For the purposes of the pendulum experiments, time was telegraphed daily, and thus the observations made for time at Ebensburg were supplemented. The latitude of the station there was determined by observations with a sextant. At that station Mr. Peirce was aided by Messrs. Henry Farquhar and Marcus Baker. The first-named aid made the observations at York under the supervision of Assistant Peirce.

At both of the stations in Pennsylvania extensive series of observations were recorded for the purpose of studying the statical and dynamical flexure of variously modified pendulum supports, and the influence of these modifications upon the period of oscillation of the pendulum. The results will be given hereafter in a separate paper by Assistant Peirce. In the course of the season experiments were made by substituting for the ordinary knife edge of the pendulum small steel cylinders. This method, proposed in a previous report by Assistant Peirce, had been independently recommended by M. Yvon Villerceau, but the trials made by Mr. Peirce proved that the friction was increased by that method of swinging the pendulum.

The measurement of the acceleration of gravity made by Assistant Peirce at Paris, France, revealed a disagreement with the measures obtained by Borda and Biot. For the investigation of the discrepancy, Mr. Peirce again visited Paris, with the sanction of the honorable Secretary of the Treasury, and by theoretical and experimental studies demonstrated from principles not known in their times that the results obtained by the two celebrated physicists were subject to certain very large corrections. These, when properly applied, brought their results into perfect accord with results already reported by Assistant Peirce, who read a memoir upon the subject before the Academy of Science of the Institute of France. The paper was printed in the Comptes Rendus for the 14th of June, 1880, and on its reference to a committee the conclusions of Assistant Peirce received the approval of the Academy.

Operations for the comparison of the meter with a wave length of light have been provisionally completed, yet certain parts of the work require verification—in particular the comparison of decimeters with the meter has been only partly made. In connection with this subject Assistant Peirce has published, with my approval, in the American Journal of Mathematics, a memoir upon certain apparitions which appear in diffraction spectra. These were shown to be consequences of eccentricity in the screw used in ruling the diffraction plates. By another observer the subject was treated in a paper presented to the American Association for the Advancement of Science at their last meeting in Boston.

Geodetic operations in Pennsylvania.—Prof. Lewis M. Haupt at the opening of the fiscal year employed his field party in the erection of signals and in examining and permanently marking the stations to be observed on in the course of the season. Near Reading, in Berks County, a station was occupied, and from it angular measurements were made on three points previously occupied to the northward.

To the southward and westward four other positions were completed in angular measurements so as to extend the work through Lancaster County to the immediate vicinity of the Susquehanna River. At Rawlinsville, in the lower part of the last-named county, a junction was made with stations of the coast triangulation near the head of Chesapeake Bay. From the records of the field work which closed in September, 1879, the Computing Division of the office has added five additional entries in the register of geographical positions.

In July the weather was unfavorable, but was much more satifactory in August and September. The statistics of the work are:

Stations occupied	4
Signals observed on	24
Directions to subsidiary objects	133
Number of observations	3,010

The field report of Professor Haupt was accompanied by complete descriptions of the stations, sketches of the horizon at each, record of the results of observed angles, and topographical sketches of the sites occupied by the theodolite.

Topography of Cape May, N. J.—This work was taken up at the opening of the fiscal year by Assistant C. M. Bache, who remained in the field until the 20th of November.

UNITED STATES
COAST AND GEODETIC SURVEY

CARLILE P. PATTERSON
SUPERINTENDENT

A TREATISE

ON

PROJECTIONS

BY

THOMAS CRAIG

WASHINGTON
GOVERNMENT PRINTING OFFICE
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Coast and Geodetic Survey.

PART I.

MATHEMATICAL THEORY OF PROJECTIONS.

Write

$$\int \sqrt{E}\,du = \int \sqrt{(dx^2+dy^2)} = \sigma$$

σ being the arc cut from the cylinder by a right section, say by the plane of (x,y); we are thus conducted immediately to the desired general equations

Assume	$F_1(\xi+i\eta)=\sigma+iz$	$F_2(\xi+i\eta)=\sigma-iz$
then	$F_1(\xi+i\eta)=\xi+i\eta$	$F_2(\xi-i\eta)=\xi-i\eta$
	$\xi=\sigma$	$\eta=z=v$

the equations for the projection by development of the cylinder.

