THE SCIENCE OF MECHANICS

A CRITICAL AND HISTORICAL EXPOSITION OF ITS PRINCIPLES

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TRANSLATED FROM THE SECOND GERMAN EDITION
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WITH TWO HUNDRED AND FIFTY CUTS AND ILLUSTRATIONS

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TRANSLATOR'S PREFACE.

The Open Court Publishing Company has acquired the sole right of English translation of this work, which in its German original formed a volume of the *Internationale wissenschaftliche Bibliothek*, of F. A. Brockhaus, of Leipsic.

In the reproduction, many textual errors and irregularities have been corrected, marginal titles have been inserted, and the index has been amplified. It is believed that the usefulness of the book has thus been increased.

No pains have been spared to render the author's meaning clearly and faithfully. In this, it has often been necessary to depart widely from the form of the original, but never, it is hoped, from its spirit.

The thanks of the translator are due to Mr. C. S. Peirce, well known for his studies both of analytical mechanics and of the history and logic of physics, for numerous suggestions and notes. Mr. Peirce has read all the proofs and has rewritten § 8 in the chapter on Units and Measures, where the original was inapplicable to this country and slightly out of date.

THOMAS J. McCORMACK.

LA SALLE, ILL., June 28, 1893.

AUTHOR'S PREFACE TO THE TRANSLATION.

Having read the proofs of the present translation of my work, *Die Mechanik in ihrer Entwickelung*, I can testify that the publishers have supplied an excellent, accurate and faithful rendering of it, as their previous translations of essays of mine gave me every reason to expect. My thanks are due to all concerned, and especially to Mr. McCormack, whose intelligent care in the conduct of the translation has led to the discovery of many errors, heretofore overlooked. I may, thus, confidently hope, that the rise and growth of the ideas of the great inquirers, which it was my task to portray, will appear to my new public in distinct and sharp outlines.

E. MACH.

PRAGUE, April 8th, 1893.

PREFACE TO THE FIRST EDITION.

The present volume is not a treatise upon the application of the principles of mechanics. Its aim is to clear up ideas, expose the real significance of the matter, and get rid of metaphysical obscurities. The little mathematics it contains is merely secondary to this purpose.

Mechanics will here be treated, not as a branch of mathematics, but as one of the physical sciences. If the reader's interest is in that side of the subject, if he is curious to know how the principles of mechanics have been ascertained, from what sources they take their origin, and how far they can be regarded as permanent acquisitions, he will find, I hope, in these pages some enlightenment. All this, the positive and physical essence of mechanics, which makes its chief and highest interest for a student of nature, is in existing treatises completely buried and concealed beneath a mass of technical considerations.

The gist and kernel of mechanical ideas has in almost every case grown up in the investigation of very simple and special cases of mechanical processes; and the analysis of the history of the discussions concern-

ing these cases must ever remain the method at once the most effective and the most natural for laying this gist and kernel bare. Indeed, it is not too much to say that it is the only way in which a real comprehension of the general upshot of mechanics is to be attained.

I have framed my exposition of the subject agreeably to these views. It is perhaps a little long, but, on the other hand, I trust that it is clear. I have not in every case been able to avoid the use of the abbreviated and precise terminology of mathematics. To do so would have been to sacrifice matter to form; for the language of everyday life has not yet grown to be sufficiently accurate for the purposes of so exact a science as mechanics.

The elucidations which I here offer are, in part, substantially contained in my treatise, *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit* (Prague, Calve, 1872). At a later date nearly the same views were expressed by Kirchhoff (*Vorlesungen über mathematische Physik: Mechanik*, Leipsic, 1874) and by Helmholtz (*Die Thatsachen in der Wahrnehmung*, Berlin, 1879), and have since become commonplace enough. Still the matter, as I conceive it, does not seem to have been exhausted, and I cannot deem my exposition to be at all superfluous.

