THE MONIST

A

QUARTERLY MAGAZINE

VOL. II

CHICAGO:
THE OPEN COURT PUBLISHING CO
1891-1892

Reprinted with the permission of the original publishers

KRAUS REPRINT CORPORATION
New York
1966

THIS PAGE LEFT BLANK INTENTIONALLY

THE LAW OF MIND.

IN an article published in *The Monist* for January 1891, I endeavored to show what ideas ought to form the warp of a system of philosophy, and particularly emphasised that of absolute chance. In the number of April 1892, I argued further in favor of that way of thinking, which it will be convenient to christen *tychism* (from τύχη, chance). A serious student of philosophy will be in no haste to accept or reject this doctrine; but he will see in it one of the chief attitudes which speculative thought may take, feeling that it is not for an individual, nor for an age, to pronounce upon a fundamental question of philosophy. That is a task for a whole era to work out. I have begun by showing that *tychism* must give birth to an evolutionary cosmology, in which all the regularities of nature and of mind are regarded as products of growth, and to a Schelling-fashioned idealism which holds matter to be mere specialised and partially deadened mind. I may mention, for the benefit of those who are curious in studying mental biographies, that I was born and reared in the neighborhood of Concord,—I mean in Cambridge,—at the time when Emerson, Hedge, and their friends were disseminating the ideas that they had caught from Schelling, and Schelling from Plotinus, from Boehm, or from God knows what minds stricken with the monstrous mysticism of the East. But the atmosphere of Cambridge held many an antiseptic against Concord transcendentalism; and I am not conscious of having contracted any of that virus. Nevertheless, it is probable that some cultured bacilli, some benignant form of the disease was implanted in my soul, unawares, and that now, after long incubation, it comes to the surface, modified by

mathematical conceptions and by training in physical investigations.

The next step in the study of cosmology must be to examine the general law of mental action. In doing this, I shall for the time drop my tychism out of view, in order to allow a free and independent expansion to another conception signalised in my first *Monist*-paper as one of the most indispensable to philosophy, though it was not there dwelt upon ; I mean the idea of continuity. The tendency to regard continuity, in the sense in which I shall define it, as an idea of prime importance in philosophy may conveniently be termed *synechism*. The present paper is intended chiefly to show what synechism is, and what it leads to. I attempted, a good many years ago, to develop this doctrine in the *Journal of Speculative Philosophy* (Vol. III.); but I am able now to improve upon that exposition, in which I was a little blinded by nominalistic prepossessions. I refer to it, because students may possibly find that some points not sufficiently explained in the present paper are cleared up in those earlier ones.

WHAT THE LAW IS.

Logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectibility. In this spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas.

I set down this formula at the beginning, for convenience ; and now proceed to comment upon it.

INDIVIDUALITY OF IDEAS.

We are accustomed to speak of ideas as reproduced, as passed from mind to mind, as similar or dissimilar to one another, and, in short, as if they were substantial things ; nor can any reasonable objection be raised to such expressions. But taking the word "idea" in the sense of an event in an individual consciousness, it is clear that an idea once past is gone forever, and any supposed recurrence of it is another idea. These two ideas are not present in the same

state of consciousness, and therefore cannot possibly be compared. To say, therefore, that they are similar can only mean that an occult power from the depths of the soul forces us to connect them in our thoughts after they are both no more. We may note, here, in passing that of the two generally recognised principles of association, contiguity and similarity, the former is a connection due to a power without, the latter a connection due to a power within.

But what can it mean to say that ideas wholly past are thought of at all, any longer? They are utterly unknowable. What distinct meaning can attach to saying that an idea in the past in any way affects an idea in the future, from which it is completely detached? A phrase between the assertion and the denial of which there can in no case be any sensible difference is mere gibberish.

I will not dwell further upon this point, because it is a commonplace of philosophy.

CONTINUITY OF IDEAS.

We have here before us a question of difficulty, analogous to the question of nominalism and realism. But when once it has been clearly formulated, logic leaves room for one answer only. How can a past idea be present? Can it be present vicariously? To a certain extent, perhaps; but not merely so; for then the question would arise how the past idea can be related its vicarious representation. The relation, being between ideas, can only exist in some consciousness : now that past idea was in no consciousness but that past consciousness that alone contained it ; and that did not embrace the vicarious idea.

Some minds will here jump to the conclusion that a past idea cannot in any sense be present. But that is hasty and illogical. How extravagant, too, to pronounce our whole knowledge of the past to be mere delusion! Yet it would seem that the past is as completely beyond the bonds of possible experience as a Kantian thing-in-itself.

How can a past idea be present? Not vicariously. Then, only by direct perception. In other words, to be present, it must be *ipso facto* present. That is, it cannot be wholly past ; it can only be

going, infinitesimally past, less past than any assignable past date. We are thus brought to the conclusion that the present is connected with the past by a series of real infinitesimal steps.

It has already been suggested by psychologists that consciousness necessarily embraces an interval of time. But if a finite time be meant, the opinion is not tenable. If the sensation that precedes the present by half a second were still immediately before me, then, on the same principle the sensation preceding that would be immediately present, and so on *ad infinitum*. Now, since there is a time, say a year, at the end of which an idea is no longer *ipso facto* present, it follows that this is true of any finite interval, however short.

