ist eine dringende Aufgabe, zu deren Lösung die algebraische Logik keinen Beitrag liefern kann. Mi.

B. Halsted. Statement and reduction of syllogism. J. specul. Phil. XII. 418.

Halsted stellt in strengem Anschluss an Boole Symbole und Grundgesetze für die logische Multiplication, Addition und Subtraction auf. Hierauf leitet er, 16 denkbare Urtheilsformen scheidend, die freilich sämmtlich in zwei Formeln ausdrückbar sind, die hieraus sich ergebenden 256 möglichen Syllogismen und die für ihre Bestimmung nothwendigen Gesetze ab. Die Arbeit ist, was die Herleitung der Schlüsse betrifft, ein selbständiger Beitrag zum logischen Calcul. Mi.

B. Halsted. Algorithmic division in logic. J. specul. Phil. XIII. 107.

Der Verfasser begründet, theilweise gegen Jevons polemisirend, dass eine logische Division zulässig ist, und stellt die beschränkenden Bedingungen fest, unter denen sie möglich erscheint. Mi.

J. Venn. On the various notations adopted for expressing the common propositions of logic. Proc. of Cambr. IV. 36-47.

Der Verfasser lenkt die Aufmerksamkeit auf die grosse Verschiedenheit der symbolischen Ausdrucksformen für die gewöhnlichen Sätze der Logik. Er benutzt als Beispiel den Satz: „Nicht S ist P" und giebt ein Verzeichnis von 24 Symbolen, die dafür von Boole, MacFarlane, Wundt, Jevons, Delboeuf, Murphy, Holland, Drobisch, Segner, Darjes, Grassmann, MacColl, Peirce, Frege, Morgan, Ploucquet, Bentham Hamilton, Leibniz, Lambert Maimon und Chase gebraucht sind. Diese theilen sich in sechs Gruppen, die näher characterisirt und betrachtet werden, wobei

P 222

Article II.—On Certain Expansion Theorems. By Emory McClintock.

"Any function fy can be expanded in terms of x, if $y = x\phi y$, provided ϕy can be expanded in terms of y, and ϕy does not vanish with y." ... the reason for this, the author derives the following theorem without difficulty, putting $D = \frac{d}{dy}$:

$$fy = fy_{[y=o]} + x.\phi y D fy_{[y=o]} + \tfrac{1}{2}x^2.D(\phi y)^2 D fy_{[y=o]} + \ldots \qquad (1)$$

Comparing this series with others already known, he concludes thus: "We have, then, in (1) a series which is essentially identical with those of Lagrange and Bürman, which is simpler in form than either, and which is at once a connecting link between them and a necessary step in the most direct demonstration of both. The inference seems warranted that we should regard it as the main proposition, of which the others are corollaries." Among these and other corollaries instanced, the following is given as new: "To expand fu in terms of fu, find some expression of the form $\psi y - a$ which will exactly divide fu. Then

$$fu = fa + fu.\frac{y-a}{f\psi y}Df\psi y + \frac{1}{1.2}(fu)^2.D\left(\frac{y-a}{f\psi y}\right)^2 Df\psi y + \ldots_{[y=a]}."$$

Another deduction from (1) is a third new series, chiefly useful in computing the coefficients required in the reversion of series.

Article III.—Some Theorems in Numbers. By O. H. Mitchell.

The first section of this article treats of the residues, mod k, of the symmetric functions of the numbers less than k, where $k =$ any integer. An abstract of this section is given in University Circular, No. 12, under proceedings of the Mathematical Seminary.

The second section extends the results of the preceding section, and the results of an article on Binomial Congruences in the preceding number of the Journal, to the theory of functions (mod. $p, f(x)$).

If $f(x) = K \equiv A^l B^m \ldots G^o H^w \ldots Q^z$, mod. p, where p is a prime number, l, m, &c. are any integers, and A, B, &c. are irreducible (mod. p) functions of x of forms $x^a + \lambda_1 x^{a-1} + \lambda_2 x^{a-2} +$ &c., $x^b + \mu_1 x^{b-1} + \mu_2 x^{b-2} +$ &c., respectively, if $S = H \ldots Q$, and $\tau_S(K)$ denote the number of functions of less degree than K which contain S (mod. p) but do not contain any factor of K not found in S, *i. e.* the number of the *S-totitives* of K, then

$$\tau_S(K) = p^k\left(1 - \frac{1}{p^a}\right)\left(1 - \frac{1}{p^b}\right) \ldots \left(1 - \frac{1}{p^g}\right),$$

where $k =$ the degree of $K, = a + b + \ldots + q$.

The properties of the roots of $X^2 \equiv X$ (mod. p, K) are shown to be the same as the properties of the roots of $x^2 \equiv x$ mod. k, where $k =$ any integer.

The analogue of the generalized Fermatian theorem is seen to be

$$X_S^{\tau_S(K)} \equiv R_S \text{ (mod. } p, K),$$

where X_S is any S-totitive of K, and R_S is that one of the roots of $X^2 \equiv X$ (mod. p, K) which is an S-totitive of K.

