

the ambassador touching the policy to be pursued towards the French parties. Von Arnim thought that Germany should throw its influence (secretly, of course) on the Orleanist side, in order to put an end to the pernicious example of republican institutions in a neighboring country. Bismarck did not see the danger which disquieted Von Arnim; he thought the example of French republicanism more likely to prove deterrent than encouraging. He was in favor of upholding Thiers and the republic, because he believed the republic to be the weakest form of government, for offensive purposes, which France could adopt. To make France monarchic would be to make it capable of European alliances. The ambassador's point of view, however, was one with which his master sympathized, and Von Arnim's "direct reports" to William I. proved for several months a serious obstacle to Bismarck's diplomacy. It was not until 1874 that Von Arnim was recalled from his post and gazetted ambassador at Constantinople. Shortly afterwards "revelations" began to appear in the Austrian papers concerning the relations between Prussia and the Catholic Church in 1869-70. These, in spite of Von Arnim's denials, were ultimately traced to him. It was also discovered that he had abstracted a number of documents from the archives of the Paris embassy. He was arrested, tried, and sentenced to nine months' imprisonment. He fled the country and published new revelations in an anonymous pamphlet. The object of this publication, as of the previous newspaper campaign, was to demonstrate the incapacity of Bismarck and the ability of Von Arnim. He was now prosecuted for treason for revealing state secrets to the prejudices of the German empire, and was found guilty and sentenced to five years' imprisonment at hard labor. Such an offence, of course, is not extraditable, and the sentence could not be enforced. Von Arnim died in exile.

It has long been known that Von Arnim, at the beginning of his conflict with the Chancellor, had powerful friends and no small degree of court favor. It has been shown, by the published correspondence of the great war minister Von Roon, that the liberal policy pursued by Bismarck from 1867 to 1877 had estranged his Conservative friends and caused the Emperor many pangs. The elevation of the National Liberal party to the position of a Government party; a series of liberal laws which swept away the remnants of feudalism in the open country; the conflict with the Catholic Church and the adoption of compulsory civil marriage, a measure as abhorrent to orthodox Lutherans as to Catholics—these things had aroused a strong religious-conservative reaction. The military party, too, which, in the great wars of 1866 and 1870, had chafed under Bismarck's theory that the army was simply an instrument of diplomacy, and that its movements were therefore to be guided and controlled from the diplomatic point of view, became still more restive under the policy of European peace. The modesty and self-control of Moltke, the personal friendship which existed between Bismarck and Von Roon, and the common sense and tact of the Emperor were all needed to ease the strain of divergent tendencies and avoid a breach.

What Dr. Blum now adds to the facts previously brought out in the Von Arnim trials and in Von Roon's correspondence is the distinct statement that there existed, in the early seventies, a Conservative-Clerical coalition, warmly supported by the Empress Augusta, the object of which was to force Bismarck from

power and put Von Arnim in his place. The conspirators counted mainly on the known sympathy of the Emperor with their political views; and, in spite of his confidence in Bismarck, they hoped to make the latter's position untenable by intensifying in every way the "friction" between the monarch and his Chancellor. It appears that the withdrawal of Bismarck from the Minister-Presidency of Prussia in favor of Von Roon was an episode in this conflict. In 1874, however, Von Roon resigned his post; Bismarck again doubled the Imperial Chancellorship with the premiership of Prussia, and Von Arnim came to grief. But the coalition was not yet defeated, and "friction" of all sorts continued, until in April, 1877, Bismarck asked the Emperor to accept his resignation. The Emperor refused. Bismarck at first persisted, but finally compromised on a temporary withdrawal from active duties. His vacation lasted ten months. In February, 1878, he resumed the active control of Prussian and German affairs, and for the next ten years we hear little of "friction" except in the Prussian and German Parliaments.