But yet consciousness must essentially cover an interval of time; for if it did not, we could gain no knowledge of time, and not merely no veracious cognition of it, but no conception whatever. We are, therefore, forced to say that we are immediately conscious through an infinitesimal interval of time.

This is all that is requisite. For, in this infinitesimal interval, not only is consciousness continuous in a subjective sense, that is, considered as a subject or substance having the attribute of duration; but also, because it is immediate consciousness, its object is *ipso facto* continuous. In fact, this infinitesimally spread out consciousness is a direct feeling of its contents as spread out. This will be further elucidated below. In an infinitesimal interval we directly perceive the temporal sequence of its beginning, middle, and end,—not, of course, in the way of recognition, for recognition is only of the past, but in the way of immediate feeling. Now upon this interval follows another, whose beginning is the middle of the former, and whose middle is the end of the former. Here, we have an immediate perception of the temporal sequence of its beginning, middle, and end, or say of the second, third, and fourth instants. From these two immediate perceptions, we gain a mediate, or inferential, perception of the relation of all four instants. This mediate perception is objectively, or as to the object represented, spread over the four instants; but subjectively, or as itself the subject of duration, it is completely embraced in the second moment. [The reader will observe that I use the word *instant* to mean a point of time, and *moment* to mean an infinitesimal duration.] If it is objected that, upon the theory proposed, we must have more than a mediate perception of the succession of the four instants, I grant it; for the sum of the two infinitesimal intervals is itself infinitesimal, so that it is immediately perceived. It is immediately perceived in the whole interval, but only mediately perceived in the last two thirds of the interval. Now, let there be an indefinite succession of these inferential acts of comparative perception; and it is plain that the last moment will contain objectively the whole series. Let there be, not merely an indefinite succession, but a continuous flow of inference through a finite time; and the result will be a mediate objective consciousness of the whole time in the last moment. In this last moment, the whole series will be recognised, or known as known before, except only the last moment, which of course will be absolutely unrecognisable to itself. Indeed, even this last moment will be recognised like the rest, or, at least be just beginning to be so. There is a little *elenchus*, or appearance of contradiction, here, which the ordinary logic of reflection quite suffices to resolve.

INFINITY AND CONTINUITY, IN GENERAL.

Most of the mathematicians who during the last two generations have treated the differential calculus have been of the opinion that an infinitesimal quantity is an absurd. ; although, with their habitual caution, they have often added "or, at any rate, the conception of an infinitesimal is so difficult, that we practically cannot reason about it with confidence and security." Accordingly, the doctrine of limits has been invented to evade the difficulty, or, as some say, to explain the signification of the word "infinitesimal." This doctrine, in one form or another, is taught in all the text-books, though in some of them only as an alternative view of the matter; it answers well enough the purposes of calculation, though even in that application it has its difficulties.

The illumination of the subject by a strict notation for the logic of relatives had shown me clearly and evidently that the idea of an infinitesimal involves no contradiction, before I became acquainted with the writings of Dr. Georg Cantor (though many of these had

already appeared in the *Mathematische Annalen* and in *Borchardt's Journal*, if not yet in the *Acta Mathematica*, all mathematical journals of the first distinction), in which the same view is defended with extraordinary genius and penetrating logic.

The prevalent opinion is that finite numbers are the only ones that we can reason about, at least, in any ordinary mode of reasoning, or, as some authors express it, they are the only numbers that can be reasoned about mathematically. But this is an irrational prejudice. I long ago showed that finite collections are distinguished from infinite ones only by one circumstance and its consequences, namely, that to them is applicable a peculiar and unusual mode of reasoning called by its discoverer, De Morgan, the "syllogism of transposed quantity."

Balzac, in the introduction of his *Physiologie du mariage*, remarks that every young Frenchman boasts of having seduced some Frenchwoman. Now, as a woman can only be seduced once, and there are no more Frenchwomen than Frenchmen, it follows, if these boasts are true, that no French women escape seduction. If their number be finite, the reasoning holds. But since the population is continually increasing, and the seduced are on the average younger than the seducers, the conclusion need not be true. In like manner, De Morgan, as an actuary, might have argued that if an insurance company pays to its insured on an average more than they have ever paid it, including interest, it must lose money. But every modern actuary would see a fallacy in that, since the business is continually on the increase. But should war, or other cataclysm, cause the class of insured to be a finite one, the conclusion would turn out painfully correct, after all. The above two reasonings are examples of the syllogism of transposed quantity.

The proposition that finite and infinite collections are distinguished by the applicability to the former of the syllogism of transposed quantity ought to be regarded as the basal one of scientific arithmetic.

If a person does not know how to reason logically, and I must say that a great many fairly good mathematicians,—yea distinguished ones,—fall under this category, but simply uses a rule of

thumb in blindly drawing inferences like other inferences that have turned out well, he will, of course, be continually falling into error about infinite numbers. The truth is such people do not reason, at all. But for the few who do reason, reasoning about infinite numbers is easier than about finite numbers, because the complicated syllogism of transposed quantity is not called for. For example, that the whole is greater than its part is not an axiom, as that eminently bad reasoner, Euclid, made it to be. It is a theorem readily proved by means of a syllogism of transposed quantity, but not otherwise. Of finite collections it is true, of infinite collections false. Thus, a part of the whole numbers are even numbers. Yet the even numbers are no fewer than all the numbers; an evident proposition since if every number in the whole series of whole numbers be doubled, the result will be the series of even numbers.

