44TH CONGRESS, 1st Session. } HOUSE OF REPRESENTATIVES. { Ex. Doc. No. 81.

REPORT OF THE SUPERINTENDENT

OF THE

UNITED STATES COAST SURVEY,

SHOWING

THE PROGRESS OF THE SURVEY

DURING

THE YEAR 1875.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1878.

THIS PAGE LEFT BLANK INTENTIONALLY

APPENDIX No. 15.

DESCRIPTION OF AN APPARATUS FOR RECORDING THE MEAN OF THE TIMES OF A SET OF OBSERVATIONS, BY C. S. PEIRCE, ASSISTANT IN THE UNITED STATES COAST SURVEY.

The object of the contrivance is to enable an observer, after he has touched a key at each one of several observations, which succeed at a constant interval of a few seconds, immediately to read off on a dial the mean of the times of the observations to hundredths or thousandths of a second, thus avoiding the delay, labor, and error involved in reading chronograph-sheets.

Suppose the number of observations in a set is n. Then, if t_i is the time of the ith observation the mean time is—

$$\frac{1}{n}\Sigma_i t_i$$

Let S_i be double the number of whole seconds in t_i, and let s_i be the fraction of a second. Then—

$$\frac{1}{n}\Sigma_i t_i = \frac{1}{n}\Sigma_i S_i + \Sigma_i \frac{s_i}{n}$$

The problem thus divides itself into two, to determine each term of the second member of this equation. This division of the problem constitutes the first essential character of the method here to be described.

If the observations occur at irregular and unknown intervals, the observer may separately note S_i for each observation, without any particular apparatus, and so calculate the first term. But if the observations occur at intervals approximately known, the first term can be determined with less trouble. Suppose, for instance, that the observations, like transits of stars, are known to occur at intervals nearly symmetrical about the middle one. Then, if there exists any easy means of determining the time of this one accurately to the one nth part of a second, this will be equal to the first term, provided the observations follow one another with sufficient regularity. But if n be too great for this, or if it be an even number, the observer may note, by any simple means, the times of the first and last observations. These times need then only be noted to two $n$ths of a second, and so for any larger numbers. A transit-observer may conveniently use seven wires, and note the times over the second and sixth wires to a quarter of a second. When n is greater, a marking-watch may be conveniently used. In using this instrument, the observer need not seek to distinguish the different observations of a set, as their order does not affect the mean value.

I have now to describe the means by which I would determine the value of the second term—

$$\Sigma_i \frac{s_i}{n}$$

Supposing that we have the means of registering the sum of the fractions s_i. Then to register, instead, their mean, we may use one of the following methods: First, we may regulate the registering apparatus by a "regulateur Villarceau." This may be made to run at any desired rate within certain limits with great accuracy; and it should be made to run at one nth of the rate required for the registry of the sum of s_i. Second, we may have a frictional connection between two solids of revolution. Third, we may perform the required division by the graduation of the dial by changing from one dial-face to another. But the simple division of Σs by n is so very easily performed that it would hardly be worth while to make the necessary adjustment of the apparatus to put any of these methods into practice. I will, therefore, proceed at once to describe a contrivance for registering Σs.

THIS PAGE LEFT BLANK INTENTIONALLY

instrument is regulated by a vibrating reed striking the teeth of a revolving wheel, we should expect the retardation to vary as the velocity, so that—

