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III.

NOTE ON GRASSMANN'S CALCULUS OF EXTENSION.

By C. S. Peirce.

Read Oct. 10, 1877.

The last "Mathematische Annalen" contains a paper by H. Grassmann, on the application of his calculus of extension to Mechanics.

He adopts the quaternion addition of vectors. But he has two multiplications, internal and external, just as the principles of logic require.

The *internal* product of two vectors, v_1 and v_2, is simply what is written in quaternions as $-S.\,v_1\,v_2$. He writes it $[v_1 \mid v_2]$. So that

$$[v_1 \mid v_2] = [v_2 \mid v_1],$$

$$v^2 = (Tv)^2.$$

The *external* product of two vectors is the parallelogram they form, account being taken of its plane and the direction of running round it, which is equivalent to its *aspect*. We therefore have: —

$$[v_1 v_2] = v_1 v_2 \sin \angle \begin{matrix} v_1 \\ v_2 \end{matrix} \cdot \mathrm{I}.$$

$$[v_1 v_2] = -[v_2 v_1], \qquad v^2 = 0,$$

where I is a new unit. This reminds me strongly of what is written in quaternions as $-V(v_1 v_2)$. But it is not the same thing in fact, because $[v_1 v_2] v_3$ is a solid, and therefore a new kind of quantity. In truth, Grassman has got hold (though he did not say so) of an eightfold algebra, which may be written in my system as follows: —

Three Rectangular Vectors.

$$i = M : A - B : Z + C : Y + X : N$$

$$j = M : B - C : X + A : Z + Y : N$$

$$k = M : C - A : Y + B : X + Z : N$$

Three Rectangular Planes.

$$I = M : X + A : N$$

$$J = M : Y + B : N$$

$$K = M : Z + C : N$$

One Solid.

$$V = M : N$$

Unity.

$$1 = M : M + A : A + B : B + C : C$$

$$+ N : N + X : X + Y : Y + Z : Z$$

This unity might be omitted.

The relation of the two multiplications is exceedingly interesting. The system seems to me more suitable to three dimensional space, and also more natural than that of quaternions. The simplification of mechanical formulæ is striking, but not more than quaternions would effect, that I see.

By means of eight rotations through two-thirds of a circumference, around four symmetrically placed axes, together with unity, all distortions of a particle would be represented linearly. I have therefore thought of the nine-fold algebra thus resulting.

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Seven hundred and fourth Meeting.

October 10, 1877. — STATED MEETING.

The PRESIDENT in the chair.

Dr. H. P. Bowditch presented his resignation of the office of Recording Secretary. The resignation was accepted, and Professor John Trowbridge was appointed to fill the vacancy.

The President announced the death of M. LeVerrier of Paris, Foreign Honorary Member, and of John H. Temple, of West Roxbury, Resident Fellows.

Mr. C. E. Norton presented, by title, a paper on the Dimensions and Proportions of the Temple of Zeus at Olympia.

Mr. Scudder exhibited a fossil butterfly from the tertiary formation of Colorado.

The following gentlemen were elected members of the Academy: —

John Rodgers, of Washington, to be an Associate Fellow in Class I., Section 4.

Arthur Searle, of Cambridge, to be a Resident Fellow in Class I., Section 2.

Charles R. Cross, of Boston, to be a Resident Fellow in Class I., Section 3.

Amos E. Dolbear, of Somerville, to be a Resident Fellow in Class I., Section 3.

George Cheyne Shattuck, of Boston, to be a Resident Fellow in Class II., Section 4.

Francis Minot, of Boston, to be a Resident Fellow in Class II., Section 4.

Charles Smith Bradley, of Cambridge, to be a Resident Fellow in Class III., Section 1.

Oliver Wendell Holmes, Jr., of Boston, to be a Resident Fellow in Class III., Section 1.

John Lowell, of Boston, to be a Resident Fellow in Class III., Section 1.

James Bradley Thayer, of Cambridge, to be a Resident Fellow in Class III., Section 1.

The following papers were presented: —

"Note on Grassmann's Calculus of Extension." By Mr. C. S. Peirce.

"On a New Form of a Dividing-Engine." By Professor W. A. Rogers. A machine built for the physical laboratory of Princeton College was exhibited.

"On the Determination of the Chances at Billiards in the Case of a 'Grand Discount.'" By Professor Benjamin Peirce.

Seven hundred and fifth Meeting.

November 14, 1877. — ADJOURNED STATED MEETING.

The PRESIDENT in the chair.

Dr. Wyman, in behalf of the Rumford Committee, asked for an appropriation of one thousand dollars ($1,000) from the income of the Rumford Fund, to be expended under the direction of the Committee on investigations in light and heat; and this appropriation was made.

Professor Watson presented to the Academy a volume containing studies of certain inventions exhibited at the late Centennial Exhibition, and also a study of engineering works upon the river Marne in France.

On the recommendation of the Rumford Committee, it was voted that the barometer belonging to the Academy should be loaned to the Institute of Technology for meteorological investigations.

The following papers were presented: —

"On the Dimensions and Proportions of the Temple of Zeus at Olympia." By Professor Charles E. Norton.

"On the Possible Affinities of a Problematical Fossil from the Carboniferous Rocks of Illinois." By Mr. S. H. Scudder.

Professor Semper made some remarks on the inhabitants of the Pelew Islands.

Mr. Trouvelot presented, by title, the following papers: —

"Undulations observed in the Light of Coggia's Comet of 1874."

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