QUINCUNCIAL PROJECTION OF THE SPHERE.

This projection was constructed by Mr. C. S. Peirce, Assistant, United States Coast and Geodetic Survey. The brief description here given of the projection is extracted from the Coast Survey Report for 1877, Appendix No. 15, and was written by Mr. Peirce himself.

For meteorological, magnetological, and other purposes, it is convenient to have a projection of the sphere which shall show the connection of all parts of the surface. It is an orthomorphic or conform projection formed by transforming the stereographic projection, with a pole at infinity, by means of an elliptic function. For that purpose, l being the latitude, and θ the longitude, we put

$$\cos^2\varphi = \frac{\sqrt{1-\cos^2 l\cos^2\theta}-\sin l}{1+\sqrt{1-\cos^2 l\cos^2\theta}}$$

and then $\frac{1}{2}F\varphi$ is the value of one of the rectangular co-ordinates of the point on the new projection. This is the same as taking

$$\cos am(x+y\sqrt{-1})\,(\text{angle of mod.}=45^\circ) = \tan\frac{p}{2}(\cos\theta+\sin\theta\sqrt{-1})$$

where x and y are the co-ordinates on the new projection, p is the north polar distance. A table of these co-ordinates is subjoined.

Upon an orthomorphic projection the parallels represent equipotential or level lines for the logarithmic potential, while the meridians are the lines of force. Consequently we may draw these lines by the method used by Maxwell in his Electricity and Magnetism for drawing the corresponding lines for the Newtonian potential. That is to say, let two such projections be drawn upon the same sheet, so that upon both are shown the same meridians at equal angular distances, and the same parallels at such distances that the ratio of successive values of $\tan\frac{p}{2}$ is constant. Then number the meridians and also the parallels. Then draw curves through the intersections of meridians with meridians, the sums of numbers of the intersecting meridians being constant on any one curve. Also do the same thing for the parallels. Then these curves will represent the meridians and parallels of a new projection having north poles and south poles wherever the component projections had such poles.

Functions may, of course, be classified according to the pattern of the projection produced by such a transformation of the stereographic projection with a pole at the tangent points. Thus we shall have:

1. Functions with a finite number of zeroes and infinites (algebraic functions).
2. Striped functions (trigonometric functions). In these the stripes may be equal, or may vary progressively or periodically. The stripes may be simple, or themselves compounded of stripes. Thus, $\sin(a\sin z)$ will be composed of stripes each consisting of a bundle of parallel stripes (infinite in number) folded over onto itself.
3. Chequered functions (elliptic functions).
4. Functions whose patterns are central or spiral.

TABLE XXX.

Rectangular co-ordinates for construction of the "quincuncial projection."

