REPORT OF THE SUPERINTENDENT

OF THE

U. S. COAST AND GEODETIC SURVEY

SHOWING

THE PROGRESS OF THE WORK

DURING THE

FISCAL YEAR ENDING WITH

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APPENDIX No. 15.

ON THE DEDUCTION OF THE ELLIPTICITY OF THE EARTH FROM PENDULUM EXPERIMENTS.

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Any correction to pendulum experiments whose magnitude changes progressively with the latitude needs to be very accurately determined, lest an error be thereby introduced into the resulting value of the ellipticity of the earth. The atmospheric pressure and the temperature both give rise to corrections of this class, and the coefficients of these corrections have been, in general, very inadequately determined. The experiments which have recently been published in the fifth volume of the India Survey, however, sufficiently determine these coefficients for the Kater invariable pendulums. For all other forms of pendulums which have been used, excepting the large-sized Repsold reversible pendulum, their values are quite uncertain. But by far the greater part of the valuable data at hand in reference to the relative force of gravity in different latitudes consist of the times of oscillation of Kater invariable pendulums. So that even were we able properly to correct the others for temperature and pressure they would have very little influence upon the resulting ellipticity, for their absolute number is small, and they are not usefully distributed in latitude. Subject to uncertainties of reduction as they are, the only appreciable effect of admitting them into our calculations would be to increase the apparent probable error without altering the result. Under these circumstances, it has seemed best to me in making a new calculation of the ellipticity, as derived from this class of experiments, to restrict myself exclusively to those made with the Kater invariable pendulums.* The coefficient of the effect of atmospheric pressure upon pendulums of this class was determined in 1829, by Sabine, and the later researches under the auspices of the Indian Survey have not seriously invalidated his determinations. Baily, in his report on Foster's pendulum experiments, undertook to correct all the former results for this effect, but his work is erroneous in several particulars. Thus, he applies the correction to Lütke's numbers, to which it had already been rightly applied; he omits to reduce Sabine's values to the level of the sea, and commits various other errors which cannot be so easily explained in a few words. For the coefficient of temperature effect a value has usually been assumed which was first obtained by Kater, and which is entirely incompatible with the known coefficient of the expansion of brass and with the experiments upon the periods of oscillation at different temperatures. The value adopted in the reductions of the India Survey for pendulum No. 4, which was one of Sabine's pendulums, coincides precisely with that found by Lütke; differs very little from that obtained by the Indian Survey for their other pendulum, satisfies well the experiments by Sabine at different temperatures, agrees with our general knowledge of the coefficient of expansion of brass, and has consequently been made use of by me in the reduction of all the experiments made with the Kater invariable pendulums. I have, therefore, made the necessary corrections to all the results with these pendulums, with the exception of those of the Indian series and of Lütke, which I have supposed to have been rightly reduced.

The corrections for elevation of the station above the level of the sea has hitherto been made by Dr. Young's rule, which is based on the assumption that all the earth and rock rising above the level of the sea is to be considered as attracting the pendulum as if it was so much additional matter beneath the pendulum in excess of what is found at the level of the sea. That this assumption, so foreign to the facts of the case, does not accord with the determinations of gravity made at the sea shore, and at great elevations in the neighborhood, has often been pointed out; in particular by M. Faye in the "Comptes Rendus," June 21, 1880.

Let us first consider the question *à priori*.

* The present computation was begun before Major Herschel's work was received. The principles of my procedure being different from his, I have thought it worth while to complete my work and see how far Major Herschel's and my results would agree.

The only geological cause of vast horizontal displacements of solid matter, such as alone could sensibly affect the pendulum, is denudation. This has always been at work to remove matter from the continents and deposit it on the bed of the ocean; and it seems probable that the general result of it must have been to diminish the amount of matter in those cones having their vertices at the center of the earth and their bases on our present land, and to increase the amount of matter in those cones having their bases on our present sea bed. But it would seem that the effect of such transfer is one of those of which we cannot at present take account. We must, then, regard continental elevations, as produced by the elevation of a certain thickness of matter, equivalent to a crust.

If we use the following notation:

g=gravity,
δ=density of the continent,
Δ=mean density of the earth,
c=radius of the earth,
u=length of the chord from the center to the circumference of a small circle,
h=elevation above the level of the sea,

then the vertical attraction of a cap of matter having the thickness h all over this small circle upon the point at its center is, according to Pratt,

$$\frac{3}{2} g \frac{\delta}{\Delta} \frac{1}{c} \left(u+h-\sqrt{u^2+h^2+\frac{uh}{2c}} \right).$$

It has always been assumed that the second term of the polynomial in the parenthesis would alone be sensible except in extreme cases. If, now, we conceive that the elevation, instead of being produced by the addition of new matter, has been produced by the upheaval of the crust of thickness t, then the quantity in the parenthesis becomes

$$u+\sqrt{u^2+(t+h)^2}-\sqrt{u^2+t^2}-\sqrt{u^2+h^2}$$

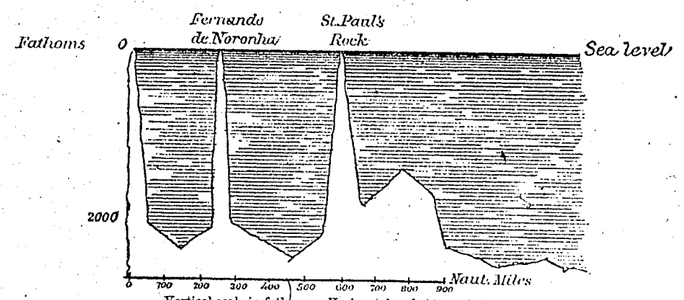
Here we perceive that that term which has generally been considered as alone sensible here disappears altogether; but a new term enters in of the first degree in h, the value of which is

$$\frac{t\,h}{\sqrt{u^2+t^2}}.$$

It will be seen that this term, which is the principal part of the whole expression, is nearly inversely proportional to the radius of the cap, so that a large cap will have less downward attraction than a small one. The reason of this is as follows:

