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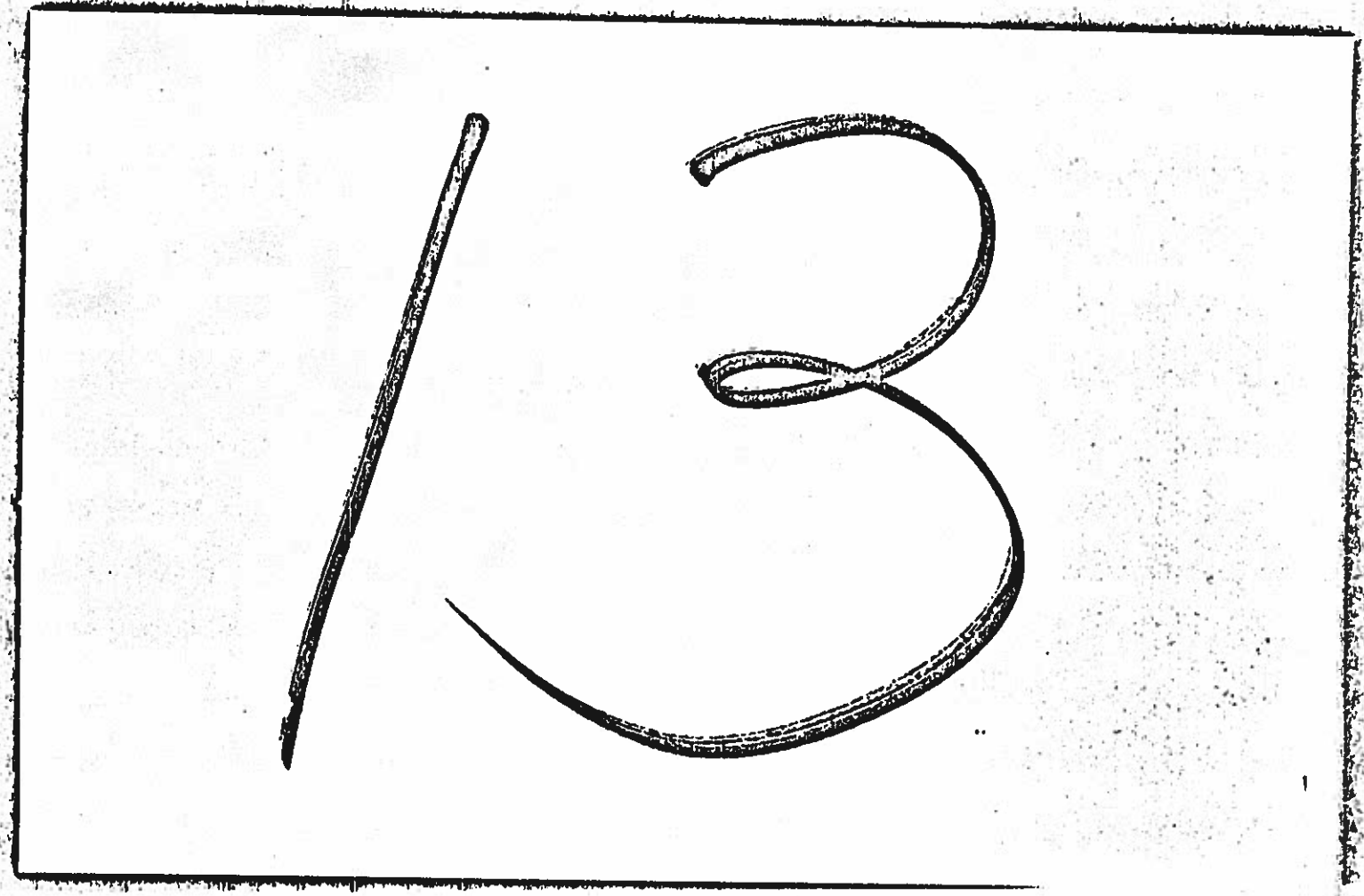

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On the Logic of Quantity.

By C. S. Peirce.

Art. 1. All the sciences may advantageously be arranged as follows:

The first prime division consists of Mathematics, which merely frames hypotheses and traces out their consequences, and alone of all the sciences does not concern itself with ~~real~~ positive truth. Although the mathematician is free to form any hypothesis which manifestly involves no contradiction and ~~is~~ which (in pure mathematics) is only so far determinate as its determinations may affect the form of the consequences which are deducible from it, yet we find that all those general hypotheses ~~which~~ each of which is carried through a whole branch of mathematics, are of a few well characterized kinds. Each deals with some system of quantity; and the systems of quantity are of three ~~sorts~~ classes, viz: 1st, the system of two values, as in ~~the most~~ demonstrative logic, where every proposition is either true or false; 2nd, systems of discrete quantity ~~greater~~ having more than two values each; 3d, systems of continuous quantity. The ~~purpose~~ problem of the present paper is to ascertain why mathematics should always deal with a system of quantity, what the different possible systems of quantity are and how they are characterized, what the logical nature of mathematical infinity is, etc.

