The Open Court.

THE CRITIC OF ARGUMENTS.

BY CHARLES S. PEIRCE.

II. THE READER IS INTRODUCED TO RELATIVES.

There is a melancholy book entitled "Astronomy Without Mathematics." The author, an F. R. A. S., presumably knew something of astronomy; therefore, I pity him. I think I hear his groans and maledictions, as he wrote the book, over the initial lie to which he had committed himself, that it is possible to convey any idea of the science of astronomy without making use of mathematics. He could tell roughly how the planets go round the sun, and make his readers think they knew what the error of the ancient system was (namely, that all went round the earth,—really, no error), and could set down surprising figures about the stars (beaten, however, by Buddhistic numbers both in magnitude and in intellectual value). A book so made might well have been called "The Story of the Heavens" (in anticipation of Dr. Ball's splendid volume, which, promising little, performs much), but it was not the "astronomy" stipulated for in the title page. When, in a neighbor's house yesterday, my eye lit upon that book, I shuddered. For I too have engaged myself by the title of these papers to produce something of solid value to my readers; but, thank God, I have not agreed to do it without the use of mathematics. I came home and pondered; and have decided that, in order to fulfil legitimate expectations, I must begin with a few chapters upon certain dry and somewhat technical matters that underlie the more interesting questions concerning reasoning. Do not fear a repetition of matter to be found in common text-books. I shall suppose the reader to be acquainted with what is contained in Dr. Watts's "Logick," a book very cheap and easily procured, and far superior to the treatises now used in colleges, being the production of a man distinguished for good sense. I mean to bring out a reprint of it, with extensive annotations, whenever I can find an eligible publisher. Though a life-long student of reasonings, I know no way of giving the reader the benefit of what I ought to have learned, without asking him to go through with some irksome preliminary thinking about relations.

For this subject, although always recognised as an integral part of logic, has been left untouched on account of its intricacy. It is as though a geographer, finding the whole United States, its topography, its population, its industries, etc., too vast for convenient treatment, were to content himself with a description of Nantucket. This comparison hardly, if at all, exaggerates the inadequacy of a theory of reasoning that takes no account of relative terms.

A *relation* is a fact about a number of things. Thus the fact that a locomotive blows off steam constitutes a relation, or more accurately a relationship (the *Century Dictionary*, under *relation*, 3, gives the terminology. See also *relativity*, etc.) between the locomotive and the steam. In reality, every fact is a relation. Thus, that an object is blue consists of the peculiar regular action of that object on human eyes. This is what should be understood by the "relativity of knowledge." Not only is every fact really a relation, but your thought of the fact *implicitly* represents it as such. Thus, when you think "this is blue," the demonstrative "this" shows you are thinking of something just brought up to your notice; while the adjective shows that you recognise a familiar idea as applicable to it. Thus, your thought, when explicated, develops into the thought of a fact concerning this thing and concerning the character of blueness. Still, it must be admitted that, antecedently to the unwrapping of your thought, you were not actually thinking of blueness as a distinct object, and therefore were not thinking of the relation as a relation.[*] There is an aspect of every relation under which it does not appear as a relation. Thus, the blowing off of steam by a locomotive may be regarded as merely an action of the locomotive, the steam not being conceived to be a thing distinct from the engine. This aspect we enphrase in saying, "the engine blows."

Thus, the question whether a fact is to be regarded as referring to a single thing or to more is a question of the form of proposition under which it suits our purpose to state the fact. Consider any argument con-

---

[*] In this connection, see James's, *Principles of Psychology*, Vol. I, pp. 237-271; *Briefer Course*, pp. 160 et seqq. James is no logician, but it is not difficult to trace a connection between the points he makes and the theory of inference.