Through any point, p, let a line be drawn in a vertical direction; through p let a sphere be described having its center upon this line. Taking p as the origin of polar co-ordinates, let a new surface be described having the radius vector in any direction equal to that of the sphere. Then this surface will be the surface upon which a particle of matter anywhere placed will exert a constant vertical attraction upon the point p. As the center of the sphere varies its position along the vertical line through p, a succession of these inclosed surfaces of equal vertical attraction will be formed all in contact with one another at the point p. It will be seen then that the nearly spherical surface of the earth cuts all these surfaces in such a manner that the uplifting of the particles anywhere on its surface other than the immediate neighborhood of p will diminish the vertical attraction upon p. The exact calculation of the effect of the elevation of a crust being extremely troublesome, and the quantity being small, I have thought it sufficient to assume that the downward attraction would be one-tenth of the correction for elevation. This quantity which was adopted as satisfying moderately well the experiments on neighboring high and low levels is equivalent to assuming that the thickness of the crust is about one-eighth of the diameter of the space arched over; and the assumption has the same effect as if it were supposed that the continental elevations were produced by additional matter having three-fourths the density of water.

We have so far only considered the solid material of the crust. In addition to this there is the water, which simply runs down to the lowest levels; so that its whole attraction is to be considered and allowed for. Let us consider a small island lying in the midst of the sea. In the first place, if there were no water about it, the gravity would be in excess upon such an island in consequence of the depression of the sea-bed around it. This excess may, as we have seen, be roughly taken as equal to what would be produced by the downward attraction of an excess of matter sufficient to build up the island and having three-fourths the density of water. In the case of a small island the effect will be greater than this. In the case of a large island, especially one lying near a continent, the effect will be increased by the surrounding deposits of matter resulting from denudation. On the whole, then, we may roughly take the excess of gravity on the island to what it would be if the island was built up of extra matter having the density of water. Then when we take into account the attraction of the ocean itself, we may say that the excess of gravity upon an island will be about equal to what would be produced by an ocean having the general depth of the ocean bed quite outside the island and extending completely over the space occupied by the island. What is meant will be understood the moment that we look at a rough profile of the equatorial Atlantic through the islands Fernando de Noronha and Saint Paul's Island.



Vertical scale in fathoms. Horizontal scale in nautical miles.

Coast stations are generally to be considered as really continental, since the true boundary of the continent at a depth of some 100 fathoms is considerably outside the coast. Stations near this boundary may be considered to be like deep-water islands surrounded by water of half the depth of that which is upon one side of the station. Taking the contour chart of the ocean bed given in Mr. Wild's book called Thalassa, I find that the following pendulum stations have to be corrected for the depth of the ocean about them:

Stations at which corrections are to be applied for attraction of water.

	Fathoms.
Hare Island	1,000
Melville Island	1,000
Galapagos	2,000
St. Thomas	3,000
Ascension	2,000
Sierra Leone	500
Bahia	1,000
Jamaica	700
Spitzbergen	500
Point Bowen	500
Valparaiso	250
Port La Coquille	3,000
Guaan	3,000
Port Lloyd	2,000
St. Helena	2,000
Montevideo	200

	Fathoms.
Staten Island	500
South Shetland	500
Cape Horn	500
Fernando de Noronha	3,000
Minicoy Island	500

Rough as these corrections are, the application of them will show what we have to expect from a more exact treatment according to the principles just laid down. I have calculated the ellipticity with these corrections, and find it to be (taking $\eta=.0052375$)

$$\frac{1}{291.5 \pm 0.9}$$

This probable error is rather smaller than that obtained by the treatment of the latest and best geodetical operations and is much smaller than that obtained hitherto from pendulum observations. Upon comparing my residuals with those of Clarke (Geodesy, p. 349), it will be observed that, as a general rule, the residuals that now remain large had previously been much larger, and are generally marine stations, from whence I infer that a more careful estimate of the attraction of the ocean would produce a still further improvement of the result. That which is most needed to improve the state of our knowledge of gravity are additional experiments in the Arctic Circle. It is desirable that these experiments should be made with the Indian apparatus, and it is also to be desired that the same apparatus should be used at Trinidad, Ascension, and Maranham, the three stations which are common to the great expeditions of Sabine and Foster.

PENDULUM EXPEDITIONS.

ABBREVIATIONS AND FORMULÆ.

$\left.\begin{matrix} E \\ A \\ T \end{matrix}\right\}$ = Author's corrections for $\left\{\begin{matrix} \text{Elevation.} \\ \text{Atmospheric effect.} \\ \text{Expansion of bar from } 62^\circ \text{ F.} \end{matrix}\right.$

$\left.\begin{matrix} \delta E \\ \delta A \\ \delta T \end{matrix}\right\}$ = Corrections to the above $\left\{\begin{matrix} E+\delta E=E' \\ A+\delta A=A' \\ T+\delta T=T' \end{matrix}\right.$

$\left.\begin{matrix} e \\ a \\ t \end{matrix}\right\}$ = corrected by $\left\{\begin{matrix} E \\ A \\ T \end{matrix}\right.$

$\left.\begin{matrix} \bar{e} \\ \bar{a} \\ \bar{t} \end{matrix}\right\}$ = uncorrected.

$\left.\begin{matrix} e' \\ a' \\ t' \end{matrix}\right\}$ = corrected for $\left\{\begin{matrix} E+\delta E \\ A+\delta A \\ T+\delta T \end{matrix}\right.$

$\left.\begin{matrix} \bar{e}' \\ \bar{a}' \\ \bar{t}' \end{matrix}\right\}$ = uncorrected.

Temperature factor = .458 oscillation per degree for 86,400 oscillations per day.

Atmospheric factor $= .345 \dfrac{\beta}{1+.0023\,(t-32^\circ)}$ or preferably $= \dfrac{1.655}{8.61} \times$ times old correc.

where sp. gr. = 8.61 β = ht. of bar. t = F. ther.