If $X'_S, X''_S, \ldots X_S^{[\tau_S(K)]}$ be the S-totitives of K, then

$$\Sigma X'_S X''_S \ldots X_S^{[m]} \equiv (-)^{\frac{m}{\tau(A)}} R_{\bar{A}}\, C^{\tau_{AS}(K)}_{\frac{m}{\tau(A)}} + \ldots$$

$$\ldots + (-)^{\frac{m}{\tau_1(G)}} R_{\bar{G}}\, C^{\tau_{GS}(K)}_{\frac{m}{\tau(G)}}, \text{ (mod. } p, K),$$

where $\bar{A} = B \ldots Q$, C is a binomial coefficient, and only those terms of the summation are to be taken for which $\frac{m}{\tau(A)}$, *i. e.* $\frac{m}{p^a - 1}$, $=$ an integer.

When $m = \tau_S(K)$ the formula becomes

$$\Sigma X'_S \ldots X_S^{[m]} \equiv R_{\bar{A}} + R_{\bar{B}} + \ldots + R_{\bar{G}} \equiv R_{\overline{AB \ldots G}} = R_S \text{ (mod. } p, K),$$

except when $\frac{AB \ldots Q}{S} =$ an irreducible function, as A, and p is not 2, in which case $\frac{\tau(K)}{p^a - 1}$ is odd, and we have

$$\Sigma X'_S \ldots X_S^{[m]} \equiv -R_S \text{ (mod. } p, K).$$

This special case of the formula constitutes the analogue of the generalized Wilsonian theorem.

Article IV.—Note on the Frequency of use of the Different Digits in Natural Numbers. By Simon Newcomb.

"That the ten digits do not occur with equal frequency must be evident to anyone making much use of logarithmic tables, and noticing how much faster the first pages wear out than the last ones." By a simple investigation it is shown that "the law of probability of the occurrence of numbers is such that all mantissae of their logarithms are equally probable;" in other words, that "every part of a table of anti-logarithms is entered with equal frequency." Hence a table is easily computed showing the relative frequency of the several digits in the first place of figures, in the second place, &c.

Article V.—Tables of the Generating Functions and Groundforms of the Binary Duodecimic, with some General Remarks, and Tables of the Irreducible Syzygies of Certain Quantics. By J. J. Sylvester.

This paper contains tables for the Binary Quantic of the 12th order, to a similar extent and in the same form as the tables for the 6^c, 7^c, 8^c, 9^c, 10^c, which have previously appeared in the Journal.

It also contains complete tables of the degrees and orders of the syzygies connecting the groundforms of the Quintic, the Sextic, and a simultaneous system of a Quadric and Cubic.

In addition to certain observations on permanent as distinguished from transient ground-differentiants, it gives an *apercu* of a general method for deducing from the generating function to any system of Binary Quantics the complete system of types of the groundforms and the syzygants of the various grades, by a uniform algebraical process which operates directly on the generating function regarded in the light of an ordinary algebraical fraction, without any regard to the particular form that may be assigned to its numerator and denominator by the introduction or suppression of a common factor.

Article VI.—A Demonstration of the Impossibility of the Binary Octavic possessing any Groundform of deg-order 10.4. By J. J. Sylvester.

In this article it is conclusively established that the covariant of order 4 and degree 10 appertaining to the Binary Octavic found by Dr. von Gall, and which he has been unable to decompose, must notwithstanding be decomposable; inasmuch as it is proved that if any groundform whatever to the Binary Octavic of such a deg-order existed, the number of linearly independent covariants of that deg-order would exceed 32, the number given by Cayley's Rule. This form then in Dr. von Gall's final enumeration being suppressed, his results are brought into exact accordance with the table given in the Journal, Vol. II, p. 233.

Article VII.—On the Logic of Number. By C. S. Peirce.

In this paper the most fundamental properties of the elementary operations with numbers are strictly deduced from a few primary propositions belonging to the more general field of the logic of relatives. The associative, commutative, and distributive laws are proved, the proof in each case consisting "in showing, 1st, that the proposition is true of the number one, and 2nd, that if true of the number n it is true of the number $1 + n$, next larger than n."

Article VIII.—On the Remainder of Laplace's Series. By Emory McClintock.

The remainder of fy after $n + 1$ terms, putting $y = f(z + x\psi y)$ and $D = \frac{d}{dz}$, is $\frac{1}{n!}D^n\int_{fz}^{y}(x\phi t + z - f^{-1}t)^n f'^1 t.dt$.

Studies from the Biological Laboratory. Edited by Professor Martin. Associate Editor, Dr. W. K. Brooks. Contents of *Vol. II, No.* 2. (In press.)

List of Medusæ found at Beaufort, N. C., during the Summers of 1880 *and* 1881. By W. K. Brooks.

On the Origin of the so-called "Test-cells" in the Ascidian Ovum. By J. Playfair McMurrich. With one plate.

A Contribution to the Study of the Bacterial Organisms commonly found upon Exposed Mucous Surfaces and in the Alimentary Canal of Healthy Persons. By George M. Sternberg. With three plates.

A Fatal Form of Septicæmia in the Rabbit, Produced by the Subcutaneous Injection of Human Saliva. By George M. Sternberg. With one plate.

223