The second prime division of the sciences consists of Philosophy, which concerns itself, indeed, with positive truths but only with such as would ~~remain~~ manifestly remain true whatever observation might reveal. Even

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if it be denied that there is any such truth, room a place in the scheme of the sciences must be accorded to this inquiry. For a department of sciences is not a body of truth but a group of possible researches after truth. But then, nobody ever maintained that philosophy was not based on observation; and it suffices to establish this department of science that there are, or seem to be, some truths which are logically involved in the general fact that any observation is made, — such as, that there seems to be a world which is not entirely homogeneous, and that there seems to be some power of observation, etc.; for these truths in seeming to be logically involved in that general fact are independent of the possible varieties of results which might be revealed by observation; and if such these truths seem to be, a branch of inquiry can be directed to them. Philosophy has two and but two branches, Logic, or the philosophy of thought, and Metaphysics, or the philosophy of being. The inquiry whether any given branch of science, such as ethics, belongs to philosophy or to any other division of science is an application of logic. Philosophy always has ~~made~~ reposed, and must ~~make~~ repose, very greatly upon ~~great use of~~ mathematics; and this is particularly true of logic, which has been mathematical from the very first. Metaphysics has always been based, and must be based, upon accepted principles of logic.

The third prime division of the sciences consists of the special observational sciences. These sciences, however observationally they may be pursued, always have depended, and must depend, to an important extent upon metaphysics for their first principles. The special observational sciences are divisible

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in two ways. In the first way they are divided into the Psychical and the Physical sciences, the latter embracing ~~the~~ all special observational study of purely material phenomena, the former all special observational study of manifestations of mind as such, including anthropology, philology, history, etc. The physical sciences perhaps depend more upon psychical principles than the psychical sciences upon physical principles. In the second way, the special observational sciences are divided into the Nomological, the Classificatory, and the Descriptive. The nomological sciences study the universal laws of mind or of matter. Nomological psychics treats of ~~the~~ time, of the laws of association, fatigue, etc. Nomological physics treats of physical geometry, or the real properties of space, for example, whether the elliptic, hyperbolic, or parabolic; also of dynamics, ~~and motion~~ elatenics and thermotics, ~~and of the pheno~~ Whether the study of the properties of the ether ought to be included under Nomological or classificatory physics may be doubted. Classificatory psychics studies ~~the~~ observationally the different kinds of manifestations of mind, such as language, business, worship, etc. describes their general properties and regularities, and seeks to explain the phenomena upon the general principles of association, etc. Classificatory physics, or chemistry, describes the different kinds of matter determines their properties, and seeks to explain their constitutions on the principles of dynamics. Thus, it treats of the crystalline forms of the different substances; and in ~~In~~ regard to ~~the~~ one great class of substances, the protoplasms it has to describe and explain the organized forms into which they grow, thus ~~producing~~ developing the science of biology. ~~Descriptive~~ The Descriptive sciences describe individual objects and seek to explain ~~them~~ their origin and history upon the principles of nomological and classificatory science. Under descriptive psychics we place history of all kinds, political geography, law, theology, etc. Under descriptive physics

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we have astronomy, geography, geology, etc.

The fourth prime division of science consists of the Arts. Every art depends, at once, upon psychical and upon physical Knowledge. Some arts are almost entirely psychical, such as rhetoric, ethics, religion (the art of propitiating the highest power), jurisprudence, etc.; others are almost entirely physical, such as glass-blowing, hydrology, and hundreds of others; while many arts require high Knowledge of both Kinds, such as painting, music, etc. But their classification of the arts is most complicated.

I purpose, in a series of papers, to examine the logic of each of the above twelve branches of science.

Art. 2. We remark in the world three categories of elements. The first comprises the qualities, such as red, bitter, tedious, hard, heart-rending, agreeable, and doubtless manifold varieties which are utterly unknown to us. They are potentialities and general in their nature, but there is no reason why any of them should be as it is. The second category comprises actual occurrences. They are individual and brutely insistent. The third category comprises laws. No fact nor collection of facts can constitute a law which goes beyond any completed facts to determine how facts that may be shall be characterized. In these, the ideal world of qualities and the actual world of facts overlap.