cerning the validity of which a person might conceivably entertain for a moment some doubt. For instance, let the premise be that from either of two provinces of a certain kingdom it is possible to proceed to any province by floating down the only river the kingdom contains, combined with a land-journey within the boundaries of one province; and let the conclusion be that the river, after touching every province in the kingdom, must again meet the one which it first left. Now, in order to show that this inference is (or that it is not) absolutely necessary, it is requisite to have something analogous to a diagram with different series of parts, the parts of each series being evidently related as those provinces are said to be, while in the different series something corresponding to the course of the river has all the essential variations possible; and this diagram must be so contrived that it is easy to examine it and find out whether the course of the river is in truth in every case such as is here proposed to be inferred. Such a diagram has got to be either auditory or visual, the parts being separated in the one case in time, in the other in space. But in order completely to exhibit the analogue of the conditions of the argument under examination, it will be necessary to use signs or symbols repeated in different places and in different juxtapositions, these signs being subject to certain "rules," that is, certain general relations associated with them by the mind. Such a method of forming a diagram is called *algebra*. All speech is but such an algebra, the repeated signs being the words, which have relations by virtue of the meanings associated with them. What is commonly called *logical algebra* differs from other formal logic only in using the same formal method with greater freedom. I may mention that unpublished studies have shown me that a far more powerful method of diagrammatisation than algebra is possible, being an extension at once of algebra and of Clifford's method of graphs; but I am not in a situation to draw up a statement of my researches.

Diagrams and diagrammatoidal figures are intended to be applied to the better understanding of states of things, whether experienced or read of or imagined. Such a figure cannot, however, show what it is to which it is intended to be applied; nor can any other diagram avail for that purpose. The where and the when of the particular experience, or the occasion or other identifying circumstance of the particular fiction to which the diagram is to be applied, are things not capable of being diagrammatically exhibited. Describe and describe and describe, and you never can describe a date, a position, or any homaloidal quantity. You may object that a map is a diagram showing localities; undoubtedly, but not until the law of the projection is understood, nor even then unless at least two points on the map are somehow previously identified with points in nature. Now, how is any diagram ever to perform that identification? If a diagram cannot do it, algebra cannot: for algebra is but a sort of diagram; and if algebra cannot do it, language cannot: for language is but a kind of algebra. It would, certainly, in one sense be extravagant to say that we can never tell what we are talking about; yet, in another sense, it is quite true. The meanings of words ordinarily depend upon our tendencies to weld together qualities and our aptitudes to see resemblances, or, to use the received phrase, upon associations by *similarity;* while experience is bound together, and only recognisable, by forces acting upon us, or, to use an even worse chosen technical term, by means of associations by *contiguity.* Two men meet on a country road. One says to the other, "that house is on fire." "What house?" "Why, the house about a mile to my right." Let this speech be taken down and shown to anybody in the neighboring village, and it will appear that the language by itself does not fix the house. But the person addressed sees where the speaker is standing, recognises his *right* hand side, (a word having a most singular mode of signification,) estimates a *mile*, (a length having no geometrical properties different from other lengths,) and looking there, sees a house. It is not the language alone, with its mere associations of similarity, but the language taken in connection with the auditor's own experiential associations of contiguity, which determines for him what house is meant. It is requisite then, in order to show what we are talking or writing about, to put the hearer's or reader's mind into real, active connection with the concatenation of experience or of fiction with which we are dealing, and, further, to draw his attention to, and identify, a certain number of particular points in such concatenation. If there be a reader who cannot understand my writings, let me tell him that no straining of his mind will help him: his whole difficulty is that he has no personal experience of the world of problems of which I am talking, and he might as well close the book until such experience comes. That the diagrammatisation is one thing and the application of the diagram quite another, is recognised obscurely in the structure of such languages as I am acquainted with, which distinguish the *subjects* and *predicates* of propositions. The subjects are the indications of the things spoken of, the predicates words that assert, question, or command whatever is intended. Only, the shallowness of syntax is manifest in its failing to recognise the impotence of mere words, and especially of common nouns, to fulfil the function of a grammatical subject. Words like *this, that, lo, hallo, hi there,* have a direct, forceful action upon the nervous system, and compel the hearer to look about him; and so they, more than ordinary words, contribute towards indicating what the speech

is about. But this is a point that grammar and the grammarians (who, if they are faithfully to mirror the minds of the language-makers, can hardly be scientific analysts) are so far from seeing as to call demonstratives, such as *that* and *this*, pronouns,—a literally preposterous designation, for nouns may more truly be called pro-demonstratives.