1, 2, 3, 4, 5, 6, etc.
2, 4, 6, 8, 10, 12, etc.

So for every number there is a distinct even number. In fact, there are as many distinct doubles of numbers as there are of distinct numbers. But the doubles of numbers are all even numbers.

In truth, of infinite collections there are but two grades of magnitude, the *endless* and the *innumerable*. Just as a finite collection is distinguished from an infinite one by the applicability to it of a special mode of reasoning, the syllogism of transposed quantity, so, as I showed in the paper last referred to, a numerable collection is distinguished from an innumerable one by the applicability to it of a certain mode of reasoning, the Fermatian inference, or, as it is sometimes improperly termed, "mathematical induction."

As an example of this reasoning, Euler's demonstration of the binomial theorem for integral powers may be given. The theorem is that $(x+y)^n$, where n is a whole number, may be expanded into the sum of a series of terms of which the first is $x^n y^0$ and each of the others is derived from the next preceding by diminishing the exponent of x by 1 and multiplying by that exponent and at the same time increasing the exponent of y by 1 and dividing by that increased exponent. Now, suppose this proposition to be true for a certain exponent, $n = M$, then it must also be true for $n = \mathrm{M} + 1$.

For let one of the terms in the expansion of $(x+y)^M$ be written $Ax^p y^q$. Then, this term with the two following will be

$$A\,x^p y^q + A\frac{p}{q-1}x^{p-1}y^{q+1} + A\frac{p}{q-1}\frac{p-1}{q+2}x^{p-2}y^{q+2}$$

Now, when $(x+y)^M$ is multiplied by $x+y$ to give $(x+y)^{M+1}$, we multiply first by x and then by y instead of by x and add the two results. When we multiply by x, the second of the above three terms will be the only one giving a term involving $x^p y^{q+1}$ and the third will be the only one giving a term in $x^{p-1}y^{q+2}$; and when we multiply by y the first will be the only term giving a term in $x^p y^{q+1}$, and the second will be the only term giving a term in $x^{p-1}y^{q+2}$. Hence, adding like terms, we find that the coefficient of $x^p y^{q+1}$ in the expansion of $(x+y)^{M+1}$ will be the sum of the coefficients of the first two of the above three terms, and that the coefficient of $x^{p-1}y^{q+2}$ will be the sum of the coefficients of the last two terms. Hence, two successive terms in the expansion of $(x+y)^{M+1}$ will be

$$A\left(1+\frac{p}{q+1}\right)x^p y^{q+1} + A\left(\frac{p}{q+1}+1+\frac{p-1}{q+2}\right)x^{p-1}y^{q+2}$$

$$= A\frac{p+q+1}{q+1}x^p y^{q+1} + A\frac{p+q+1}{q+1}\frac{p}{q+2}x^{p-1}y^{q+2}.$$

It is, thus, seen that the succession of terms follows the rule. Thus if any integral power follows the rule, so also does the next higher power. But the first power obviously follows the rule. Hence, all powers do so.

Such reasoning holds good of any collection of objects capable of being ranged in a series which though it may be endless, can be numbered so that each member of it receives a definite integral number. For instance, all the whole numbers constitute such a numerable collection. Again, all numbers resulting from operating according to any definite rule with any finite number of whole numbers form such a collection. For they may be arranged in a series thus. Let F be the symbol of operation. First operate on 1, giving F(1). Then, operate on a second 1, giving F(1,1). Next, introduce 2, giving 3rd, F(2); 4th, F(2,1); 5th, F(1,2); 6th, F(2,2). Next use a third variable giving 7th, F(1,1,1); 8th, F(2,1,1); 9th, F(1,2,1); 10th, F(2,2,1); 11th, F(1,1,2); 12th, F(2,1,2); 13th, F(1,2,2); 14th, F(2,2,2). Next introduce 3, and so on, alternately introducing new variables and new figures: and in this way it is plain that every arrangement of integral values of the variables will receive a numbered place in the series.*

The class of endless but numerable collections (so called because they can be so ranged that to each one corresponds a distinct whole number) is very large. But there are collections which are certainly innumerable. Such is the collection of all numbers to which endless series of decimals are capable of approximating. It has been recognised since the time of Euclid that certain numbers are surd or incommensurable, and are not exactly expressible by any finite series of decimals, nor by a circulating decimal. Such is the ratio of the circumference of a circle to its diameter, which we know is nearly 3.1415926. The calculation of this number has been carried to over 700 figures without the slightest appearance of regularity in their sequence. The demonstrations that this and many other numbers are incommensurable are perfect. That the entire collection of incommensurable numbers is innumerable has been clearly proved by Cantor. I omit the demonstration; but it is easy to see that to discriminate one from some other would, in general, require the use of an endless series of numbers. Now if they cannot be exactly expressed and discriminated, clearly they cannot be ranged in a linear series.