Qualities are selfcontained. The fact that a thing is green, so long as the green is looked upon as a mere quality, does not logically involve anything else being the subject of any other quality. Actuality is double in its nature. In the act of will, which is its type, there is the acting subject and the reacting object. The happening of an event consists in the previous existence of one state of things, and the subsequent existence of

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another. To think of anything as having individual existence involves to in every case two a pair of factors. Law has a triple nature. It supposes, in the first place, a subject which it affects. Then, there must be a cause. That is, that affected subject must fulfil the condition of the law; and this it can only do by a relation to something else. Then the effect of the law is realized in another relation to a third thing. There are other ways in which this element of the world may be conceived; but in every case, three things are involved.

The result of this universal phenomenon is that the conceptions of First, Second, and Third present themselves in Logic at every turn. It is well to reflect upon the true nature of these conceptions. The One that we hear so much in philosophy is a synthetical unity, that is, a totality, or something collected with a collection. That is as far as possible from the pure idea of the First. The First is that which springs out of nothing, the prominent element in the conception of spontaneity. It is so utterly freed from all trammels that it does not even ~~think~~ involve the conception of a second. It stands on its own feet and is what it is, — red is red and green is green, — without reference to anything else. The Second, on the contrary, is so only by virtue of another thing which is its second. The pair react. But they do so brutally, without a reason; for a reason would be a mediating third. Tertium in Latin is nearly synonymous with Medium. That is the conception of the category. It is the continuous ever mediating line which leads from one extremity to the other. The ideas are, then, ~~Firstness, Thirdness,~~ Mediation; or the Idea, the Fact; and the Evolution.

~~So by: treats of Terms, Propositions, and Inferences. Taking the second of these Terms may be appears in different lights in different ways developing Logic, but the most convenient as well as the most philosophical way is to regard the terms as ors. Verbs~~

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Art. 3. Any world, or product of experience, may be studied in three ways, according as the mind gives prominence to its firstness, to its secondness, or to its thirdness.

It may, in the first place, be scrutinized in respect to its qualities, regardless of anything more than how each object appears. That is the way in which the Artist, who relatively to other non-artists is always an impressionist, looks upon an experiential presentment. His conversation shows how he values each object for itself, how he delights in the exceptional and in the original, and how his distrusts, and even dislikes, mental arrangements in sets that are not to be seen in nature.

The world may, in the second place, be examined with reference to the actions and reactions of different objects upon one another. This is the way the Practical Man studies the world. How that man's mind runs upon pairs will be manifest as soon as he talks. His divisions are almost all dichotomies: every transaction is either honest or dishonest; those he deals with are either friends or enemies; doctrines are either commendable or reprehensible.

Finally the world may be studied in order to discover how things are brought about, what the laws of the phenomena are, what idea governs the courses. This is the way in which the Philosopher, or man of intellect, is interested in the world. He loves to group objects in threes, and the nature of his pursuit leads him often to do so without remarking it.

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Art. 4. Singular facts, – facts about single subjects, – are of only one type; but what this means must be gathered from comparison with dual and with plural facts.

Dual facts, – facts about pairs of subjects, – are of two types, first, those which imply no more about their subjects than that they have certain qualities between which some comparison is made; and second, those which imply more about their subjects than is true of them in their separate existences, something real and positive which could not be true of one if the other did not exist to enable it to be true, in other words a real action and reaction between the individuals. This is the old distinction between real relations and relations of reason. Real relations exhibit the fully developed type of duality. In this case, it is what happens to the individuals, the actions of them, which pairs them. Relations of reason imply no other facts than the possession of two qualities severally by two individuals; and it is primarily the qualities that are paired. One individual can only act upon a different individual; the individual things must be two. One part of a thing may act upon another part, or the individual as it is at one instant upon the individual as he is at another instant. But these are different individuals; for an individual is strictly that which is here and now, – a particle at a given instant. But relations of reason, such as identity, likeness, etc. may subsist between an individual taken as first of the pair and the very same individual taken as second of the pair.