$$D_t^2\, s = a - v\, D_t\, s$$

the integral of which is of the form—

$$s = A\, t + B\,(1 - 6^{-ct})$$

Here t denotes the time and s the reading of the chronoscope. The existence of this correction is also shown by attentively listening to the note of the reed, which is distinctly heard to be lowered when the hands are geared in. It must be confessed, however, that the measured times of known intervals are not accounted for by this correction; for example, at Berlin, in 1876, May 10 and 11, I experimented on the fall of a ball. The length of the seconds' pendulum at Berlin, according to Bessel, as stated by Bruhns, is $0^m.9942318$. This is for the level of the sea; for my point it is .994223. The reciprocal of this, or 1.00581, is the square of the time in mean seconds of the vibration of a metre-pendulum. The square root of this, or 1.00290, is the time itself. Multiplying by $\frac{366}{365}$ we get 1.00565, the time in sidereal seconds. The square of this, or 1.01133, is then to be multiplied by $\frac{1}{\partial^2}$ or, .101321 to get the velocity square in sidereal seconds after a fall of one metre. This is .102469. If, therefore, the true squares by the chronoscope of a fall through 5 centimetres be divided by .0102469, we get the square time by the chronoscope divided by the sidereal seconds. Now, then, A being nearly 40.1 millimetres and B being nearly 19.6 millimetres, I found—

Heights.	Chronoscope.	(Chron.)²	Δ	Δ / .01025	Chron. / Sid. sec.
A.	.0919	.00845	.01090	1.064	1.032
A + 5 cen.	.1391	.01935	.01037	1.012	1.006
A + 10 cen.	.1724	.02972	.01076	1.050	1.025
A + 15 cen.	.2012	.04048	.01010	.986	.993
A + 20 cen.	.2249	.05058	.01048	1.023	1.011
A + 25 cen.	.2471	.06106	.01028	1.003	1.001
A + 30 cen.	.2671	.07134	.01097	1.071	1.035
A + 35 cen.	.2869	.08231	.00986	.962	.981
A + 40 cen.	.3036	.09217	.01094	1.068	1.034
A + 45 cen.	.3211	.10311	.01046	1.021	1.010
A + 50 cen.	.3370	.11357	....	....	....
B.	.0637	.00406	.00101	.980	.993
B + 5 m.	.0712	.00507	.00108	1.054	1.027
B + 10 m.	.0784	.00615	.00104	1.015	1.007
B + 15 m.	.0848	.00719	....	....	....

I also measured some clock-intervals, with the following results:

Interval in sid. sec.	Chronoscope.	Δ	Chron. / sid. sec.
1	.996	.996	0.996
2	1.993	.997	0.997
10	9.956	7.963	0.9954
50	49.745	39.789	0.9947

If we take the times of falling through A and B, we have—

Height.	(Chron.)²	(Chron.²) / Height.	(Chron.)² / 205 height.	Chron. / sid. sec.
.0401	.00845	.211	1.03	1.02
.0196	.00406	.207	1.01	1.01

If we compare the time of falling A + 50 cen. with the record of 1 second, we have $54.01 \times .00205 = .1083 = (.3291)^2$. Hence the difference of time is 0.6709 sidereal second. The difference of the chronoscope results is 0.659, and the ratio is only .982.

That there really is a retardation is certain both *à priori* and from the sound; and it is shown by the rates from the clock-seconds. But this can hardly account for the discrepancy between the measures with the clock and fall-apparatus, for the last given rate connecting the two is too small. I am inclined to think that there may be a correction of the fall-experiments, proportional to the momentum at impact, and, therefore, to the time. Experiments should be made with pendulums of different lengths. The relay above described, with the addition of a circuit-reverser, will render such experiments easy.

In regard to the accuracy of Hipp's chronoscope, I may mention that the chronoscope-times given above for the falls are the means of ten observations each. It may, then, be calculated from the agreement of the resulting ratios in the last column that the probable error of a single observation but slightly exceeds one thousandth of a second. In using the instrument for the automatic record of pendulum-transits, then, it will be quite sufficient to have ten observations in a set. This will give the intervals accurately to a thousandth of a second, or as accurately as the method of coincidences.