It is evident that there are as many points on a line or in an interval of time as there are of real numbers in all. These are, therefore, innumerable collections. Many mathematicians have incautiously assumed that the points on a surface or in a solid are more than those on a line. But this has been refuted by Cantor. Indeed, it is obvious that for every set of values of coördinates there is a single distinct number. Suppose, for instance, the values of the coördinates all lie between 0 and + 1. Then if we compose a number by putting in the first decimal place the first figure of the first coördinate, in the second the first figure of the second coördi-

* This proposition is substantially the same as a theorem of Cantor, though it is enunciated in a much more general form.

nate, and so on, and when the first figures are all dealt out go on to the second figures in like manner, it is plain that the values of the coördinates can be read off from the single resulting number, so that a triad or tetrad of numbers, each having innumerable values, has no more values than a single incommensurable number.

Were the number of dimensions infinite, this would fail; and the collection of infinite sets of numbers having each innumerable variations, might, therefore, be greater than the simple innumerable collection, and might be called *endlessly infinite*. The single individuals of such a collection could not, however, be designated, even approximately, so that this is indeed a magnitude concerning which it would be possible to reason only in the most general way, if at all.

Although there are but two grades of magnitudes of infinite collections, yet when certain conditions are imposed upon the order in which individuals are taken, distinctions of magnitude arise from that cause. Thus, if a simply endless series be doubled by separating each unit into two parts, the successive first parts and also the second parts being taken in the same order as the units from which they are derived, this double endless series will, so long as it is taken in that order, appear as twice as large as the original series. In like manner the product of two innumerable collections, that is, the collection of possible pairs composed of one individual of each, if the order of continuity is to be maintained, is, by virtue of that order, infinitely greater than either of the component collections.

We now come to the difficult question, What is continuity? Kant confounds it with infinite divisibility, saying that the essential character of a continuous series is that between any two members of it a third can always be found. This is an analysis beautifully clear and definite; but unfortunately, it breaks down under the first test. For according to this, the entire series of rational fractions arranged in the order of their magnitude, would be an infinite series, although the rational fractions are numerable, while the points of a line are innumerable. Nay, worse yet, if from that series of fractions any two with all that lie between them be excised, and any number of such finite gaps be made, Kant's definition is still true of the series, though it has lost all appearance of continuity.

Cantor defines a continuous series as one which is *concatenated* and *perfect*. By a concatenated series, he means such a one that if any two points are given in it, and any finite distance, however small, it is possible to proceed from the first point to the second through a succession of points of the series each at a distance from the preceding one less than the given distance. This is true of the series of rational fractions ranged in the order of their magnitude. By a perfect series, he means one which contains every point such that there is no distance so small that this point has not an infinity of points of the series within that distance of it. This is true of the series of numbers between 0 and 1 capable of being expressed by decimals in which only the digits 0 and 1 occur.

It must be granted that Cantor's definition includes every series that is continuous; nor can it be objected that it includes any important or indubitable case of a series not continuous. Nevertheless, it has some serious defects. In the first place, it turns upon metrical considerations; while the distinction between a continuous and a discontinuous series is manifestly non-metrical. In the next place, a perfect series is defined as one containing "every point" of a certain description. But no positive idea is conveyed of what all the points are: that is definition by negation, and cannot be admitted. If that sort of thing were allowed, it would be very easy to say, at once, that the continuous linear series of points is one which contains every point of the line between its extremities. Finally, Cantor's definition does not convey a distinct notion of what the components of the conception of continuity are. It ingeniously wraps up its properties in two separate parcels, but does not display them to our intelligence.

Kant's definition expresses one simple property of a continuum; but it allows of gaps in the series. To mend the definition, it is only necessary to notice how these gaps can occur. Let us suppose, then, a linear series of points extending from a point, A, to a point, B, having a gap from B to a third point, C, and thence extending to a final limit, D; and let us suppose this series conforms to Kant's definition. Then, of the two points, B and C, one or both must be excluded from the series; for otherwise, by the definition, there

would be points between them. That is, if the series contains C, though it contains all the points up to B, it cannot contain B. What is required, therefore, is to state in non-metrical terms that if a series of points up to a limit is included in a continuum the limit is included. It may be remarked that this is the property of a continuum to which Aristotle's attention seems to have been directed when he defines a continuum as something whose parts have a common limit. The property may be exactly stated as follows: If a linear series of points is continuous between two points, A and D, and if an endless series of points be taken, the first of them between A and D and each of the others between the last preceding one and D, then there is a point of the continuous series between all that endless series of points and D, and such that every other point of which this is true lies between this point and D. For example, take any number between 0 and 1, as 0.1; then, any number between 0.1 and 1, as 0.11; then any number between 0.11 and 1, as 0.111; and so on, without end. Then, because the series of real numbers between 0 and 1 is continuous, there must be a *least* real number, greater than every number of that endless series. This property, which may be called the Aristotelicity of the series, together with Kant's property, or its Kanticity, completes the definition of a continuous series.

The property of Aristotelicity may be roughly stated thus: a continuum contains the end point belonging to every endless series of points which it contains. An obvious corollary is that every continuum contains its limits. But in using this principle it is necessary to observe that a series may be continuous except in this, that it omits one or both of the limits.

Our ideas will find expression more conveniently if, instead of points upon a line, we speak of real numbers. Every real number is, in one sense, the limit of a series, for it can be indefinitely approximated to. Whether every real number is a limit of a *regular* series may perhaps be open to doubt. But the series referred to in the definition of Aristotelicity must be understood as including all series whether regular or not. Consequently, it is implied that between any two points an innumerable series of points can be taken.