The parliamentary difficulties of the following decade were due to Bismarck's new financial policy. During his period of retirement he had become a protectionist. Poschinger's "Bismarck als Volkswirt" has thrown much light upon this transformation; and Blum, like Poschinger, ascribes it to study and reflection. The impartial observer will be inclined to think that Bismarck's failure to secure the financial independence of the empire by taxes on brandy or tobacco, had much to do with his new convictions. This would not imply their insincerity, for all practical statesmen have doubtless something of that power of "extemporizing convictions" which Lowell noted in Gladstone. A protectionist, at all events, Bismarck became; and in 1879 he secured a majority of agricultural and manufacturing votes for a protective tariff. This triumph, however, destroyed the majority with which he had previously worked. It split the National Liberal party and drove half its members into opposition. But the tariff question also divided the Ultramontane or Centre party, and by gradual concessions in the field of ecclesiastical politics the Chancellor contrived to carry most of the governmental measures until 1887, when the conflict over the army question and an appeal to the country once more brought in a majority of Conservatives and National Liberals.

In this decade there was, apparently, but one serious difference of opinion between Bismarck and the Emperor, but the point at issue was of the first importance. It concerned the international policy of the German Empire. Russia, as everybody knows, was dissatisfied with the settlement of the Oriental question in the Congress of Berlin (1878). She ascribed her diplomatic defeat—unjustly, as Bismarck always insisted—to the half-hearted official support and secret opposition of the German diplomacy. We now learn for the first time (pages 219, 220) how serious was the tension and how great the risk of war. In the summer of 1879 a European "commission of delimitation" was at work in Novi-Bazar. The Czar demanded of the German Emperor, in direct and unofficial correspondence, that the German commissioner be instructed to act in accord with the Russian. This demand, by the advice of Bismarck, the Emperor declared himself unable to grant. Alexander replied that his assent of the Emperor to his request was necessary to the maintenance of peace between the two countries. Bismarck advised

the Emperor to adhere to his refusal, and to request the Czar to carry on any further discussion through the usual official channels. The Emperor did so; but at the same time, on his own motion, he arranged a personal interview with the Czar. The results were not wholly satisfactory. Bismarck now entered into negotiations with Austria. He revealed the situation to Count Andrassy, and expressed his fear that a Franco-Russian alliance was under arrangement. Andrassy responded that the only possible counterpoise to such an alliance was a German-Austrian treaty, and that he believed his master ready to sign such a treaty. Bismarck declared himself unable to pledge the assent of his Emperor, and the latter, in fact, at first absolutely refused to consider such an alliance, and maintained his refusal for a fortnight. It required all Bismarck's exertions and a special embassy from Francis Joseph to secure William's consent. The alliance was obviously directed against Russia, and would, William believed, be a provocation to war. Bismarck's view, on the contrary, was that it would effectually deter Russia from war; and subsequent history seems to have justified his opinion.

Four days after the signing of this treaty, another agreement was executed between the two powers, in which Austria annulled the fifth article of the Peace of Prague, and thus released Germany from all obligation to cede any part of Schleswig to Denmark. This fact Dr. Blum mentions in another part of the book (page 447), and he indicates no connection between the two agreements; but there can be no doubt that this concession was one of the levers which Bismarck used to move his wavering master.

MATHEMATICAL FUNCTIONS.

Theory of Functions of a Complex Variable. By A. R. Föreyth. Cambridge: University Press; New York: Macmillan. 1893.

A Treatise on the Theory of Functions. By James Harkness, Associate Professor of Mathematics in Bryn Mawr College, Pa., and Frank Morley, Professor of Pure Mathematics in Haverford College, Pa. Macmillan & Co. 1893.

Traité d'analyse. Par E. Picard, Membre de l'Institut. Paris: Gauthier-Villars. Tome I. 1891. Tome II. 1893.

MANY good people fancy that the advances of mathematics, like those of jurisprudence, become manifest only when the state of things in one generation is compared with that in another; and that they are merely in the nature of extensions of old methods to new cases. In point of fact, there is probably no science in which the rate of acceleration of discovery, of the proportion of excess of the discoveries of one year over those of the year before, is so great as it is in mathematics, and no science but mathematics in which discovery seems to be becoming continually more and more fundamental. We are speaking of pure mathematics, not celestial mechanics.