EXPEDITION No. I.

[Kater's, 1818–'19.—Pendulum not numbered; coefficient for expansion of bar .423 oscillation per degree Fahrenheit; specific gravity of pendulum taken at 8.61; no atmospheric effect except buoyancy allowed for. Phil. Trans. 1819.]

Stations.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$	Mean temp.	A	δT	A′	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$
	° ′ ″		°				
London (Mr. Browne's)	51 31 08	86,055.36	70.5	+5.91	+0.32	+ 9.85	86,065.53
Unst	60 45 28	90.77	57.8	6.07	− .16	10.02	100.63
Portsoy	57 40 59	79.77	60.5	6.05	− .06	10.00	089.71
Leith Fort	55 58 41	73.16	57.0	6.05	− .18	10.00	082.98
Clifton	53 27 45*	62.01	55.0	5.94	− .26	9.80	071.55
Arbury Hill	52 12 56	56.88	52.9	6.04	− .34	9.97	066.51
London (Mr. Browne's)	51 31 8	55.12	51.8	6.18	− .23	10.20	065.09
Shanklin Farm	50.37 24	51.28	60.9	+6.09	− .04	+10.05	061.29

* See Phil. Trans., 1823, p. 325.

EXPEDITION No. II.

[Sabine's first voyage. Pendulum No. 2 in Shelton clock No. 2. Temp. coefficient 0.439 oscillations per degree Fahr.; specific gravity pendulum, 8.4. Correction for buoyancy only. Phil. Trans., 1821.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 50°	Mean temp.	E	A	T	δA	T′	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		°						
Brassa	60 09 42	86,530.51	54.6	+.10	+6.24	+2.02	+4.09	−3.39	86,529.09
Hare Island	70 26 19	562.64	44.7	+.18	+6.45	−2.29	+4.22	−7.92	561.05
London (Mr. Browne's)	51 31 08	497.38	44.5	+.35	+6.46	−2.39	+4.23	−8.01	495.64

EXPEDITION No. III.

[Sabine's second voyage. Pendulums Nos. 1 and 2 in Shelton clocks Nos. 1 and 2. Survey each in its own clock and each in other's clock. Same reductions as above. Same memoir.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 45°	Mean temp.	E	A	T	δA	T′	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
		Each pendulum in its own clock.							
	° ′ ″		°						
London (Mr. Browne's)	51 31 08	86,444.78	47.7	+.35	+6.39	+1.41	+4.19	−6.55	86,440.66
Melville Island	74 47 12	519.17	46.5	+.14	+6.39	+0.67	+4.19	−6.70	515.70
London on return	51 31 08	444.65	51.4	+.35	+6.32	+2.81	+4.14	−4.85	440.78
		Each pendulum in the other's clock.							
London	51 31 08	86,446.58	48.4	+.35	+6.41	+1.40	+4.20	−6.23	456.30
Mellville Island	74 47 12	541.46	45.7	+.14	+6.36	+0.30	+4.17	−7.47	531.36

EXPEDITION No. IV.

[Hall's, 1820–'23.—Number of pendulum not given. Coefficient for expansion of bar, .423 oscillation per degree Fahr. Specific gravity of pendulum not given. Reduced by author to 68° Fahr. Phil. Trans., 1823.]

Stations.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 68°	Bar.	Mean temp.	δT.	A′	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 68°	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		in.	°				
London (Mr. Browne's), rejected by author	51 31 8	86,229.78	30.08	66.5	−.05	+9.87	86,239.60	86,242.35
Earl of Abingdon's Island	0 32 19	095.54	29.93	80.9	+.45	9.51	105.50	108.25
San Blas de California	21 32 24	119.00	29.80	83.0	+.23	9.45	129.57	132.32
Rio de Janeiro	22 55 22	125.62	29.84	74.6	+.23	9.60	135.45	138.20
London	51 31 8	230.80	29.83	66.9	−.04	+9.77	240.53	243.28

EXPEDITION No. V.

[Goldingham's, 1820–'23. Pendulum not numbered. Sp. gr. taken at 8.02; Baily says it was 7.97. Coefficient for expansion of bar = .423 oscillation per degree Fahr. Phil. Trans., 1822.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 70°	Temp.	Bar ured.	δT	A′	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 70°	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		°	in.				
London	51 31 8	86,293.14	67.6	29.97	−.08	+9.77	86,302.83	86,306.49
Madras	13 4 9	163.08	84.5	30.19	+.51	+9.67	86,176.27	86,179.93

EXPEDITION No. VI.

[Brisbane's, 1821–'22. Pendulum not numbered. Coefficient for expansion of bar = .423 oscillation per degree Fahr. Specific gravity of pendulum taken at 8.00. Reduced by author to 60° Fahr. Phil. Trans., 1823.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 60°	Mean temp.	A	δT	δA	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 60°	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		°					
London	51 31 8	86,083.84	61.8	+6.42	+.06	+3.90	86,094.22	86,093.30
Paramatta	33 48 43	15.42	56.8	+6.47	−.11	+3.93	25.71	24.79

EXPEDITION No. VII.