In my fundamental conception of the nature of science as Economy of Thought,—a view which I indicated both in the treatise above cited and in my pamphlet, *Die Gestalten der Flüssigkeit* (Prague, Calve, 1872), and which I somewhat more extensively developed in my academical memorial address, *Die ökonomische Natur der physikalischen Forschung* (Vienna, Gerold, 1882,—I no longer stand alone. I have been much gratified to find closely allied ideas developed, in an original manner, by Dr. R. AVENARIUS (*Philosophie als Denken der Welt, gemäss dem Princip des kleinsten Kraftmaasses*, Leipsic, Fues, 1876). Regard for the true endeavor of philosophy, that of guiding into one common stream the many rills of knowledge, will not be found wanting in my work, although it takes a determined stand against the encroachments of metaphysical methods.

The questions here dealt with have occupied me since my earliest youth, when my interest for them was powerfully stimulated by the beautiful introductions of LAGRANGE to the chapters of his *Analytic Mechanics*, as well as by the lucid and lively tract of JOLLY, *Principien der Mechanik* (Stuttgart, 1852). If DUEHRING'S estimable work, *Kritische Geschichte der Principien der Mechanik* (Berlin, 1873), did not particularly influence me, it was that at the time of its appearance, my ideas had been not only substantially worked out, but actually published. Nevertheless, the reader will, at least on the destructive side, find many points of agreement between Dühring's criticisms and those here expressed.

The new apparatus for the illustration of the subject, here figured and described, were designed entirely by me and constructed by Mr. F. Hajek, the mechanician of the physical institute under my control.

In less immediate connection with the text stand the fac-simile reproductions of old originals in my possession. The quaint and naïve traits of the great inquirers, which find in them their expression, have always exerted upon me a refreshing influence in my studies, and I have desired that my readers should share this pleasure with me.

E. MACH.

PRAGUE, May, 1883.

PREFACE TO THE SECOND EDITION.

In consequence of the kind reception which this book has met with, a very large edition has been exhausted in less than five years. This circumstance and the treatises that have since then appeared of E. Wohlwill, H. Streintz, L. Lange, J. Epstein, F. A. Müller, J. Popper, G. Helm, M. Planck, F. Poske, and others are evidence of the gratifying fact that at the present day questions relating to the theory of cognition are pursued with interest, which twenty years ago scarcely anybody noticed.

As a thoroughgoing revision of my work did not yet seem to me to be expedient, I have restricted myself, so far as the text is concerned, to the correction of typographical errors, and have referred to the works that have appeared since its original publication, as far as possible, in a few appendices.

E. Mach.

Prague, June, 1888.

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take the metre as our unit of length, and the second as our unit of time, the acceleration of a falling body for example is 9·81, or as it is customary to write it, indicating at once the dimensions and the fundamental measures: 9·81 (metre/second²). If we pass now to the kilometre as our unit of length ($\lambda = 1000$), and to the minute as our unit of time ($\tau = 60$), the value of the same acceleration of descent is $(60 \times 60/1000)\, 9{\cdot}81$, or 35·316 (kilometre/minute²).

The International Bureau of Weights and Measures.

[8. The following statement of the mechanical units at present in use in the United States and Great Britain is substituted for the statement by Professor Mach of the units formerly in use on the continent of Europe. All the civilised governments have united in establishing an International Bureau of Weights and Measures in the Pavillon de Breteuil, in the Parc of St. Cloud, at Sèvres, near Paris. In some countries, the standards emanating from this office are exclusively legal; in others, as the United States and Great Britain, they are optional in contracts, and are usual with physicists. These standards are a standard of length and a standard of *mass* (not *weight.*)

The international unit of length.

The unit of length is the International Metre, which is defined as the distance at the melting point of ice between the centres of two lines engraved upon the polished surface of a platiniridium bar, of a nearly X-shaped section, called the International Prototype Metre. Copies of this, called National Prototype Metres, are distributed to the different governments. The international metre is authoritatively declared to be identical with the former French metre, used until the adoption of the international standard; and it is impossible to ascertain any error in this statement, be-

cause of doubt as to the length of the old metre, owing partly to the imperfections of the standard, and partly to obstacles now intentionally put in the way of such ascertainment. The French metre was defined as the distance, at the melting-point of ice, between the ends of a platinum bar, called the *mètre des archives*. It was against the law to touch the ends, which made it difficult to ascertain the distance between them. Nevertheless, there was a strong suspicion they had been dented. The *mètre des archives* was intended to be one ten-millionth of a quadrant of a terrestrial meridian. In point of fact such a quadrant is, according to Clarke, 32814820 feet, which is 10002015 metres.

