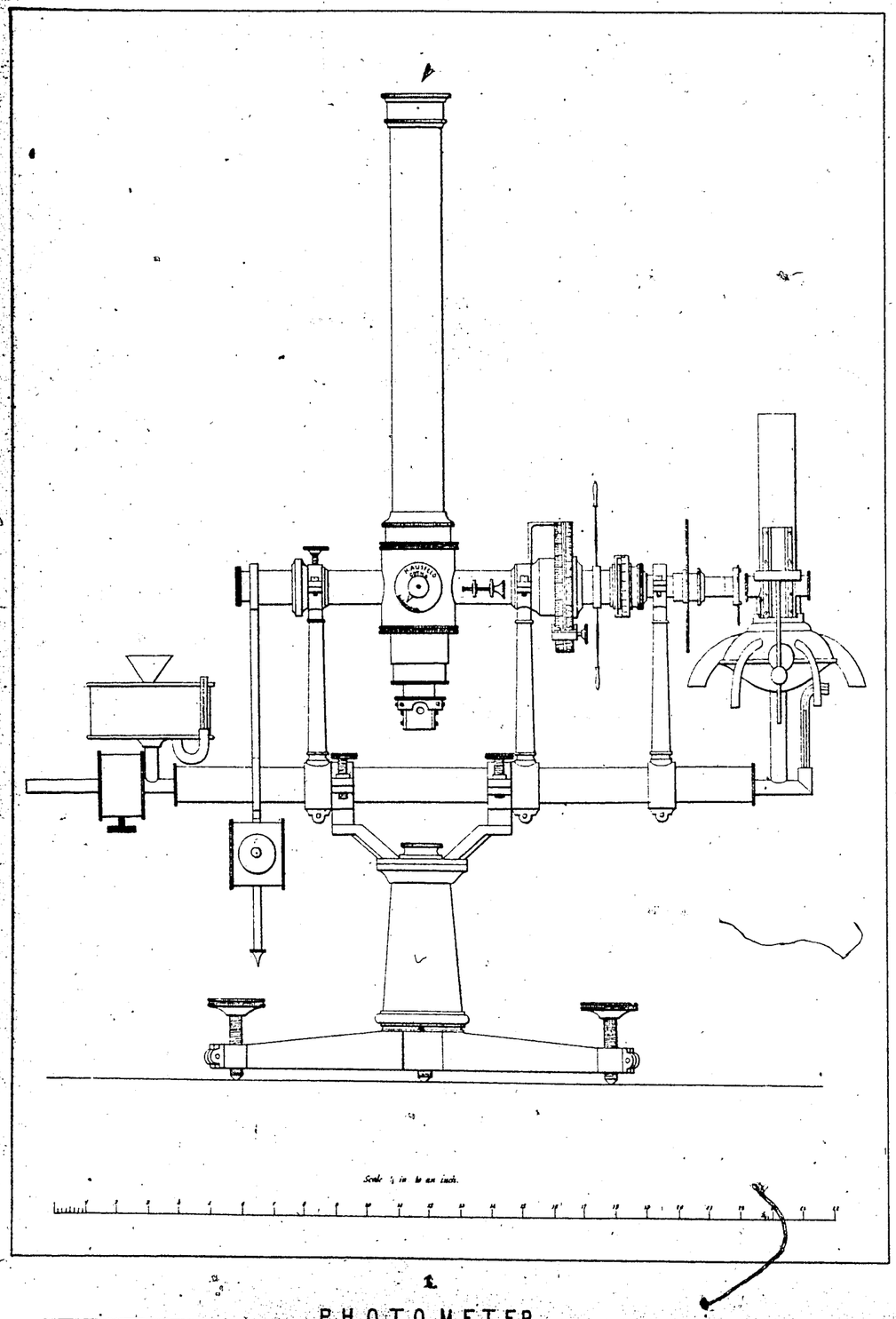
Although the reasoning by which equation (5) has been obtained is open to some criticism, I think this equation exhibits the manner in which the variation of the intrinsic light of the stars affects inferences regarding their density at different distances. It shows that although any numerical results for the values of the density may require considerable modification, yet the general relations between the inferred densities in different directions will not be completely changed.

In order to gain some idea of the variations in intrinsic light of the stars, I have compared the photometric measures of Seidel with the proper motions of the same stars, as given by Mädler. This astronomer has himself pointed out that the mean distances of the stars of the different magnitudes, as derived from the mean proper motions of each magnitude, have very different ratios from the distances as calculated from the photometric measures. In fact, the mean distances of the first six magnitudes, as calculated from the proper motions by Mädler, are nearly as the square roots of the distances photometrically deduced.