[Sabine's (in the Pheasant), 1821–'24. Pendulums Nos. 3 and 4. Coefficient for expansion of bar = .42 (p. 16) oscillation per degree Fahr. Specific gravity of the pendulum taken at 8.61. From "Sabine's Experiments to determine the figure of the Earth." London, 1825, p. 236. The corrections in the last column but one are given by Sabine. Phil. Trans., 1828, p. 77.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 6.°	Mean temp.	A +	δT ±	δA +	Corr.	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		°					
St. Thomas	0 24 41	86,029.40	81.5	+8.24	5.70	+.75	3.79	.....	86,033.90
Maranham	− 2 31 43	019.78	80.9	+7.96	5.78	+.72	3.79	−.01	024.28
Ascension	− 7 55 48	033.11	81.1	+8.06	5.80	+.73	3.80	.....	037.64
Sierra Leone	+ 8 29 28	028.14	80.5	+7.80	5.75	+.71	3.77	.....	032.62
Trinidad	+10 38 56	027.31	82.8	+8.78	5.75	+.79	3.77	+.04	031.91
Bahia	−12 59 21	032.81	73.2	+4.71	5.88	+.43	3.85	+.09	037.18
Jamaica	17 56 7	045.27	81.6	+8.27	5.78	+.75	3.79	−.08	049.73
New York	40 42 43	117.97	34.6	−11.51	6.43	−1.04	4.22	−.05	121.10
*London	51 31 8	159.79	62.0	.....	.....	.....	.....	.....	163.20
Drontheim	63 25 54	198.52	46.7	−6.43	6.16	−.58	4.03	.....	201.97
Hammerfest	70 40 5	320.96	49.2	−6.36	6.15	−.49	4.03	.....	224.50
Greenland	74 32 10	230.44	42.3	−8.28	6.26	−.75	4.10	.....	233.79
Spitzbergen	79 49 58	242.93	41.0	−8.78	6.27	−.79	4.11	.....	246.25

* Specially computed from data given by Sabine.

EXPEDITION No. VIII.

[Foster's first, 1824–'25. Pendulum No. 3. Coefficient for expansion of bar = .423 oscillation per degree Fahr. Specific gravity of pendulum, 8.61. Phil. Trans., 1826, IV.]

Station.	Latitude.	Oscil. per day. $\bar{e}$ $\bar{a}$ $\bar{t}$ 50°	Temp.	A	δT	δA	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 50°	Oscil. per day. $\bar{e}'$ $\bar{a}'$ $\bar{t}'$ 62°
	° ′ ″		°					
Greenwich	51 28 40	86,152.87	56.05	+6.06	+.21	+3.97	86,163.11	86,157.61
Port Bowen	73 13 30	86,223.87	50.70	+6.14	+.02	+4.02	86,234.05	86,228.55
Greenwich	51 28 40	86,153.13	44.50	+6.04	−.10	+3.96	86,162.94	86,157.44

Expedition No. IX.

[Luetke, 1829–'31.—Hall's pendulums. Reductions correct. St. Petersburg Acad. Sci. Divers Savans, Vol. III, 1837.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$ 62°		Oscil. per day. $\delta\ a\ t$ 62°
	° ′ ″			
Petersburg	59 56 30	86, 268. 86	+.28	86, 269. 14
Greenwich	51 28 40	86, 236. 09	.....	86, 236. 09
Valparaiso	−33 2 30	86, 165. 33	+.21	86, 165. 54
New Archangel	57 02 59	86, 257. 33	+.14	86, 257. 47
Petropolowski	53 0 59	86, 245. 50	+.07	86, 245. 57
Port La Coquille	5 21 16	86, 112. 64	±.00	86, 112. 64
Guahan	13 26 18	86, 117. 84	−.07	86, 117. 77
Port Lloyd	27 4[illegible]9	86, 159. 00	−.14	86, 158. 86
St. Helena	−15 55 3	86, 125. 19	−.21	86, 124. 98
By Captain Reinecke. (Edges reground).				
Kandalachka	67 7 43	86, 300. 34	.....	86, 300. 34
Petersburg	59 56 30	86, 279. 17	.....	86, 279. 17

Expedition No. X.

[Sabine's, 1827. Pendulums Nos. 7 and 8. Coefficient for expansion of bar = .421 oscillation per degree Fahr. Specific gravity of pendulums, 8. 61. Reduced to 58° at Paris and 60° at London. Phil. Trans., 1828.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$ 60°	Mean temp.	A +	δT	δA	Oscil. per day. $\delta\ a'\ t'$ 60°	Oscil. per day. $\delta\ a'\ t'$ 62°
	° ′ ″		°					
Paris	48 50 14	85, 927. 52	56. 88	6. 04	−.12	10. 00	85, 937. 40	85, 936. 48
London	51 31 8	85, 939. 55	63. 36	6. 00	+.12	9. 93	85, 949. 60	85, 948. 62

Expedition No. XI.

[Sabine's, 1829. Pendulum No. 12. Coefficient of expansion of bar = .43 oscillation per degree Fahr. for London-Greenwich, and 0.44 for Greenwich-Altona. Atmospheric reduction correctly made. Phil. Trans., 1829 and 1830.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$ 62°	Mean temp.	δT	Oscil. per day. $\delta\ a'\ t'$ 62°
	° ′ ″		°		
London (Mr. Browne's)	51 31 8	85, 973. 57	62. 5	+.01	85, 973. 58
Greenwich	51 28 40	85, 974. 09	58. 92	−.09	74. 00
Knife reground.					
London	51 31 8	85, 969. 34	71. 9	+.28	85, 969. 62
Greenwich	51 28 40	85, 969. 78	61. 5	−.01	69. 77
		To 60°			
Greenwich	51 28 40	85, 970. 77	63. 53	+.03	85, 969. 88
Altona	53 32 45	85, 979. 10	58. 32	−.06	978. 12

Expedition No. XII.

[Fallow's, 1828–'29. Pendulum No. 4, as in Expedition No. VII. Coefficient for expansion of bar = .421 oscillation per degree Fahr. Specific gravity of pendulum. 8.60. Phil. Trans., 1830.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$ 62°	Mean temp.	Bar.	A +	δT	δA	Oscil. per day. $\delta\ a'\ t'$ 62°
	° ′ ″		°	in.				
London (Mr. Browne's)	51 31 8	86, 164. 62	70. 82	29. 80	5. 87	+.33	3. 84	86, 168. 79
Cape of Good Hope	33 55 56	83, 097. 72	71. 44	30. 03	5. 01	+.25	3. 87	86, 101. 94

Expedition No. XIII.

[Foster's, 1828–'31. Pendulums Nos. 10 and 11. Coefficient for expansion of bar = .422 oscillation per degree Fahr. Atmospheric effect correctly taken into account. Memoirs Royal Astron. Society, Vol. VII, 1834.]