Time was when geometry absorbed so overwhelming a proportion of the studies of mathematicians that "geometer" was understood to be synonymous with "mathematician"; and even to-day geometry is the most studied branch of mathematics. There are various reasons for this. One is that it is a comparatively easy subject. Besides that, the sensuous element of it seduces the mind, and carries it into excesses of study; and other causes there are. But next after geometry, in respect to

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the quantity of researches annually published about it, and far superior to geometry in its intellectual rank, is the subject of the theory of functions. For the last twenty years and more there has been a perfect freshet of original work in this line. Every year its tide is rising; every year increases the force and value of the new discoveries, which sweep on faster than they can be taken account of. In early days, enthusiasts would sacrifice necks to celebrate the solutions of problems. Later, problems appeared less sublime; theorems were requisite to excite admiration. Now, theorems are as the sands of the sea; original methods alone can command mathematical dithyrambs.

At a not remote period in the history of mathematical thought, a Mystery (with a big M and in the darkest of black-letter) hung over the imaginary unit. It used to be written $\sqrt{-1}$; and what was the square root of a negative? But when it was found that the imaginary unit of algebra was only one of a class of units which, operating upon themselves, in a generalized sense of multiplication, produce -1 , the mystery lost its capital; and after the philosophy of ordinary quantity had become better comprehended, the mysteriousness of the imaginary had vanished. The numbers, one, two, three, four, etc., are sounds which we have learned to pronounce in a certain order of succession, and which we do pronounce in telling over the individuals of collections: If such a collection is finite, we reach a last individual; and the number pronounced on coming to this last one affords the means of determining whether the individuals of two collections can be made to correspond, one to one, or, if not, in what manner they fail of that. Sometimes the things counted are really in succession like the numbers. Such are trees in a row, degrees of temperature, and years. In other cases, the succession of counting is purely artificial, as in enumerating the population, or the pounds of flour in a barrel. But the counting does not, on that account, cease to be useful, because, in whatever order the individuals are counted, the final number will, in counts of any one collection, always be the same. Even the separation into discrete units (as the gallons of water in a lake) may be artificial, provided that, if it were effected in various ways, it would always lead to the same resulting number. It will be noticed that this is not a nominalistic account of numbers—it does not make them *status vocis*, only—but it makes their existence *in re* consist in an experiential constancy; that is, it assigns to reality three elements, (1) sensuous quality, (2) compulsiveness, (3) generality. Besides the system of whole numbers, we often make use of a scheme of quantity connected together like the points on a line. This is useful even when there is no perfect continuity in the things to which it is applied. The scheme of imaginary quantity is simply one that is connected like the points of a plane. Certain natural phenomena, especially in hydrodynamics, correspond exactly, in theory at least, with such a scheme. But since any line upon such a plane is connected like ordinary real, continuous quantity, the usefulness of imaginary quantities extends to almost all cases in which real quantity is used.

The questions to which the theory of functions, so far as yet developed, chiefly addresses itself arise out of the supposition that a correspondence between the points of two different planes of quantity has been established by an equation. It considers the nature of the resulting continuity (so far as this is not resolved in

the theory of plane curves), and, more especially, the modes of representing the relation, both geometrical and analytical. The main object of the whole study is to find out how to make use of differential equations, especially such as are the immediate dicta of mechanical laws.

The disciplinary value of the theory of functions is superior to that of any other branch of mathematics. For many minds elementary geometry serves, directly or indirectly, as their model of reasoning. But elementary geometry is so artificial, and is so permeated with fallacies and caprices, that it must be and ought to be difficult to a healthy and ingenuous young mind; and much of the perverse logic that is current in the world is to be laid at its door. Algebra has done much for every educated man; it has given him an exemplar of perfectly accurate abstraction. It would put a mighty weapon in his hands if the application of it, in the elementary books, were not pretty much restricted to two problems, elimination between linear equations and the solution of the quadratic. The theory of probabilities is most instructive and useful, but that is only applied algebra. The theory of numbers is an admirable school of reasoning, as far as it goes, and it goes so far that reflection upon it will counteract much of the poison that the text-books of logic inject into the current of thought. Projective geometry imparts the most precious secrets in generalization while making no fundamental analyses. As for analytical geometry and the calculus, all that ought to be taught (as in Prof. Benjamin Peirce's 'Curves and Functions' it was taught nearly fifty years ago) as part of the general theory of functions. The theory of functions is, in the first place, intrinsically, quite easy—we mean to follow, not to invent. Of course, it is capable of being obscurely stated. Its logic is the most fundamental conceivable, and, at the same time, is the very subtlest that can anywhere be found; so that no man is too eminent never to have made a slip in it. The outlines of the theory ought to be known to every educated person.