Mag.	Dist. from proper motions.	$\sqrt{\text{Dist.}}$ from the brightness.
1	1.00	1.00
2	1.32	1.35
3	1.62	1.62
4	2.00	1.91
5	2.45	2.29
6	2.56	2.88

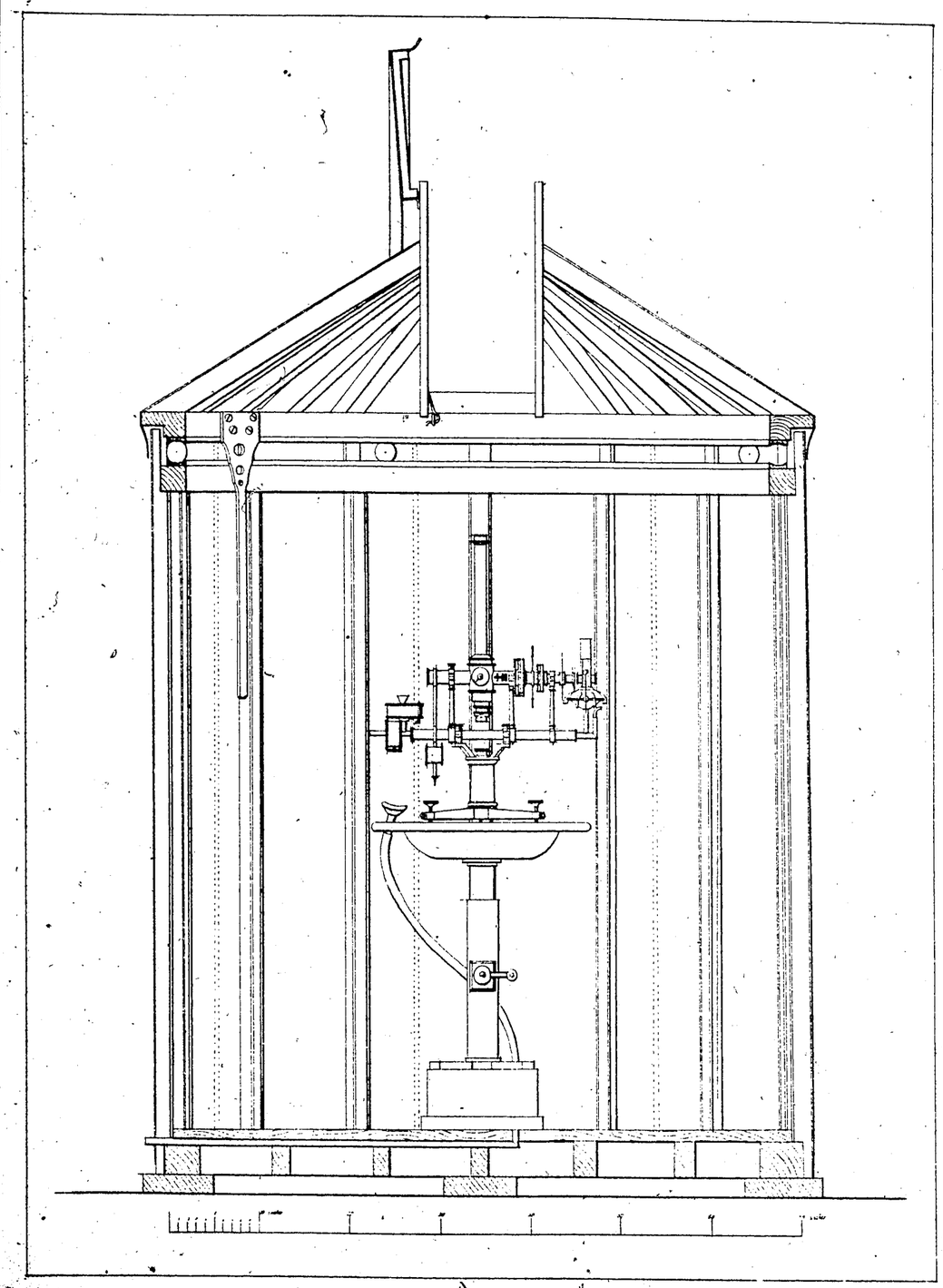
There are various ways of accounting for this difference; but, however it is to be accounted for, it would seem that if the light of each star is divided by such a power of the proper motion that the ratio tends neither to a continual increase nor a continual decrease with l, the mean values of this ratio must be independent of the radius vector, and that its variations can depend only on the variations of the intrinsic light, the variations of the real motions of the stars in space, and the errors of observation. In the case of the stars of Seidel, I find that such a ratio exhibits enormous variations, which may be due in large measure to the errors in those proper motions which are very small. But, on the other hand, it is nearly certain that the intrinsic light of the stars does vary enormously. Supposing that it has some maximum value, never surpassed, the mean value of L^x would on the supposition of enormous variations depend almost entirely on this maximum value; and consequently the effect upon equation (5) would be nearly the same as if all the stars had the same intrinsic light.

On the whole, therefore, I believe that considerable weight should be attached to general conclusions respecting the shapes of the surfaces of equal condensation, calculated on the supposition of equal intrinsic light; but such conclusions are greatly strengthened, in the case of the bright stars, by the circumstance that at a distance of some 22° from the milky way the apparent density of these stars reaches a value which is nowhere much surpassed.



1

PHOTOMETER



PHOTOMETER OBSERVATORY