The international unit of mass.

The international unit of mass is the kilogramme, which is the mass of a certain cylinder of platiniridium called the International Prototype Kilogramme. Each government has copies of it called National Prototype Kilogrammes. This mass was intended to be identical with the former French kilogramme, which was defined as the mass of a certain platinum cylinder called the *kilogramme des archives*. The platinum being somewhat spongy contained a variable amount of occluded gases, and had perhaps suffered some abrasion. The kilogramme is 1000 grammes; and a gramme was intended to be the mass of a cubic centimetre of water at its temperature of maximum density, about 3·93° C. It is not known with a high degree of precision how nearly this is so, owing to the difficulty of the determination.

The British unit of length.

The regular British unit of length is the Imperial Yard which is the distance at 62° F. between the centres of two lines engraved on gold plugs inserted in a bronze bar usually kept walled up in the Houses of Parliament in Westminster. These lines are cut rela-

Conditions of comparison of the Imperial Yard with other measures.

tively deep, and the burr is rubbed off and the surface rendered mat, by rubbing with charcoal. The centre of such a line can easily be displaced by rubbing; which is probably not true of the lines on the Prototype metres. The temperature is, by law, ascertained by a mercurial thermometer; but it was not known, at the time of the construction of the standard, that such thermometers may give quite different readings, according to the mode of their manufacture. The quality of glass makes considerable difference, and the mode of determining the fixed points makes still more. The best way of marking these points is first to expose the thermometer for several hours to wet aqueous vapor at a known pressure, and mark on its stem the height of the column of mercury. The thermometer is then brought down to the temperature of melting ice, as rapidly as possible, and is immersed in pounded ice which is melting and from which the water is not allowed to drain off. The mercury being watched with a magnifying glass is seen to fall, to come to rest, and to commence to rise, owing to the lagging contraction of the glass. Its lowest point is marked on the stem. The interval between the two marks is then divided into equal degrees. When such a thermometer is used, it is kept at the temperature to be determined for as long a time as possible, and immediately after is cooled as rapidly as it is safe to cool it, and its zero is redetermined. Thermometers, so made and treated, will give very constant indications. But the thermometers made at the Kew observatory, which are used for determining the temperature of the yard, are otherwise constructed. Namely the melting-point is determined first and the boiling-point afterwards; and the thermometers are exposed to both tempera-

tures for many hours. The point which upon such a thermometer will appear as 62° will really be considerably hotter (perhaps a third of a centigrade degree) than if its melting-point were marked in the other way. If this circumstance is not attended to in making comparisons, there is danger of getting the yard too short by perhaps one two-hundred-thousandth part. General Comstock finds the metre equal to 39·36985 inches. Several less trustworthy determinations give nearly the same value. This makes the inch 2·540014 centimetres.

Relative lengths of the metre and yard.

At the time the United States separated from England, no precise standard of length was legal*; and none has ever been established. We are, therefore, without any precise legal yard; but the United States office of weights and measures, in the absence of any legal authorisation, refers standards to the British Imperial Yard.*

The American unit of length.

The regular British unit of mass is the Pound, defined as the mass of a certain platinum weight, called the Imperial Pound. This was intended to be so constructed as to be equal to 7000 grains, each the 5760th part of a former Imperial Troy pound. This would be within 3 grains, perhaps closer, of the old avoirdupois pound. The British pound has been determined by Miller to be 0·4535926525 kilogramme; that is the kilogramme is 2·204621249 pounds.

The British unit of mass.

At the time the United States separated from Great Britain, there were two incommensurable units of weight, the *avoirdupois pound* and the *Troy pound*. Con-[illegible]s since established a standard Troy pound, [illegible]pt in the Mint in Philadelphia. It was a [illegible]e old Imperial Troy pound which had been [illegible]n England after American independence. It

* The so-called standard of 1758 had not been legalised.