Every number whose expression in decimals requires but a finite number of places of decimals is commensurable. Therefore, incommensurable numbers suppose an infinitieth place of decimals. The word infinitesimal is simply the Latin form of infinitieth; that is, it is an ordinal formed from *infinitum*, as centesimal from *centum*. Thus, continuity supposes infinitesimal quantities. There is nothing contradictory about the idea of such quantities. In adding and multiplying them the continuity must not be broken up, and consequently they are precisely like any other quantities, except that neither the syllogism of transposed quantity, nor the Fermatian inference applies to them.

If A is a finite quantity and i an infinitesimal, then in a certain sense we may write $A + i = A$. That is to say, this is so for all purposes of measurement. But this principle must not be applied except to get rid of *all* the terms in the highest order of infinitesimals present. As a mathematician, I prefer the method of infinitesimals to that of limits, as far easier and less infested with snares. Indeed, the latter, as stated in some books, involves propositions that are false; but this is not the case with the forms of the method used by Cauchy, Duhamel, and others. As they understand the doctrine of limits, it involves the notion of continuity, and therefore contains in another shape the very same ideas as the doctrine of infinitesimals.

Let us now consider an aspect of the Aristotelical principle which is particularly important in philosophy. Suppose a surface to be part red and part blue; so that every point on it is either red or blue, and, of course, no part can be both red and blue. What, then, is the color of the boundary line between the red and the blue? The answer is that red or blue, to exist at all, must be spread over a surface; and the color of the surface is the color of the surface in the immediate neighborhood of the point. I purposely use a vague form of expression. Now, as the parts of the surface in the immediate neighborhood of any ordinary point upon a curved boundary are half of them red and half blue, it follows that the boundary is half red and half blue. In like manner, we find it necessary to hold that consciousness essentially occupies time; and what is

present to the mind at any ordinary instant, is what is present during a moment in which that instant occurs. Thus, the present is half past and half to come. Again, the color of the parts of a surface at any finite distance from a point, has nothing to do with its color just at that point; and, in the parallel, the feeling at any finite interval from the present has nothing to do with the present feeling, except vicariously. Take another case: the velocity of a particle at any instant of time is its mean velocity during an infinitesimal instant in which that time is contained. Just so my immediate feeling is my feeling through an infinitesimal duration containing the present instant.

ANALYSIS OF TIME

One of the most marked features about the law of mind is that it makes time to have a definite direction of flow from past to future. The relation of past to future is, in reference to the law of mind, different from the relation of future to past. This makes one of the great contrasts between the law of mind and the law of physical force, where there is no more distinction between the two opposite directions in time than between moving northward and moving southward.

In order, therefore, to analyse the law of mind, we must begin by asking what the flow of time consists in. Now, we find that in reference to any individual state of feeling, all others are of two classes, those which affect this one (or have a tendency to affect it, and what this means we shall inquire shortly), and those which do not. The present is affectible by the past but not by the future.

Moreover, if state A is affected by state B, and state B by state C, then A is affected by state C, though not so much so. It follows, that if A is affectible by B, B is not affectible by A.

If, of two states, each is absolutely unaffectible by the other, they are to be regarded as parts of the same state. They are contemporaneous.

To say that a state is *between* two states means that it affects one and is affected by the other. Between any two states in this sense lies an innumerable series of states affecting one another; and

if a state lies between a given state and any other state which can be reached by inserting states between this state and any third state, these inserted states not immediately affecting or being affected by either, then the second state mentioned immediately affects or is affected by the first, in the sense that in the one the other is *ipso facto* present in a reduced degree.

These propositions involve a definition of time and of its flow. Over and above this definition they involve a doctrine, namely, that every state of feeling is affectible by every earlier state.

THAT FEELINGS HAVE INTENSIVE CONTINUITY.

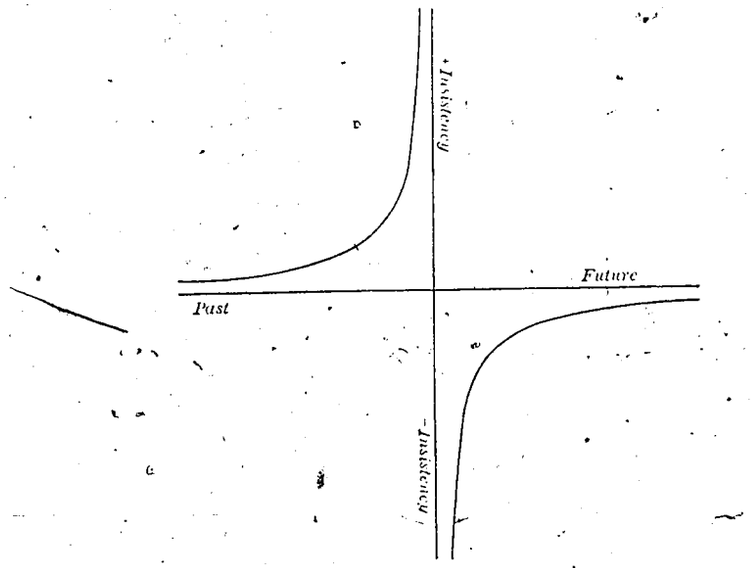
Time with its continuity logically involves some other kind of continuity than its own. Time, as the universal form of change, cannot exist unless there is something to undergo change, and to undergo a change continuous in time, there must be a continuity of changeable qualities. Of the continuity of intrinsic qualities of feeling we can now form but a feeble conception. The development of the human mind has practically extinguished all feelings, except a few sporadic kinds, sound, colors, smells, warmth, etc., which now appear to be disconnected and disparate. In the case of colors, there is a tridimensional spread of feelings. Originally, all feelings may have been connected in the same way and the presumption is that the number of dimensions was endless. For development essentially involves a limitation of possibilities. But given a number of dimensions of feeling, all possible varieties are obtainable by varying the intensities of the different elements. Accordingly, time logically supposes a continuous range of intensity in feeling. It follows, then, from the definition of continuity, that when any particular kind of feeling is present, an infinitesimal continuum of all feelings differing infinitesimally from that is present.