	x (for longitudes in upper line). y (for longitudes in lower line).																			
Lat.	0° 90	5° 85	10° 80	15° 75	20° 70	25° 65	30° 60	35° 55	40° 50	45° 45	50° 40	55° 35	60° 30	65° 25	70° 20	75° 15	80° 10	85° 5	Lat.
85°	.033	.033	.033	.032	.031	.030	.029	.027	.025	.024	.021	.019	.017	.014	.011	.009	.006	.003	85°
80	.067	.066	.066	.064	.063	.061	.058	.055	.051	.047	.043	.038	.033	.028	.023	.017	.012	.006	80
75	.100	.100	.099	.097	.094	.091	.087	.082	.077	.071	.065	.058	.050	.042	.034	.026	.017	.009	75
70	.135	.134	.133	.130	.127	.122	.117	.110	.103	.095	.087	.077	.067	.057	.046	.035	.023	.012	70
65	.169	.169	.167	.163	.159	.154	.147	.139	.130	.120	.109	.097	.085	.072	.058	.044	.029	.015	65
60	.205	.204	.201	.198	.192	.185	.177	.168	.157	.145	.131	.117	.102	.086	.070	.053	.036	.018	60
55	.241	.240	.237	.232	.226	.218	.208	.197	.184	.170	.154	.138	.120	.102	.082	.062	.042	.021	55
50	.278	.277	.274	.269	.261	.251	.240	.227	.212	.196	.178	.159	.139	.117	.095	.072	.048	.024	50
45	.317	.316	.312	.306	.297	.286	.273	.258	.241	.223	.202	.181	.158	.134	.109	.083	.055	.028	45
40	.357	.356	.351	.344	.334	.321	.307	.290	.270	.250	.228	.204	.179	.151	.123	.094	.063	.032	40
35	.400	.398	.393	.384	.373	.358	.341	.322	.301	.279	.254	.228	.200	.170	.139	.106	.071	.036	35
30	.446	.443	.437	.427	.413	.396	.377	.356	.332	.308	.281	.253	.222	.190	.155	.119	.081	.041	30
25	.495	.492	.484	.471	.455	.435	.414	.391	.365	.338	.309	.279	.246	.211	.174	.134	.091	.046	25
20	.548	.545	.534	.518	.498	.476	.452	.426	.398	.369	.339	.307	.272	.235	.195	.151	.104	.053	20
15	.609	.604	.589	.568	.544	.517	.490	.461	.432	.401	.369	.336	.300	.262	.219	.173	.121	.062	15
10	.681	.672	.649	.620	.590	.559	.528	.497	.466	.434	.401	.367	.330	.291	.248	.200	.143	.076	10
5	.775	.752	.713	.673	.635	.600	.566	.532	.500	.467	.433	.399	.363	.324	.282	.234	.177	.102	5
0	1.000	.841	.774	.723	.679	.639	.602	.567	.533	.500	.467	.433	.398	.361	.321	.277	.226	.159	0

TABLE XXXI.

Preceding table enlarged for the spaces surrounding infinite points.

	x (for longitudes in upper line). y (for longitudes in lower line).																					
Lat.	0° 90	1° 89	2° 88	3° 87	4° 86	5° 85	6° 84	8° 82	10° 80	12½° 77½	15 75	75° 15	77½° 12½	80° 10	82° 8	84° 6	85° 5	86° 4	87° 3	88° 2	89° 1	Lat.
15°	.609	.609	.608	.607	.606	.604	.602	.596	.589	.579	.568	.173	.147	.121	.093	.074	.062	.050	.038	.025	.013	15
12½	.643	.643	.642	.641	.639	.636	.634	.627	.618	.606	.594	.185	.159	.131	.107	.082	.069	.055	.042	.028	.014	12½
10	.681	.681	.680	.678	.675	.672	.668	.659	.649	.635	.620	.200	.173	.143	.118	.091	.076	.062	.047	.031	.016	10
8	.715	.714	.713	.710	.706	.702	.697	.686	.674	.658	.641	.213	.185	.155	.129	.100	.085	.069	.052	.035	.018	8
6	.753	.752	.750	.746	.741	.735	.728	.714	.700	.681	.662	.227	.199	.169	.142	.112	.095	.078	.060	.040	.020	6
5	.775	.774	.770	.765	.759	.752	.745	.729	.713	.692	.673	.234	.207	.177	.150	.119	.102	.084	.065	.044	.022	5
4	.798	.797	.793	.786	.779	.770	.761	.743	.725	.704	.683	.242	.215	.185	.158	.128	.110	.092	.071	.049	.025	4
3	.825	.823	.817	.808	.798	.788	.778	.757	.738	.715	.693	.250	.224	.194	.168	.137	.120	.101	.079	.055	.029	3
2	.857	.853	.843	.831	.819	.806	.794	.772	.750	.726	.703	.259	.233	.204	.178	.148	.131	.112	.090	.065	.035	2
1	.899	.889	.872	.854	.839	.824	.810	.785	.763	.737	.713	.268	.243	.215	.190	.161	.144	.126	.105	.079	.046	1
0	1.000	.929	.899	.877	.857	.841	.825	.798	.774	.747	.723	.277	.253	.226	.202	.175	.159	.143	.123	.101	.071	0