There has hitherto been no treatise in our language on the modern development of the theory. At length the same year presents us with two. Though the first has 700 pages of royal octavo, and the other 500 of common octavo, yet the subject is so vast that a considerable part of the contents of either is excluded from the other, and much that we might desire to see is absent from both. Dr. Forsyth has been well known for some years less than half a generation as an indefatigable investigator of functions, and he has already produced two profound treatises on differential equations. The present work contains many not unimportant contributions of his own. Messrs. Harkness and Morley are younger men, but, as this volume shows, thoroughly versed in their subject. Dr. Forsyth keeps as much as he can to the general theory, treating such a special subject as elliptic integrals, for instance, with the greatest brevity possible, and at the same time in such a way as to afford a bird's-eye view of it. Messrs. Harkness and Morley, on the other hand, seem to have been of the opinion that it was better to go somewhat deeply into a smaller selection of topics. Many things are crowded out to make room for long chapters on Elliptic and Abelian Functions, while, at the same time, these very subjects are not treated with all the fulness which is requisite for the practical applications of them. This, at least, is certainly true of elliptics. Practical applications of Abelian

functions ought, perhaps, in the present state of things, not to be thought of. Certain preliminary branches, absolutely indispensable to the comprehension of the theory of functions, such as the logic of infinity, continuity, etc., and the doctrine of the convergency of series, are entirely omitted by Forsyth, while he inserts matter about substitutions which the reader will be glad to find thus at his hand, but which really belongs in a treatise on algebra. The other writers have followed the opposite course in these respects, though we cannot quite content ourselves with their attempted reproduction of Cantor's logical ideas. Dr. Forsyth imitates, in a general way, the French lucid style of exposition, though the French accuracy of statement and neatness of demonstration are often wanting in his book. Messrs. Harkness and Morley express themselves in the German manner, which makes the exposition as easy as possible for the writer—and never mind the reader. For an illustration of what we mean it is sufficient to open the book at random. At the top of p. 352, we read (with a slight modification of notation for our printer's sake):

"The symbol MN equals 1 or 0, according as M and N do or do not contain a common letter."

Now, it is inaccurate to speak of a symbol being equal to a number; and since M and N are single letters, there can be no question of their containing a common letter. But the authors mean that when they are replaced by the duadio symbols which form one of four or five different ways of expressing the same thing, those symbols have or have not a common letter, according as the corresponding quantities equal 1 or 0. The opposite page, 353, presents several singular instances of saying one thing while meaning another; and it is stated that a certain notation "will be" used, which notation is incontinently dropped without another word, and another one, not defined, is constantly used for many pages. So, on p. 355, a notation is defined, in no apparent connection with anything in the vicinity, and is never used for many pages, until it suddenly springs up after we have forgotten all about it. These examples are not culled.

It would be unfair to convey the idea that Forsyth is quite impeccable in his expressions. This is far from being true. Thus, at the beginning of chapter iii., in enunciating Cauchy's fundamental theorem on the expansion of a holomorphic function, the important words "unconditionally and uniformly" as describing the mode of convergence, are omitted, as they are overlooked in the proof given. The first page of chapter vii. has but sixteen lines of text, yet they contain no less than three faults of expression, if not of logic. Indeed, Forsyth is really too negligent in regard to terminology. Thus, that category of surfaces, curves, etc., which the Germans call *Geschlecht*, the French *genre*, and which we should do well to term *genus*, instead of the usual word "deficiency," Forsyth most confusingly designates as the *class*. Both books will be found serviceable to students, alike to those of higher and of lower grades. We may mention, by the way, that Forsyth is rich in illustrative examples, Harkness and Morley pretty poor. But we cannot sincerely pronounce either of them quite satisfactory, whether as a handbook or as a textbook, and both handbook and textbook are certainly needed. The latter ought to be so clear of all pedantic details as to be fit for the use of every young person who seeks a broad intellectual education. Picard's very admirable work is not mentioned.