THAT FEELINGS HAVE SPATIAL EXTENSION.

Consider a gob of protoplasm, say an amœba or a slime-mould. It does not differ in any radical way from the contents of a nerve-cell, though its functions may be less specialised. There is no

Second, in the presence of this continuity of feeling, nominalistic maxims appear futile. There is no doubt about one idea affecting another, when we can directly perceive the one gradually modified and shaping itself into the other. Nor can there any longer be any difficulty about one idea resembling another, when we can pass along the continuous field of quality from one to the other and back again to the point which we had marked.

Third, consider the insistency of an idea. The insistency of a past idea with reference to the present is a quantity which is less the further back that past idea is, and rises to infinity as the past idea



is brought up into coincidence with the present. Here we must make one of those inductive applications of the law of continuity which have produced such great results in all the positive sciences. We must extend the law of insistency into the future. Plainly, the insistency of a future idea with reference to the present is a quantity affected by the minus sign; for it is the present that affects the future, if there be any effect, not the future that affects the present. Accordingly, the curve of insistency is a sort of equilateral hyperbola. [See the figure.] Such a conception is none the less mathematical, that its quantification cannot now be exactly specified.

Now consider the induction which we have here been led into. This curve says that feeling which has not yet emerged into immediate consciousness is already affectible and already affected. In fact, this is habit, by virtue of which an idea is brought up into present consciousness by a bond that had already been established between it, and another idea while it was still *in futuro*.

We can now see what the affection of one idea by another consists in. It is that the affected idea is attached as a logical predicate to the affecting idea as subject. So when a feeling emerges into immediate consciousness, it always appears as a modification of a more or less general object already in the mind. The word suggestion is well adapted to expressing this relation. The future is suggested by, or rather is influenced by the suggestions of, the past.

IDEAS CANNOT BE CONNECTED EXCEPT BY CONTINUITY.

That ideas can nowise be connected without continuity is sufficiently evident to one who reflects upon the matter. But still the opinion may be entertained that after continuity has once made the connection of ideas possible, then they may get to be connected in other modes than through continuity. Certainly, I cannot see how anyone can deny that the infinite diversity of the universe, which we call chance, may bring ideas into proximity which are not associated in one general idea. It may do this any times. But then the law of continuous spreading will produce a mental association: and this I suppose is an abridged statement of the way the universe has been evolved. But if I am asked whether a blind *ἀνάγκη* cannot bring ideas together, first I point out that it would not remain blind. There being a continuous connection between the ideas, they would infallibly become associated in a living, feeling, and perceiving general idea. Next, I cannot see what the mustness or necessity of this *ἀνάγκη* would consist in. In the absolute uniformity of the phenomenon, says the nominalist. Absolute is well put in: for if it merely happened so three times in succession, or three million times in succession, in the absence of any reason, the coincidence could only be attributed to chance. But absolute uniformity must extend over the whole infinite future: and it is idle to

talk of that except as an idea. No; I think we can only hold that wherever ideas come together, they tend to weld into general ideas; and wherever they are generally connected, general ideas govern the connection; and these general ideas are living feelings spread out.

MENTAL LAW FOLLOWS THE FORMS OF LOGIC

The three main classes of logical inference are Deduction, Induction, and Hypothesis. These correspond to three chief modes of action of the human soul. In deduction the mind is under the dominion of a habit or association by virtue of which a general idea suggests in each case a corresponding reaction. But a certain sensation is seen to involve that idea. Consequently, that sensation is followed by that reaction. That is the way the hind-legs of a frog, separated from the rest of the body, reason, when you pinch them. It is the lowest form of psychical manifestation.

By induction, a habit becomes established. Certain sensations, all involving one general idea, are followed each by the same reaction: and an association becomes established, whereby that general idea gets to be followed uniformly by that reaction.

Habit is that specialisation of the law of mind whereby a general idea gains the power of exciting reactions. But in order that the general idea should attain all its functionality, it is necessary, also, that it should become suggestible by sensations. That is accomplished by a psychical process having the form of hypothetic inference. By hypothetic inference, I mean, as I have explained in other writings, an induction from qualities. For example, I know that the kind of man known and classed as a "mugwump" has certain characteristics. He has a high self-respect and places great value upon social distinction. He laments the great part that rowdyism and unrefined good-fellowship play in the dealings of American politicians with their constituency. He thinks that the reform which would follow from the abandonment of the system by which the distribution of offices is made to strengthen party organisations and a return to the original and essential conception of office-filling would be found an unmixed good. He holds that monetary considerations should usually be the decisive ones in questions of public policy.