Let us now briefly consider the effect of the resistance of the lever shown in the plate upon the motion of the pendulum. Owing to the elasticity of the material, we may consider the impact to be instantaneous and to produce a reduction of the velocity of the pendulum in a fixed ratio. Let t be the variable time which is occupied by the pendulum in swinging from the vertical position to one having an angle, φ, from the vertical; let v be the angular velocity at that instant; T, the period of oscillation; Φ, the amplitude of oscillation; π, the ratio of the circumference to the diameter, and $\frac{1}{1+i}$ the ratio of v just before the impact to v just after. Let δt and $\delta \Phi$ denote variations of t and Φ produced by the impact. Then we have, from the common theory of the pendulum—

$$t = \frac{\mathrm{T}}{\pi} \text{ arc tan } \frac{\varphi \pi}{v \mathrm{T}}$$

$$\Phi = \sqrt{\frac{v^2 \mathrm{T}^2}{\pi^2} + \varphi^2}.$$

In these equations we are to multiply v by $(1 + i)$ and subtract from the products the above unchanged values to obtain δt and $\delta \Phi$. Developed by Taylor's theorem, and neglecting all but the first two terms, we have—

$$\delta \Phi = \frac{v^2 \mathrm{T}^2}{\Phi \pi^2} i = \Phi \left(\cos \frac{\pi t}{\mathrm{T}} \right)^2 i = \frac{\Phi^2 - \varphi^2}{\Phi} i$$

$$\delta t = \frac{\frac{\varphi \pi}{v \mathrm{T}}}{\sqrt{1 + \left(\frac{\varphi \pi}{v \mathrm{T}}\right)^2}} i = \frac{\mathrm{T}}{\pi} \sin \frac{t}{\mathrm{T} \pi} \cdot i = \frac{\mathrm{T}}{\pi} \sqrt{1 - \frac{\delta \Phi}{\Phi i}} \cdot i = \frac{\mathrm{T}}{\pi} \frac{\varphi}{\Phi^2 - \varphi^2} \delta \Phi = \frac{\mathrm{T}}{\pi} \frac{\varphi}{\Phi} i$$

The quantity i is twice the ratio of the virtual mass of the lever to the sum of those of pendulum and lever. By the virtual mass, I mean the square of the moment of the momentum divided by the moment of inertia. It is safe to say that i does not exceed $\frac{1}{1000}$. And as $\frac{\varphi}{\Phi}$ can easily be reduced to $\frac{1}{20}$, the effect of the resistance can hardly in ten vibrations be perceptible. However, its amount may be calculated by observing φ and $\delta \Phi$.

It will be observed that the resistance shortens the time of oscillation if the impact occurs while the pendulum is moving upward, and lengthens it in the reverse case. Hence, there would be no accumulation of the effect in ten transits over what there would be in two, were it not for the thickness of the agate.

The following results of successive series of 298 swings each, measured with the above-described instrument, by ten transits every five minutes, are a fair specimen of the results obtained.

APRIL 8.

Heavy end up.	Heavy end down.
299s. 9463	299s. 9162
299 . 9436	299 . 9153
299 . 9457	299 . 9147
299 . 9427	299 . 9129
299 . 9439	299 . 9132

APRIL 7.

Heavy end down.	Heavy end up.
299s. 9215	299s. 9549
299 . 9176	299 . 9552
299 . 9167	299 . 9564
299 . 9179	299 . 9513
299 . 9176	299 . 9525

The weights to be assigned to successive intervals of a set of 5 are 5, 8, 9, 8, 5, and this gives for April 8—

Heavy end up $\mathrm{T}_2 = 1^s.006525$ $\mathrm{T}_2^2 = 1.013093$
Heavy end down $\mathrm{T} = 1.006424$ $\mathrm{T}^2 = 1.012889$

$$\mathrm{T}^2 \text{ (corr'd for resistance of air, etc.)} = \frac{39\,\mathrm{T}_1^2 - 17\,\mathrm{T}_2^2}{22} = 1.012731$$

For April 7—

$$\mathrm{T}_1 = 1.006436 \quad \mathrm{T}^2 = 1.012913$$
$$\mathrm{T}_2 = 1.006558 \quad \mathrm{T}_2^2 = 1.013159$$
$$\mathrm{T}^2 \text{ (corr'd)} = 1.012731$$

This is expressed in chronograph-seconds. The results of the two days are identical to the ast figure.