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ly a treatise on the theory of functions. Indeed, when the first volume appeared, the author's purpose was to treat this subject rather slightly, and we were informed that volume ii. would deal with Differential Equations. Instead of that, only one of the seventeen chapters relates to that subject. It is evident the theory of functions has been growing in importance in the author's mind. Hence it is, probably, that instead of embarking frankly in the vehicle of imaginaries, the author thinks it more philosophical to deal chiefly with real functions, thus making many things difficult and crabbed which would in Cauchy's hands appear delightfully facile. The work is one of truly considerable power. It cannot justly be called a classic. Members of the French Institute are apt, for an obvious reason, to be over-praised. While we admit the great value of this work, we must say that some comments upon it we have seen from men to whose opinions we should naturally be inclined to defer, appear to touch the point of extravagant laudation.

Selected Letters of Malcolm Kingsley Macmillan. Privately Printed. London. 1893.

This beautiful volume makes a certain appeal to the general public in the prefatory note, which says: "To his friends the volume will assuredly be welcome; and even to those who did not know him personally, it may be of interest as a record of the impressions of a mind singularly gifted and alert." There is so little doubt of this that we cannot but hope that the book will soon be placed by Macmillan & Co. on the list of their regular publications. There are no privacies which it would be presumptuous for any lover of good reading to invade; and for the untimely death of one so rarely gifted, but who had hardly begun to use his gifts in any public way, there might be some little consolation in the wide enjoyment of his letters and of the contact with a personality so frank and cordial, so genial and robust, as that which they reveal. The book as now presented assumes the reader's knowledge of Mr. Macmillan's blood and state; the stranger finds out as he goes on that he was a son of Mr. Alexander Macmillan, the present head of the great house, that he studied at Oxford, and that he was about thirty-eight years old when he was cut off by some accident on an expedition to Mount Olympus in 1889. As he appears in these letters he was about equally insatiable as a traveller and as a reader. He was away from England much of the time during the last years of his life. Once within a few months we find him in Greece, in Egypt, in Italy and Switzerland, then off to Greece again. He was planning his first visit to America in the immediate future, after the journey from which he came not back.

He wrote a book called 'Dagonet the Jester' and a few things for the London weeklies, but somehow it was difficult for him to put his gifts and culture to any practical account. It was not that the native hue of resolution was sicklied o'er in the *Hamlet* way. We get everywhere the impression of a singularly healthy mind. Two lines of study had for him a particular attraction--Richardson, in connection with the early English novel, and certain forms of mediæval superstition. He meant to write on them, but apparently he never did. Yet he was one of the busiest of men; there was so much to read, of so many kinds, in so many languages; and then the architecture to study, the music to hear, the pictures and the plays to see, the friends to meet. To these last

he must have been a treasure. One can taste their hearty liking for him in his replies to their affectionate greetings, and his picture shows a free and open countenance that was like the nature of the man.

Many of his letters, if not the most of them, were written to his sister, Miss Margaret Macmillan, now Mrs. Louis Dyer; many others to Prof. Dyer. One of his best friends was Mr. Marion Crawford, and, writing to his brother, he gives an account of the novelist that exhibits him as a man of parts:

"Crawford was up a day or two lately, and I am more than ever struck with the fact that he is far more remarkable than his books. He speaks four languages so that the natives cannot detect him for a foreigner. He knows a good deal of Sanskrit, though he hardly ever refers to it. He learnt Norwegian, so as to pronounce it properly, in about three lessons from Ross. He is a good fencer, a good sailor, and can do silver repousse work. With no training, he has designed the entire reconstruction of his house at Sorrento. Both in mathematics and draughtsmanship he is more than mediocre. He seems able to do almost anything he turns his attention to. The one thing he has almost entirely neglected is modern literature; and he always says that he is not really a literary man. In this there is some truth, though he has a kind of imagination that he throws into everything."

No disposition could be further from the *nil admirari* than Mr. Macmillan's. He much preferred Landor's "delights of admiration" to being pleased with nothing. The breadth of his sympathies may have done something to abridge his positive activity: it was so hard for him to take a side, or, taking one, it was so natural to see the other. What could the Gothic or the Renaissance fanatic do with a man who wrote of Wren, Pugin, and Barry as the three English architects of genius? Fancy Pugin's immeasurable disgust at such a collocation! But if it makes the purist in these things question the writer's penetration, it has the advantage of being a personal opinion--always we feel that the man is looking through his own eyes and not trying to see things as he has been told he ought to. As with seeing, so with hearing: his admiration for Wagner is immense, but you will not convince him that Liszt is not a kind of "complicated tin kettle."