He respects the principle of individualism and of *laisser-faire* as the greatest agency of civilisation. These views, among others, I know to be obtrusive marks of a "mugwump." Now, suppose I casually meet a man in a railway-train, and falling into conversation find that he holds opinions of this sort: I am naturally led to suppose that he is a "mugwump." That is hypothetic inference. That is to say, a number of readily verifiable marks of a mugwump being selected, I find this man has these, and infer that he has all the other characters which go to make a thinker of that stripe. Or let us suppose that I meet a man of a semi-clerical appearance and a sub-pharisaical sniff, who appears to look at things from the point of view of a rather wooden dualism. He cites several texts of scripture and always with particular attention to their logical implications; and he exhibits a sternness, almost amounting to vindictiveness, toward evil-doers, in general. I readily conclude that he is a minister of a certain denomination. Now the mind acts in a way similar to this, every time we acquire a power of coördinating reactions in a peculiar way, as in performing any act requiring skill. Thus, most persons have a difficulty in moving the two hands simultaneously and in opposite directions through two parallel circles nearly in the medial plane of the body. To learn to do this, it is necessary to attend, first, to the different actions in different parts of the motion, when suddenly a general conception of the action springs up and it becomes perfectly easy. We think the motion we are trying to do involves this action, and this, and this. Then, the general idea comes which unites all those actions, and thereupon the desire to perform the motion calls up the general idea. The same mental process is many times employed whenever we are learning to speak a language or are acquiring any sort of skill.

Thus, by induction, a number of sensations followed by one reaction become united under one general idea followed by the same reaction; while by the hypothetic process, a number of reactions called for by one occasion get united in a general idea which is called out by the same occasion. By deduction, the habit fulfils its function of calling out certain reactions on certain occasions.

UNCERTAINTY OF MENTAL ACTION

The inductive and hypothetic forms of inference are essentially probable inferences, not necessary; while deduction may be either necessary or probable.

But no mental action seems to be necessary or invariable in its character. In whatever manner the mind has reacted under a given sensation, in that manner it is the more likely to react again; were this, however, an absolute necessity, habits would become wooden and ineradicable, and no room being left for the formation of new habits, intellectual life would come to a speedy close. Thus, the uncertainty of the mental law is no mere defect of it, but is on the contrary of its essence. The truth is, the mind is not subject to "law," in the same rigid sense that matter is. It only experiences gentle forces which merely render it more likely to act in a given way than it otherwise would be. There always remains a certain amount of arbitrary spontaneity in its action, without which it would be dead.

Some psychologists think to reconcile the uncertainty of reactions with the principle of necessary causation by means of the law of fatigue. Truly for a *hare*, this law of fatigue is a little lawless. I think it is merely a case of the general principle that an idea in spreading loses its insistency. Put me tarragon into my salad, when I have not tasted it for years, and I exclaim "What nectar is this!" But add it to every dish I taste for week after week, and a habit of expectation has been created; and in thus spreading into habit, the sensation makes hardly any more impression upon me; or, if it be noticed, it is on a new side from which it appears as rather a bore. The doctrine that fatigue is one of the primordial phenomena of mind I am much disposed to doubt. It seems a somewhat little thing to be allowed as an exception to the great principle of mental uniformisation. For this reason, I prefer to explain it in the manner here indicated, as a special case of that great principle. To consider it as something distinct in its nature, certainly somewhat strengthens the necessitarian position; but even if it be distinct, the hypothesis that all the variety and apparent arbitrariness of mental action ought

to be explained away in favor of absolute determinism does not seem to me to recommend itself to a sober and sound judgment, which seeks the guidance of observed facts and not that of prepossessions.

RESTATEMENT OF THE LAW

Let me now try to gather up all these odds and ends of commentary and restate the law of mind, in a unitary way.

First, then, we find that when we regard ideas from a nominalistic, individualistic, sensualistic way, the simplest facts of mind become utterly meaningless. That one idea should resemble another or influence another, or that one state of mind should so much as be thought of in another is, from that standpoint, sheer nonsense.

Second, by this and other means we are driven to perceive, what is quite evident of itself, that instantaneous feelings flow together into a continuum of feeling, which has in a modified degree the peculiar vivacity of feeling and has gained generality. And in reference to such general ideas, or continua of feeling, the difficulties about resemblance and suggestion and reference to the external, cease to have any force.

Third, these general ideas are not mere words, nor do they consist in this, that certain concrete facts will every time happen under certain descriptions of conditions; but they are just as much, or rather far more, living realities than the feelings themselves out of which they are concreted. And to say that mental phenomena are governed by law does not mean merely that they are describable by a general formula; but that there is a living idea, a conscious continuum of feeling, which pervades them, and to which they are docile.

Fourth, this supreme law, which is the celestial and living harmony, does not so much as demand that the special ideas shall surrender their peculiar arbitrariness and caprice entirely; for that would be self-destructive. It only requires that they shall influence and be influenced by one another.

Fifth, in what measure this unification acts, seems to be regulated only by special rules; or, at least, we cannot in our present

knowledge say how far it goes. But it may be said that, judging by appearances, the amount of arbitrariness in the phenomena of human minds is neither altogether trifling nor very prominent.

PERSONALITY.

