In order to show the linear relations between the squares of the P-functions with quadruple indices, we will begin with the 84 groups, of which

$$kl \quad lm \quad mk \quad klm \quad klmn \quad klmp \quad klmq \quad klmr \quad klms \quad klmt$$

is the first. Take the functions

$$P_{klm}, \quad P_{klmn}, \quad P_{klmp}, \quad P_{klmq},$$

and find the values of A, B, C, D, so that the radicals $\sqrt{R(x_1)R(x_2)}$, etc., shall vanish in the sum $\quad AP^2_{klm} + BP^2_{klmn} + CP^2_{klmp} + DP^2_{klmq}$.

Dropping out a common factor

$$[a_k][x_3, x_4][a_l][x_3, x_4][a_m][x_3, x_4]$$

the necessary condition is easily seen to be

$$A + B[a_n][x_3, x_4] + C[a_p][x_3, x_4] + D[a_q][x_3, x_4] = 0;$$

this is equivalent to $\quad\quad\quad\quad B + C + D = 0,$

$$a_n B + a_p C + a_q D = 0,$$

$$A + a_n^2 B + a_p^2 C + a_q^2 D = 0,$$

and these are easily seen to be satisfied by the values

$$A = -|a_n a_p a_q|, \quad B = |a_p a_q|, \quad C = |a_q a_n|, \quad D = |a_n a_p|.$$

Introducing these values in the above sum we have

$$- |a_n a_p a_q| P^2_{klm} + |a_p a_q| P^2_{klmn} + |a_q a_n| P^2_{klmp} + |a_n a_p| P^2_{klmq}$$

$$= \frac{1}{|x_1 x_2 x_3 x_4|^2}$$

$\{-|a_n a_p a_q|[|x_2 x_3 x_4|^2[a_n, a_p, a_q, a_r, a_s, a_t][x_1][a_k][x_2, x_3, x_4][a_l][x_2, x_3, x_4][a_m][x_2, x_3, x_4]+..]$
$+|a_p a_q|[|x_2 x_3 x_4|^2[a_p, a_q, a_r, a_s, a_t][x_1][a_k][x_2, x_3, x_4][a_l][x_2, x_3, x_4][a_m][x_2, x_3, x_4]+..]$
$+|a_q a_n|[|x_2 x_3 x_4|^2[a_q, a_r, a_s, a_t, a_n][x_1]$ "   "   "   "   "   "   " $]+..]$
$+|a_n a_p|[|x_2 x_3 x_4|^2[a_r, a_s, a_t, a_n, a_p][x_1]$ "   "   "   "   "   "   " $]+..\}$

The right-hand side of this equation may be written in the form

$$\frac{1}{|x_1 x_2 x_3 x_4|^2} [|a_p a_q||x_2 x_3 x_4|[a_r, a_s, a_t, a_p, a_q][x_1][a_k][x_2, x_3, x_4][a_l][x_2, x_3, x_4][a_m][x_2, x_3, x_4]$$

$$\times \{|x_2 x_3 x_4|[a_n][x_2, x_3, x_4] - |x_3 x_4 x_1|[a_n][x_3, x_4, x_1] + |x_4 x_1 x_2|[a_n][x_2, x_3, x_4]$$

$$- |x_1 x_2 x_3|[a_n][x_1, x_3, x_3]\} + ...$$

$$+ \text{(two similar terms containing } [a_q, a_n][x_1], [a_n, a_p][x_1] \text{ respectively)}$$

$$- |a_n a_p a_q|[a_r, a_s, a_t, a_n, a_p, a_q][x_1][a_k][x_2, x_3, x_4][a_l][x_2, x_3, x_4][a_m][x_2, x_3, x_4]$$

$$\times \{|x_2 x_3 x_4| - |x_3 x_4 x_1| + |x_4 x_1 x_2| - |x_1 x_2 x_3|\} + ...]$$

The omitted terms are easily supplied by symmetry. Now

$$\Sigma |x_2 x_3 x_4| = 0, \quad \Sigma |x_2 x_3 x_4|[a_n][x_2, x_3, x_4] = |x_1 x_2 x_3 x_4|$$

The fourth line of the last equation vanishes and the first terms of the first three

$$|x_2 x_3 x_4|[a_r, a_s, a_t][x_1][a_k][x_2, x_3, x_4][a_l][x_2, x_3, x_4][a_m][x_2, x_3, x_4]$$

The Coast Survey Investigation.

We give publicity to the following circular letter:

ANN ARBOR, MICH., August 10, 1885.

DEAR SIR: I just learn from a newspaper that the Commission which investigated the Coast Survey reflected upon me in their report as follows:

"That for several years, beginning in 1873, C.S. Peirce, assistant, has been making experimental researches with pendulums, without restriction or limitation as to times and places; that since 1879 expenditures on account of those experiments, aside from salaries of chiefs and assistants, amount to about $31,000; that the meagre value of those experiments to the bureau have been substantially destroyed."

I have immediately addressed a letter to the Secretary of the Treasury, of which the following is the substance:

1st. My expenditures, aside from compensation of myself and my assistants, during the period specified, have not amounted to one-third of the sum named; and I appeal to the Secretary to ascertain this by the addition of the amounts of my original accounts now on file in the Department.

2d. All my operations have been carried on under specific instructions, and therefore have not been "without restriction or limitation as to times and places." I ask to be informed what operation does not appear to be covered by instructions on file in the office, and promise to show, in any instance, that it really is so covered.

3d. No records have been destroyed.

4th. I maintain the value of determinations of gravity in general, and the excellence of mine in particular.

5th. I tender my resignation if the opinion expressed as to the meagre value of my services is accepted by the Department.

Until my letter is acted upon it might perhaps be considered a breach of official etiquette for me to make it public; but I wish you, as a friend and scientific man, to know that I have a defence against the accusations made.

Yours very truly,
C. S. PEIRCE.