The American unit of mass.

is a hollow brass weight of unknown volume; and no accurate comparisons of it with modern standards have ever been published. Its mass is, therefore, unknown. The mint ought by law to use this as the standard of gold and silver. In fact, they use weights furnished by the office of weights and measures, and no doubt derived from the British unit; though the mint officers profess to compare these with the Troy pound of the United States, as well as they are able to do. The old avoirdupois pound, which is legal for most purposes, differed without much doubt quite appreciably from the British Imperial pound; but as the Office of Weights and Measures has long been, without warrant of law, standardising pounds according to this latter, the legal avoirdupois pound has nearly disappeared from use of late years. The makers of weights could easily detect the change of practice of the Washington Office.

Measures of capacity are not spoken of here, because they are not used in mechanics. It may, however, be well to mention that they are defined by the weight of water at a given temperature which they measure.

The unit of time.

The universal unit of time is the mean solar day or its one 86400th part, which is called a second. Sidereal time is only employed by astronomers for special purposes.

Whether the International or the British units are employed, there are two methods of measurement of mechanical quantities, the *absolute* and the *gravitational*. The *absolute* is so called because it is not relative to the acceleration of gravity at any station. This method was introduced by Gauss.

The special absolute system, widely used by physicists in the United States and Great Britain, is called

The absolute system of the United States and Great Britain.

the Centimetre-Gramme-Second system. In this system, writing C for centimetre, G for gramme mass, and S for second,

the unit of length is . . . . C;
the unit of mass is . . . . G;
the unit of time is . . . . S;
the unit of velocity is . . . . C/S;
the unit of acceleration (which might be called a "galileo," because Galileo Galilei first measured an acceleration) is . . . . C/S^2;
the unit of density is . . . . G/C^3;
the unit of momentum is . . . . GC/S;
the unit of force (called a *dyne*) is . . . GC/S^2;
the unit of pressure (called one millionth of an absolute atmosphere) is . . G/CS^2;
the unit of energy (*vis viva*, or work, called an *erg*) is . . . . $\frac{1}{2}GC^2/S^2$;
etc.

The Gravitational System.

The gravitational system of measurement of mechanical quantities, takes the kilogramme or pound, or rather the attraction of these towards the earth, compounded with the centrifugal force,—which is the acceleration called gravity, and denoted by g, and is different at different places,—as the unit of force, and the foot-pound or kilogramme-metre, being the amount of gravitational energy transformed in the descent of a pound through a foot or of a kilogramme through a metre, as the unit of energy. Two ways of reconciling these convenient units with the adherence to the usual standard of length naturally suggest themselves, namely, first, to use the pound weight or the kilogramme weight divided by g as the unit of mass, and, second, to adopt such a unit of time as will make the acceleration of g at an initial station, unity. Thus, at Washington, the acceleration of gravity is 980·05 galileos. If, then, we take the centimetre as the unit of length, and the 0·031943 second as the unit of time, the acceleration of gravity will be 1 centimetre for such unit of time squared. The latter system would be for most purposes the more convenient; but the former is the more familiar.

Comparison of the absolute and gravitational systems.

In either system, the formula $p = mg$ is retained; but in the former g retains its absolute value, while in the latter it becomes unity for the initial station. In Paris, g is 980·96 galileos; in Washington it is 980·05 galileos. Adopting the more familiar system, and taking Paris for the initial station, if the unit of force is a kilogramme's weight, the unit of length a centimetre, and the unit of time a second, then the unit of mass will be 1/981·0 kilogramme, and the unit of energy will be a kilogramme-centimetre, or (1/2)·(1000/981·0) G C²/S². Then, at Washington the gravity of a kilogramme will be, not 1, as at Paris, but 980·1/981·0 = 0·99907 units or Paris kilogramme-weights. Consequently, to produce a force of one Paris kilogramme-weight we must allow Washington gravity to act upon 981·0/980·1 = 1·00092 kilogrammes.]

In mechanics, as in some other branches of physics closely allied to it, our calculations involve but three fundamental quantities, quantities of space, quantities of time, and quantities of mass. This circumstance is a source of simplification and power in the science which should not be underestimated.

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