If upon a diagram we mark two or more points to be identified at some future time with objects in nature,[*] so as to give the diagram at that future time its meaning; or if in any written statement we put dashes in place of two or more demonstratives or pro-demonstratives, the professedly incomplete representation resulting may be termed a *relative rhema*. It differs from a relative *term* only in retaining the "copula," or signal of assertion. If only one demonstrative or pro-demonstrative is erased, the result is a *non-relative rhema*. For example, "— buys — from — for the price —," is a relative rhema; it differs in a merely secondary way from

"— is bought by — from — for —,"
from "— sells — to — for —,"
and from "— is paid by — to — for —."

On the other hand, "— is mortal" is a non-relative rhema.

A rhema is somewhat closely analogous to a chemical atom or radicle with unsaturated bonds. A non-relative rhema is like a univalent radicle; it has but one unsaturated bond. A relative rhema is like a multivalent radicle. The blanks of a rhema can only be filled by terms, or, what is the same thing, by "something which" (or the like) followed by a rhema; or, two can be filled together by means of "itself " or the like. So, in chemistry, unsaturated bonds can only be saturated by joining two of them, which will usually, though not necessarily, belong to different radicles. If two univalent radicles are united, the result is a saturated compound. So, two non-relative rhemas being joined give a complete proposition. Thus, to join "— is mortal" and "— is a man," we have "X is mortal and X is a man," or some man is mortal. So likewise, a saturated compound may result from joining two bonds of a bivalent radicle;[†] and, in the same way, the two blanks of a dual rhema may be joined to make a complete proposition. Thus, "— loves —," "X loves X," or something loves itself. A univalent radicle united to a bivalent radicle gives a univalent radicle (as H·O·); and, in like manner, a non-relative rhema, joined to a dual rhema, gives a non-relative rhema. Thus, "— is mortal " joined to "— loves—" gives "— loves something that is mortal," which is a

non-relative rhema, since it has only one blank. Two, or any number of bivalent radicles united, give a bivalent radicle (as·O·O·S·O·O·), and so two or more dual rhemata give a dual rhema; as "— loves somebody that loves somebody that serves somebody that loves —." Non-relative and dual rhemata only produce rhemata of the same kind, so long as the junctions are by twos; but junctions of triple rhemata (or junctions of dual rhemata by threes), will produce all higher orders. Thus, "— gives — to —" and "— takes — from —," give "— gives — to somebody who takes — from —," a quadruple rhema. This joined to another quadruple rhema, as "— sells — to — for —," gives the sextuple rhema "— gives — to somebody who takes — from somebody who sells — to — for —." Accordingly, all rhemata higher than the dual may be considered as belonging to one and the same order; and we may say that all rhemata are either singular, dual, or plural.

Such, at least, is the doctrine I have been teaching for 25 years, and which, if deeply pondered, will be found to enwrap an entire philosophy. Kant taught that our fundamental conceptions are merely the ineluctable ideas of a system of logical forms; nor is any occult transcendentalism requisite to show that this is so, and must be so. Nature only appears intelligible so far as it appears rational, that is, so far as its processes are seen to be like processes of thought. I must take this for granted, for I have no space here to argue it. It follows that if we find three distinct and irreducible forms of rhemata, the ideas of these should be the three elementary conceptions of metaphysics. That there are three elementary forms of categories is the conclusion of Kant, to which Hegel subscribes; and Kant seeks to establish this from the analysis of formal logic. Unfortunately, his study of that subject was so excessively superficial that his argument is destitute of the slightest value. Nevertheless, his conclusion is correct; for the three elements permeate not only the truths of logic, but even to a great extent the very errors of the profounder logicians. I shall return to them next week. I will only mention here that the ideas which belong to the three forms of rhemata are firstness, secondness, thirdness; firstness, or spontaneity; secondness, or dependence; thirdness, or mediation.

But Mr. A. B. Kempe, in his important memoir on the "Theory of Mathematical Forms,"[*] presents an analysis which amounts to a formidable objection to my views. He makes diagrams of spots connected by lines; and it is easy to prove that every possible system of relationship can be so represented, although he does not perceive the evidence of this. But he

---

[*] *Nature*, in connection with a picture, copy, or diagram, does not necessarily denote an object not fashioned by man, but merely the object represented, as something existing apart from the representation.

[†] Thus, *CO*, which appears as such a radicle in formic acid, makes of itself a saturated compound.