"My own style," he writes, "through want of practice in composition, is getting to be an admirable exemplification of the faults I dislike." But it is evident that much of the phrasing of his letters was wrought out with literary conscience, and that they suffer most from his inclination to the mock-heroic. Many of his judgments are very happily expressed, and from page to page one gets all the pleasure of conversing with a highly cultivated, active, earnest mind about the music, books, and art that have impressed it one way or another. The Cicero of the Capitoline Museum is described as "pompous, impressive, thoughtful, with a dignified purity of aspiration, a most weak flutter about the mouth--in a word, *ecclesiastical*." He "cannot help thinking that the highly idealized portrait of Shelley has been made purposely to wear the same expression" as the Beatrice Cenci of the Barberini Palace. As to the scepticism whether she is a Beatrice or the work of Guido, he says nothing. Fresh from the Parthenon, he goes to Venice, and writing of St. Mark's he says:

"One wonders how the Parthenon in all its glory would have compared with it. I mean, of course, simply as to decoration and color. After looking at all the architectural glory of that building for many months, and constantly craving some adequate notion of its former complete effect, it is a relief to have here those signs of a more recent life and glory far less gone towards extinction. Of course, the Greeks

would not have used those red and variegated marbles so freely even if they had had them in the same abundance. But I think, in spite of all, it is the nearest surviving thing to the dead Athenian glory."

Greek things suggest Schliemann, of whom Mr. Macmillan saw a good deal. He was reputed to have a Homer printed upon India-rubber, so that he might read it while bathing in Greek waters. He thought "that Alcibiades should have been allowed to conquer Italy, and that then we should have had a Græcized Europe." There is an interesting appendix concerning Charles Kingsley, for whom Mr. Macmillan was named; a second, criticising with enthusiasm the acting of Eleonora Duse-Checchi; and a third, pathetically incomplete, which is a dialogue between Wordsworth and Blake.

Reed's Rules: A Manual of General Parliamentary Law, with notes of changes made by the House of Representatives, and suggestions for special rules. By Thomas B. Reed, ex-Speaker of the House of Representatives. Chicago: Rand, McNally & Co. 1894. 12mo, pp. 221.

Parliamentary Tactics, for the use of the Presiding Officer and Public Speakers. Arranged by Harry W. Hoot. New York: The Scientific Publishing Co. 1893. Pp. 51.

EX-SPEAKER REED'S manual is very entertaining reading, and it will be very useful to the student who desires to understand the proceedings in the lower branch of Congress. For general use it is no improvement on our old, trustworthy Cushing. The practice at Washington does not govern parliamentary law elsewhere, especially in regard to special rules. Mr. Reed naturally speaks of his famous method of counting a quorum, and states, on p. 23, "Those who sit silent are regarded as consenting to the result. Such was the recent decision of the U. S. Supreme Court." To us this seems to be only a half-truth. That court decided at the October term, 1891, in the case of United States vs. Ballin (U. S. Reports, vol. 144, pp. 1-11), that the House had a rule that the names of members present and not voting should be added to those voting and "be counted and announced in determining the presence of a quorum to do business." Also, that this rule was "a constitutional mode of ascertaining the presence of a quorum empowered to act as the House." With all due respect to the court, no one can read that opinion without seeing that it was a foregone conclusion. By the Constitution the House could make rules, and it had made one. Confessedly the Constitution failed to provide a mode of ascertaining a quorum or of compelling members to act when present. The court clearly could not call a rule contrary to the Constitution on a point whereon that instrument was silent. It was necessary for the court, if it gave reasons at all, to fall back on general principles, and its opinion is not fortified by citing Mansfield, Dane, Dillon, or various courts. The simple question remains, Shall members sitting silent be held constructively to be acting? Until a supreme law pronounces expressly on one side or the other, each will find advocates as a matter of opinion. That such a law is needed is shown by the provision in the constitutions of many of our States, that to pass a bill there shall be required the recorded vote in the affirmative of a clear majority of the members. This provision is the simplest and clearest reply to the unnecessary argument of the court that the act of a mino-