MEANS FOR PENDULUMS 10 AND 11.

Station	Latitude.	Oscil. per day. $\delta\ a\ t$ 62°	T	δT	Oscil. per day. $\delta\ a'\ t'$ 62°
	° ″ ′				
London (Mr. Browne's; on Kater's support)	51 31 8	86, 066. 48	− 3. 24	−.28	86, 066. 20
Greenwich	51 28 40	86, 065. 59	− 6. 31	−.54	86, 065. 05
Montevideo	−34 54 26	86, 001. 70	− 3. 69	−.31	86, 001. 39
Staten Island	−54 46 23	86, 082. 04	− 5. 87	−.50	86, 081. 59
*South Shetland	−62 56 11	86, 111. 53	−10. 76	−.92	86, 110. 65
Cape Horn	−55 51 20	86, 084. 92	− 8. 95	−.76	86, 084. 17
Cape of Good Hope	−33 54 37	85, 998. 21	− 1. 64	−.14	85, 998. 07
St. Helena	−15 56 7	85, 954. 76	+ 6. 12	+.52	84, 955. 29
Ascension	− 7 55 23	85, 939. 03	+ 8. 49	+.72	85, 939. 74
†Green Mountains	− 7 58 00	85, 930. 68	+ 5. 31	+.45	85, 931. 13
Fernando de Noronha	− 3 49 59	85, 938. 70	+ 7. 98	+.68	85, 939. 38
Maranham	− 2 31 35	85, 925. 17	+ 8. 66	+.74	85, 925. 38
Para	− 1 27 00	85, 927. 31	+ 8. 84	+.75	85, 928. 05
Trinidad	+10 38 55	85, 934. 54	+ 9. 08	+.77	85, 935. 29
Porto Bello	+ 9 32 30	85, 939. 59	+ 6. 76	+.58	85, 940. 17
‡London (Mr. Baily's)	+51 31 8	86, 066. 50	− 4. 03	−.34	86, 066. 10

[One-half mean difference of pendulums = 51. 71 St. Helena to Para.]

* No. 10 reduced to mean. † Pendulum No. 10 only used here. ‡ Pendulum No. 10 rejected, and also those of 1832 with pendulum No. 11.

Expedition No. XIV.

Murphy's, 1835. Pendulum No. 10. Coefficient for expansion of bar = .423 oscillations per degree Fahr. Atmospheric effect correctly taken into account. Memoirs Ast. Soc., XII.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$	T	δT	Oscil. per day. $\delta\ a'\ t'$ 62°
	° ′ ″				
London (Baily's house)	51 31 8	86, 021. 13	− 7. 80	−.65	86, 020. 48
Bir	37	85, 960. 55	− 7. 36	−.61	85, 959. 94
Bussora	30 30	85, 940. 73	+11. 13	+.93	85, 941. 66

Expedition No. XV.

[Maclear's, 1839. Pendulum No. 11. Coefficient for expansion of bar = .423 oscillations per degree Fahr. Atmospheric effect correctly taken into account. Memoirs Ast. Soc., XII.]

Station.	Latitude.	Oscil. per day. $\delta\ a\ t$ 62°	T	δT	Oscil. per day. $\delta\ a\ t$ 62°
	° ′ ″				
London (Baily's house)	51 31 8	86, 116. 07	+1. 31	+.11	86, 116. 18
Cape of Good Hope	33 56 3	86, 049. 40	−1. 57	−.13	86, 049. 27

Expedition No. XVI.

Basevi and Heaviside, 1865 to 1874. Pendulums Nos. 4 and 1821. Coefficients of expansion = 0.458 for pendulum No. 4 and 0.442 for pendulum No. 1821. G. T. Survey of India, Vol. 5 [p. 120], 1879.

Station.	Latitude.	Oscil. per day. $\frac{\varepsilon}{t}\frac{a}{62°}\frac{t}{}$
	° ′ ″	
Pannæ	8 09 28	85,982.75
Kúdankolam	9 10 21	82.56
Minicoy	8 17 01	87.01
Mallapatti	9 28 45	82.60
Alleppy	9 29 39	85.89
Pachapaliam	10 59 40	82.28
Aden	12 46 53	91.67
Mangalore	12 51 37	88.87
Bangalore, South	13 00 41	78.49
Bangalore, North	13 04 56	79.38
Madras	13 04 08	89.03
Namthábád	15 05 52	87.71
Cocanáda	16 56 21	98.23
Kodangal	17 07 57	91.01
Damargida	18 03 17	91.04
Colaba (Bombay)	18 53 46	86,005.19
Somtana	19 05 00	85,996.27
Badgaon	20 44 23	86,002.26
Calcutta	22 32 55	12.69
Ahmadpúr	23 36 21	08.21
Kaliánpúr	24 07 11	10.36
Pahárgarh	24 56 07	11.10
Usira	26 57 06	21.31
Datairi	28 44 05	26.73
Kaliana	29 30 55	27.25
Nojli	29 53 28	27.62
Dehra	30 19 29	20.86
Mussoorie	30 27 41	11.59
Ismailia	30 35 55	35.93
Meean Meer	31 31 37	34.55
Moré	33 15 39	85,984.62
Kew	51 28 00	86,118.15

The results of Expeditions Nos. IX, XI, XII, XIII, XV have been reduced to VII by solving by least squares the following equations of condition:

		Obs.	*Calc.*
VII at Greenwich	— reduction of IX	= 4.9356890	6880
VII at St. Helena	— reduction of IX	= 4.9351291	1302
VII at London	— reduction of XI	= 4.9353550	3560
VII at Greenwich	— reduction of XI	= 4.9353564	3554
VII at London	— reduction of XII	= 4.9353500	3510
VII at C. G. H.	— reduction of XII	= 4.9350129	0119
VII at London	— reduction of XIII	= 4.9348326	8296
VII at Greenwich	— reduction of XIII	= 4.9348269	8289
VII at C. G. H.	— reduction of XIII	= 4.9344887	4904
VII at St. Helena	— reduction of XIII	= 4.9342726	2714
VII at Maranham	— reduction of XIII	= 4.9341242	1261
VII at Ascension	— reduction of XIII	= 4.9341940	1948
VII at Trinidad	— reduction of XIII	= 4.9341715	1691
VII at London	— reduction of XV	= 4.9350847	0855
VII at C. G. H.	— reduction of XV	= 4.9347472	7404
VII at London		= 4.9353222	3224
VII at Maranham		= 4.9346210	6190
VII at Ascension		= 4.9346885	6877
VII at Trinidad		= 4.0346596	6619

The following are the values of the unknown quantities:

Reduction of Expedition No. IX	=	− 3661
No. XI	=	+ 9664
No. XII	=	− 284
No. XIII	=	+ 4926
No. XIV	=	+ 2371

Oscillations of VII at London	=	4.9353224
Greenwich	=	3218
C. G. H.	=	4.9349833
St. Helena	=	7642
Maranham	=	6190
Ascension	=	6877
Trinidad	=	6619

Expedition No. I.

Reduction to No. VII by London = + 4,943.

Station.	Log No. oscillations.	Red. to VII −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced.
London	4.9348281	3,224	6,448	39	13,916	2,571
Unst	50063	5,006	10,012	12	17,283	2,741
Portsoy	49512	4,455	8,910	39	16,215	2,734
Leith Fort	49173	4,116	8,232	28	15,597	2,663
Clifton	48597	3,540	7,080	141	14,659	2,562
Arbury Hill	48342	3,285	6,570	306	14,184	2,692
Shanklin Farm	48078	3,021	6,042	100	13,570	2,572

Expedition No. II.

Reduction to No. VII from experiments in London = − 16,718.

Station.	Log No. oscillations.	Red. to VII −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced.
London	4.9369942	3,224	6,448	39	13,916	2,571
Brassa	4.9371622	4,904	9,808	10	17,081	2,737
Hare Island	4.9373225	6,507	13,014	18	20,146	2,876

Expedition No. III.

Reduction to No. VII from experiments in London: For each pendulum in its own clock = − 13,957; for each pendulum in the other clock = − 14,742.

Station.	Log No. oscillations.	Red. to VII −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced.
London	4.9367181	3,224				
London	4.9367066					
Melville Island	4.9370954	6,997	13,994	14	21,129	2,879
Melville Island	4.9371735	6,993	13,986	14	21,129	2,871

Expedition No. IV.

Reduction to No. VII from experiments in London: Before St. Blas — 3,982; after Abingdon's Island — 4,029.

Station.	Log No. oscillations.	Red. to VII —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to No. VII.
London	4.9357206	3,224				
Earl of Abingdon's Island	4.9350448	6,466	2,932	5	2	2,935
San Blas	1662	7,633	5,266	48	3,065	2,249
Rio de Janeiro	1958	7,929	5,858	29	3,449	2,438
London	7253	3,224				

Expedition No. V.

Reduction to No. VII from experiments at London = — 7,210.

Station.	Log No. oscillations.	Red. to VII —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to No. VII.
London	4.9360434	3,224				
Madras	4.9354061	6,851	3,702	11	1,163	2,550

Expedition No. VI.

Reduced to No. VII from experiments in London = + 3,531.

Station.	Log No. oscillations.	Red. to VII —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
London	4.9349693	3,224				
Paramatta	4.9346236	9,767	9,535	31	7,038	2,528

Expedition No. VII.

Station.	Log No. oscillations.	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced.
St. Thomas	4.9346696	3,392	9	1	3,400
Maranham	46210	2,421	32	44	2,409
Ascension	46885	3,770	7	432	3,345
Sierra Leone	46631	3,263	79	496	2,846
Trinidad	46595	3,191	9	776	2,424
Bahia	46861	3,723	89	1,149	2,663
Jamaica	47495	4,990	3	2,156	2,837
New York	51095	2,101	28	9,666	2,553
London	53222	6,445	39	13,916	2,568
Drontheim	55172	0,344	51	18,158	2,237
Hammerfest	56307	2,614	12	20,206	2,420
Greenland	56775	3,550	13	21,078	2,485
Spitzbergen	57402	4,805	9	21,982	2,832

Expedition No. VIII.

Reduction = + 286.

Station.	Log No. oscillations.	[Red. to Sabine] —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
Greenwich	4.9352932	3,218	6,436	66	13,900	2,602
Port Bowen	6511	6,797	3,594	49	20,802	2,841

Expedition No. IX.

Reductions = — 3,661 and = — 4,165.

Station.	Log No. oscillations.	[Red. to Sabine] —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
Petersburg	4.9358555	4,894	4,788	26	17,006	2,808
Greenwich	6890	3,229	6,459	66	13,900	2,625
Valparaiso	3336	9,675	9,350	5	6,757	2,558
New Archangel	7967	4,306	8,612	6	15,085	2,633
Petropaulowski	7368	3,707	7,414	32	14,490	2,956
Port La Coquille	0669	7,008	4,016	2	199	3,819
Guahan	0927	7,266	4,533	2	1,223	3,709
Port Lloyd	2999	9,398	8,676	6	4,730	3,952
St. Helena	1293	7,630	5,260	15	1,714	3,561
Kandalachka	4.9360125	5,960	1,920	12	10,266	2,666
Petersburg	4.9359059	4,894				

Expedition No. X.

Reductions = + 10,834. Derived from London values.

Station.	Log No. oscillations.	Red. to Sabine 21—24 —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
Paris	4.9341775	2,609	5,218	100	12,873	2,445
London	4.9342389	3,223	6,447	39	13,916	2,570

Expedition No. XI.

Reductions = { + 9,561 / + 9,767 } Derived from London values.