Having thus endeavored to state the law of mind, in general, I descend to the consideration of a particular phenomenon which is remarkably prominent in our own consciousnesses, that of personality. A strong light is thrown upon this subject by recent observations of double and multiple personality. The theory which at one time seemed plausible that two persons in one body corresponded to the two halves of the brain will, I take it, now be universally acknowledged to be insufficient. But that which these cases make quite manifest is that personality is some kind of coördination or connection of ideas. Not much to say, this, perhaps. Yet when we consider that, according to the principle which we are tracing out, a connection between ideas is itself a general idea, and that a general idea is a living reality, it is plain that we have at least taken an appreciable step toward the understanding of personality. This personality, like any general idea, is not a thing to be apprehended in an instant. It has to be lived in time; nor can any finite time embrace it in all its fulness. Yet in each infinitesimal interval it is present and living, though specially colored by the immediate feelings of that moment. Personality, so far as it is apprehended in a moment, is immediate self-consciousness.

But the word coördination implies somewhat more than this; it implies a teleological harmony in ideas, and in the case of personality this teleology is more than a mere purposive pursuit of a predeterminate end; it is a developmental teleology. This is personal character. A general idea, living and conscious now, it is already determinative of acts in the future to an extent to which it is not now conscious.

This reference to the future is an essential element of personality. Were the ends of a person already explicit, there would be no room for development, for growth, for life; and consequently there would be no personality. The mere carrying out of predetermined purposes is mechanical. This remark has an application to the philosophy of religion. It is that a genuine evolutionary philosophy, that is, one that makes the principle of growth a primordial element of the universe, is so far from being antagonistic to the idea of a personal creator, that it is really inseparable from that idea; while a necessitarian religion is in an altogether false position and is destined to become disintegrated. But a pseudo-evolutionism which enthrones mechanical law above the principle of growth, is at once scientifically unsatisfactory, as giving no possible hint of how the universe has come about, and hostile to all hopes of personal relations to God.

COMMUNICATION

Consistently with the doctrine laid down in the beginning of this paper, I am bound to maintain that an idea can only be affected by an idea in continuous connection with it. By anything but an idea, it cannot be affected at all. This obliges me to say, as I do say, on other grounds, that what we call matter is not completely dead, but is merely mind hide-bound with habits. It still retains the element of diversification; and in that diversification there is life. When an idea is conveyed from one mind to another, it is by forms of combination of the diverse elements of nature, say by some curious symmetry, or by some union of a tone, color with a refined odor. To such forms the law of mechanical energy has no application. If they are eternal, it is in the spirit they embody; and their origin cannot be accounted for by any mechanical necessity. They are embodied ideas; and so only can they convey ideas. Precisely how primary sensations, as colors and tones, are excited, we cannot tell, in the present state of psychology. But in our ignorance, I think that we are at liberty to suppose that they arise in essentially the same manner as the other feelings, called secondary. As far as sight and hearing are in question, we know that they are only excited by vibrations of inconceivable complexity: and the chemical senses are probably not more simple. Even the least psychical of peripheral sensations, that of pressure, has in its excitation conditions which, though apparently simple, are seen to be

complicated enough when we consider the molecules and their attractions. The principle with which I set out requires me to maintain that these feelings are communicated to the nerves by continuity, so that there must be something like them in the excitants themselves. If this seems extravagant, it is to be remembered that it is the sole possible way of reaching any explanation of sensation, which otherwise must be pronounced a general fact absolutely inexplicable and ultimate. Now absolute inexplicability is a hypothesis which sound logic refuses under any circumstances to justify.

I may be asked whether my theory would be favorable or otherwise to telepathy. I have no decided answer to give to this. At first sight, it seems unfavorable. Yet there may be other modes of continuous connection between minds other than those of time and space.

The recognition by one person of another's personality takes place by means to some extent identical with the means by which he is conscious of his own personality. The idea of the second personality, which is as much as to say that second personality itself, enters within the field of direct consciousness of the first person, and is as immediately perceived as his ego, though less strongly. At the same time, the opposition between the two persons is perceived, so that the externality of the second is recognised.

The psychological phenomena of intercommunication between two minds have been unfortunately little studied. So that it is impossible to say, for certain, whether they are favorable to this theory or not. But the very extraordinary insight which some persons are able to gain of others from indications so slight that it is difficult to ascertain what they are, is certainly rendered more comprehensible by the view here taken.

A difficulty which confronts the synechistic philosophy is this. In considering personality, that philosophy is forced to accept the doctrine of a personal God; but in considering communication, it cannot but admit that if there is a personal God, we must have a direct perception of that person and indeed be in personal communication with him. Now, if that be the case, the question arises how it is possible that the existence of this being should ever have been

doubted by anybody. The only answer that I can at present make is that facts that stand before our face and eyes and stare us in the face are far from being, in all cases, the ones most easily discerned. That has been remarked from time immemorial.

CONCLUSION

I have thus developed as well as I could in a little space the *synechistic* philosophy, as applied to mind. I think that I have succeeded in making it clear that this doctrine gives room for explanations of many facts which without it are absolutely and hopelessly inexplicable; and further that it carries along with it the following doctrines: 1st, a logical realism of the most pronounced type; 2nd, objective idealism; 3rd, tychism, with its consequent thoroughgoing evolutionism. We also notice that the doctrine presents no hindrances to spiritual influences, such as some philosophies are felt to do.

C. S. PEIRCE.