[*] *Philosophical Transactions* for 1886. No logician should fail to study this memoir.

shows (§ 68) that every such form can be represented by spots indefinitely varied, some of them being connected by lines, all of the same kind. He thus represents every possible relationship by a diagram consisting of only *two* different kinds of elements, namely, spots and lines between pairs of spots. Having examined this analysis attentively, I am of opinion that it is of extraordinary value. It causes me somewhat to modify my position, but not to surrender it. For, in the first place, it is to be remarked that Mr. Kempe's conception depends upon considering the diagram purely in its self-contained relations, the idea of its representing anything being altogether left out of view; while my doctrine depends upon considering how the diagram is to be connected with nature. It is not surprising that the idea of thirdness, or mediation, should be scarcely discernible when the representative character of the diagram is left out of account. In the second place, while it is not in the least necessary that the spots should be of different kinds, so long as each is distinguishable* from the others, yet it is necessary that the connections between the spots should be of two different kinds, which, in Mr. Kempe's diagrams, appear as lines and as the absence of lines. Thus, Mr. Kempe has, and must have, three kinds of elements in his diagrams, namely, one kind of spots, and two kinds of connections of spots. In the third place, the spots, or units, as he calls them, involve the idea of firstness; the two-ended lines, that of secondness; the attachment of lines to spots, that of mediation.

My position has been modified by the study of Mr. Kempe's analysis. For, having a perfect algebra for dual relations, by which, for instance, I could express that "A is at once lover of B and servant of C," I declared that this was inadequate for the expression of plural relations; since to say that A gives B to C is to say more than that A gives something to C, and gives to somebody B, which is given to C by somebody. But Mr. Kempe (§ 330) virtually shows that my algebra is perfectly adequate to expressing that A gives B to C; since I can express each of the following relations:

In a certain act, D, something is given by A;
In the act, D, something is given to C;
In the act, D, to somebody is given B.

This is accomplished by adding to the universe of concrete things the abstraction "this action." But I remark that the diagram fails to afford any formal representation of the manner in which this abstract idea is derived from the concrete ideas. Yet it is precisely in such processes that the difficulty of all difficult reasoning lies. We have an illustration of this in the circumstance that I was led into an error about the capa-

* I use this word in its proper sense, and not to mean unlike, as Mr. Kempe does.

bility of my own algebra for want of just the idea that process would have supplied. The process consists, psychologically, in catching one of the transient elements of thought upon the wing and converting it into one of the resting places of the mind. The difference between setting down spots in a diagram to represent recognised objects, and making new spots for the creations of logical thought, is huge. To include this last as one of the regular operations of logical algebra is to make an intrinsic transmutation of that algebra. What that mutation was I had already shown before Mr. Kempe's memoir appeared.

THE PSYCHOLOGY OF BUDDHISM.
BY PROF. H. H. WILLIAMS.
[CONCLUDED.]

What was this knowledge? It is this: Physical life and suffering are one thing. To be born is to suffer. The cause of suffering is the cause of physical existence. And the cause of physical existence is desire. Why does the soul desire? It is through ignorance. Ignorance, not knowing, is a positive thing, a power. This can be displaced by knowledge, a positive thing, a power. Knowledge is the only thing that will overcome ignorance. It will uproot the cause of desire. Knowledge thus destroys the cause of physical existence. It breaks the ceaseless chain of transmigration. Knowledge is salvation.

From the viewpoint of Gotama, this solution was the only one possible. For to him, the cause of the child is not the parents. These are only incidents in a process. The cause of the child lies in the child itself, in the ego, as we say. The physical form is but one of a thousand others, and the essential child has persisted through all these physical forms. What then is there within the child itself that could be the cause of its physical existence? This was Gotama's problem. And the deepest answer he could give was the one he gave, viz., desire. (Desire is used in an inclusive sense.) He reached this doctrine through an analysis of conduct and easily identified the ground of conduct with the ground of physical existence, being a Hindoo.

The third crisis involves the resolution to preach. Gotama had won his own salvation. The knowledge that brought him salvation made him Buddha. His doctrine was won at high cost. It was not easy to be understood. Should he preach it to men? He debated as follows: "This profound truth, I have, after "many struggles, perceived; a truth difficult to discern; a truth difficult to understand; a truth fraught "with blessings; supreme, transcending all thought; "a truth, teeming with meaning; one that the sage "alone can grasp. The race of man is of the earth: "there it moves and has its being, there is its abode,