Station.	Log No. oscillations.	Red. to Sabine 21—24 —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
London	4.9343650	3,211	6,422	39	13,916	2,545
Greenwich	3671	3,232	6,464	66	13,900	2,630
London	3450	3,217	6,434	39	13,916	2,557
Greenwich	3457	3,224	6,448	66	13,900	2,619
Greenwich	4.9343463	3,230	6,460	66	13,900	2,626
Altona	3879	3,646	7,202	41	14,691	2,642

Expedition No. XII.

Reductions = — 284.

Station.	Log No. oscillations.	[Red. to Sabine] —4.935	Mult. by 2.	Elevation corr. +	Latitude corr. —	Reduced to VII.
Portland Place	4.9353500	3,216	6,432	39	13,916	2,525
Cape of Good Hope	4.9350129	9,845	9,690	14	7,073	2,631

Expedition No. XIII.

Reduction = + 4,926.

Station.	Log No. oscillations.	[Red. to Sabine] −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced to VII.
London	4.9348326	3,252	6,504	39	13,916	2,627
Greenwich	48268	3,194	6,388	66	13,900	2,554
Montevideo	45055	9,981	9,962	5	7,456	2,511
Staten Island	49103	4,029	8,058	7	15,151	2,914
S. Shetland	50567	5,493	0,986	10	18,000	2,996
Cape Horn	49232	4,158	8,316	17	15,550	2,783
Cape of Good Hope	44888	9,814	9,628	14	7,073	2,569
St. Helena	42726	7,652	5,304	13	1,714	3,603
Ascension	41940	6,866	3,732	7	432	3,307
Green Mount	41506	6,432	2,864	026	437	3,353
Fernando de Noronha	41922	6,848	3,696	14	102	3,608
Maranham	41242	6,166	2,332	32	44	2,320
Para	41350	6,276	2,552	17	15	2,554
Trinidad	41715	6,641	3,282	9	776	2,515
Porto Bello	41962	6,888	3,776	5	625	3,156
London	48321	3,247	6,494	43	13,916	2,621

Expedition No. XIV.

Reduction = + 7,205.

Station.	Log No. oscillations.	[Red. to Sabine] −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced to VII.
London (Mr. Baily's house)	4.9346018	3,223	6,447	39	13,916	2,570
Bir	4.9342961	0,166	0,332	270	8,230	2,372
Bassorah	4.9342037	9,242	8,485	1	5,856	2,630

Expedition No. XV.

Reduction = + 2,371.

Station.	Log No. oscillations.	[Red. to Sabine] −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced to VII.
London (Mr. Baily's house)	4.9350847	3,218	6,446	39	13,916	2,564
Cape of Good Hope	4.9347471	9,842	9,695	14	7,073	2,631

Expedition No. XVI.

Reduction = + 2,421. Reduction derived from Madras.

Station.	Log No. oscillations.	[Red. to Sabine] −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Reduced to VII.
Punnae	4.9344113	6,534	3,068	20	458	2,630
Kudankolam	4.9344103	6,524	3,049	70	400	2,659
Minicoy	4.9344328	6,749	3,499	3	472	3,030
Mallapatti	4.9344105	6,526	3,053	120	617	2,556
Allepy	4.9344271	6,692	3,385	3	619	2,769
Pachapaliam	4.9344089	6,510	3,020	4	827	2,197
Aden	4.9344563	6,984	3,969	2	1,113	2,858
Mangalore	4.9344422	6,843	3,686	3	1,126	2,563
Bangalore South	4.9343808	6,310	2,638	1,295	1,151	2,782
Bangalore North	4.9343943	6,364	2,728	1,250	1,165	2,803
Madras	4.9344430	6,851	3,703	11	1,163	2,551

Expedition No. XVI—Continued.

Reduction = + 2,421. Reduction derived from Madras.

Station.	Log No. oscillations.	[Red. to Sabine] −4.935	Mult. by 2.	Elevation corr. +	Latitude corr. −	Sabine.
Namthábád	4.9344363	6,884	3,769	487	1,542	2,714
Cocanáda	4.9344895	7,316	4,633	3	1,931	2,705
Kodangal	4.9344530	6,951	3,903	796	1,974	2,725
Damargida	4.9344532	6,953	3,906	809	2,184	2,531
Colaba-Bombay	4.9345246	7,667	5,335	14	2,385	2,964
Somtana	4.9344796	7,217	4,435	711	2,430	2,716
Badgaon	4.9345699	7,520	5,040	465	2,852	2,653
Calcutta	4.9345625	8,046	6,092	8	3,422	2,678
Ahmadpúr	4.9345398	7,819	5,639	704	3,638	2,705
Kaliánpur	4.9345507	7,928	5,856	732	3,792	2,796
Pahárgarh	4.9345545	7,966	5,932	682	4,041	2,573
Usira	4.9346060	8,481	6,963	337	4,670	2,630
Datairi	4.9346334	8,755	7,510	298	5,254	2,554
Kallána	4.9346360	8,781	7,563	337	5,517	2,383
Nojli	4.9346379	8,800	7,600	365	5,645	2,320
Dehra	4.9346037	8,458	6,917	932	5,795	2,254
Mussoorie	4.9345569	7,990	5,981	2,875	5,842	3,014
Ismailia	4.9346798	9,219	8,439	13	5,890	2,562
Meean Meer	4.9346729	9,150	8,300	294	6,214	2,380
Moré	4.9344207	6,628	3,257	6,429	6,836	2,850
Kew	4.9340997	3,418	6,837	6	13,896	2,947

Table of residuals.

No.	Station.	No. of expedition.	Elevation corr.	Latitude corr.	Logarithm relative gravity.	Same corr. for sea-depth.	o−c
1	Spitzbergen	VII	9	21,982	2,832	2,657	+ 28
2	Melville Island	III	14	21,129	2,875	2,525	− 104
3	Greenland	VII	13	21,078	2,485	.....	− 144
4	Port Bowen	VIII	49	20,802	2,841	2,666	+ 37
5	Hammerfest	VII	12	20,206	2,420	.....	− 209
6	Hare Island	II	18	20,146	2,876	2,526	− 103
7	Kandalachka	IX	12	19,266	2,666	.....	+ 37
8	Drontheim	VII	51	18,158	2,237	.....	− 392
9	Unst	I	39	17,283	2,741	.....	+ 112
10	Brassa	II	10	17,081	2,737	.....	+ 108
11	St. Petersburg	IX	26	17,006	2,808	.....	+ 179
12	Portsoy	I	39	16,215	2,734	.....	+ 105
13	Sitka (New Archangel)	IX	6	15,985	2,633	.....	+ 4
14	Leith Fort	I	28	15,597	2,663	.....	+ 34
15	Altona	XI	41	14,691	2,642	.....	+ 13
16	Clifton	I	141	14,659	2,562	.....	− 67
17	Petropaulowski	IX	32	14,490	2,956	.....	+327
18	Arbury Hill	I	306	14,184	2,692	.....	+ 73
19	London	I, II, III, IV, V, VI, VII, X, XI, XII, XIII, XIV, XV	39	13,916	2,571	.....	− 58
20	Greenwich	VIII, IX, XI, XIII	66	13,900	2,602	.....	− 27
21	Kew	XVI	6	13,896	2,247	.....	+ 318
22	Shanklin Farm	I	100	13,570	2,572	.....	− 57
23	Paris	X	100	12,873	2,445	.....	− 184
24	New York	VII	28	9,666	2,553	.....	− 76
25	Bir	XIV	270	8,230	2,372	.....	− 257
26	Moré	XVI	6,429	6,836	2,850	.....	+ 221
27	Meean Meer	XVI	294	6,214	2,380	.....	− 249
28	Ismailia	XVI	13	5,890	2,562	.....	− 67
29	Bassorah	XIV	1	5,856	2,630	.....	+ 1
30	Mussoorie	XVI	2,875	5,842	3,014	.....	+ 385

Table of residuals—Continued.

No.	Station.	No. of expedition.	Elevation corr.	Latitude corr.	Logarithm relative gravity.	Same corr. for sea-depth.	o—c
41	Dehra	XVI	932	5,795	2,254	.....	− 375
42	Neill	XVI	365	5,645	2,920	.....	− 309
43	Kalkan	XVI	337	5,517	2,383	.....	− 246
44	Tuticorin	XVI	298	5,254	2,554	.....	− 75
45	Jamis Island (Port Lloyd)	IX	6	4,730	3,952	3,252	+ 523
46	Dodra	XVI	337	4,670	2,630	.....	+ 1
47	Chakragarh	XVI	682	4,041	2,573	.....	− 56
48	Kallanpur	XVI	732	3,792	2,796	.....	+ 167
49	Ahmedabad	XVI	704	3,638	2,705	.....	+ 76
50	Calcutta	XVI	8	3,422	2,678	.....	+ 49
51	San Blas de California	IV	48	3,065	2,249	.....	− 380
52	Dodgeon	XVI	465	2,852	2,653	.....	+ 24
53	Sherkana	XVI	711	2,430	2,716	.....	+ 87
54	Bombay	XVI	14	2,885	2,964	.....	+ 335
55	Dematgoda	XVI	809	2,184	2,531	.....	− 98
56	Pamban	VII	3	2,156	2,837	2,592	− 37
57	Kadangal	XVI	796	1,974	2,725	.....	+ 96
58	Gonhada	XVI	3	1,931	2,705	.....	+ 76
59	Ramchabdal	XVI	487	1,542	2,714	.....	+ 85
60	Gokhan	IX	2	1,228	3,709	2,659	+ 30
61	Madras	V, XVI	11	1,163	2,550	.....	− 79
62	Bangalore North	XVI	1,250	1,165	2,803	.....	+ 174
63	Bangalore South	XVI	1,295	1,151	2,782	.....	+ 153
64	Mangalore	XVI	3	1,120	2,503	.....	− 66
65	Aden	XVI	2	1,113	2,858	.....	+ 229
66	Tachagallam	XVI	4	827	2,197	.....	− 432
67	Trinidad	VII, VIII	2	776	2,448	.....	− 181
68	Porto Bello	XIII	5	625	3,156	.....	+ 527
69	Alleppy	XVI	3	619	2,769	.....	+ 140
70	Mallapatti	XVI	120	617	2,556	.....	− 73
71	Sierra Leone	VII	79	496	2,846	2,671	+ 42
72	Minicoy	XVI	3	472	3,030	2,855	+ 226
73	Kodaikolam	XVI	70	460	2,659	.....	+ 30
74	Punnai	XVI	20	458	2,630	.....	+ 1
75	Port la Cruille (Oualan)	IX	2	199	3,819	2,769	+ 140
76	Seal of Abingdon's Island (Galapagos)	IV	5	2	2,935	2,235	− 384
77	St. Thomas	VII	9	1	3,400	2,850	− 279
78	Para	XIII	17	15	2,554	.....	− 75
79	Maranhan	VII	32	44	2,369	.....	− 260
80	Fernando de Noronha	XIII	14	102	3,608	2,558	− 71
81	Green Mountain	XIII	926	437	3,253	.....	+ 624
82	Ascension	VII, XIII	7	432	3,337	2,637	+ 8
83	Bahia	VII	89	1,149	2,663	2,313	− 316
84	St. Helena	IX, XIII	14	1,714	3,584	2,884	+ 255
85	Rio de Janeiro	IV	29	3,449	2,438	.....	− 191
86	Valparaiso	IX	5	6,757	2,558	.....	− 71
87	Paramatta	VI	31	7,038	2,528	.....	− 101
88	Cape of Good Hope	XII, XIII, XV	14	7,073	2,607	.....	− 22
89	Montevideo	XIII	5	7,456	2,511	2,441	− 188
90	Staten Island	XIII	7	15,151	2,914	2,739	+ 110
91	Cape Horn	XIII	17	15,550	2,783	2,608	− 21
92	South Shetland	XIII	10	18,000	2,996	2,821	+ 192

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