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Plural facts, — facts about sets of objects greater than pairs, — are of three types, first, those which imply no more about their subjects as they exist than that they severally possess certain qualities (colors) which comparison is made; secondly, those which imply no more about their subjects than that actions take place between the different pairs of them; and thirdly, those which imply more about their subjects than is true of them as separate units or as pairs, something real and positive concerning each which could not be true of it were not two others there to enable it to be true. For example, if the color of A is intermediate between the color of B and the color of C, this is a triple fact about A, B, and C; but all that is true of those objects is that they have severally certain colors, one or two of the objects might be imaginary and only one existent, and still the triple fact would remain true of that one. Next suppose the waters of two streams to join. Then of the three stretches of river, two above and one below the junction, any two pairs (real) act upon one another. The two above actually join, which would be impossible unless both were existent; and each of those above flows into the one below. These three actions constitute the whole fact; but the fact that A and B join to form C is a triple fact. Finally, suppose two men to agree that a certain river shall be dammed. This is something done; it (does) not consist in the possession of three independent qualities by the two men and by the river. But though it is something done, it does not consist of three actions between pairs of objects. The two men agree, and that they cannot do without agreeing upon something (result) to be brought about. So what happens to the river is that it is made the subject

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of a bargain; and it takes two persons to make a bargain.

Art. 5. It thus appears that qualities, or some genus of elements of which qualities form a species, are requisite for the existence of singular facts; that individual or material things, or some genus of which these form a species, are requisite for the existence of real dual facts; and that thoughts, or some genus of which these form a species, are requisite for the existence of real plural facts.

Note. In order to ascertain what the limits of those genera are, it will be necessary to consider just how those different categories of objects become requisite for to those kinds of facts.

The Babylon Chaldean metaphysicians were quite right in holding that if the world has come about as the result of any process, there must have been, in the beginning, — or immediately after absolute nothingness, — a state of Chaos in which the real state of things corresponded to our state of mind when we only ask questions but assert nothing. Nothing, then, existed; for existence supposes some determinacy. From this Chaos the first thing that arose made must have been a partial determination. Now a partial determination precisely defines a quality, in the sense in which a quality is the requisite of a singular fact. That, then, must have been the first product of the evolution.

It is interesting, and not wholly without importance to note that these primitive thinkers without any bias from previous metaphysical systems, made the work of the first day of creation to be Light, by which, no doubt, they meant appearance, — but appearance in existence, not to vision; — and appearance and other sensible phenomena, abstracted from the

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faculty of sensation, causes nothing but partial determination sui generis. The Chaldeans also dimly conceived that the next step must have been toward duality; for they declare that on the second day the world was segmentation separated into two parts. Whether or not it was the step that came next, it is the one most conveniently considered next.

As long as nothing existed but partial determinations, there could be no indecomposible duality. For elementary, or indecomposible duality supposes something to be true of a thing which is not true of it in itself. Now nothing is true of a partial determination but the determinations that go to make up elements of that determination. Suppose any thing a new thing, not the further true, and you simply have a further determination, which is a different thing with something more true of it. In order, then, to make elementary true duality possible, it was necessary that something should exist which could have different determinations while and yet retaining its identity. That is, there must be something possessing self-identity and otherness from all else, wholly independent of what its determinations may be. Now that which preserves identity and otherness from all else, whatever partial determinations it may receive, is as good a definition of matter as ever was proposed. Yet it is not a sufficient definition. For it is further requisite that something should be really (yet not essentially but) accidentally true of matter A which by virtue of matter B, this something being such as could not be true of identical matter in itself. This while ipso facto something becomes really and accidentally true of matter B by virtue of matter A, which could not be true of any identical matter in itself. Whatever What is true of matter A is necessarily of the nature of partial determination; and what is true of matter B is necessarily of the nature of a partial determination. But that is not all; for a partial determination which is true of A is could be true of A or B, i.e. if. It must, therefore, be something of that nature, yet but different from

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that owing to duality. It must be that ~~A~~ what is true of A is a junction of two partial determinations (not a mere sum, for that would make one partial determinations) which remain two; and the same with B. And further these two junctions must be joined, and mutually produce each other. All of that is logically necessary from the essence of duality. It describes the action of one material substance on another, — as for example in impact. There is a change of quality in each of the pair and these two changes are brought about together. Thus we see how and in what sense it is that elementary duality requires matter and further requires that material substances should be capable of acting upon each other in pairs.

x centre

~~This completes the description of matter~~ it is which we said offered an alternative definition of matter. Physicists often ~~say that matter is~~ define matter as the centre of force. But the idea of force implies law and ~~thus~~ supposes something beyond mere matter. That this is so is shown by the aversion of materialists [illegible]. There is they say, nothing but actions. Mere matter, in the purity of the conception is not a centre of force but a congeries of actions. And where over there is action there ipso facto there is matter as the subject.

~~Material substances are incapable of elemental triplicity~~

Pure

~~Pure~~ Mere matter is incapable of producing elementary triplicity, so long as all its action is by pairs. For elementary triplicity supposes something to be really true of A which could neither be ~~is not and could not be~~ true of anything essentially, i.e. from the ~~for of its own~~ nature ~~that substance~~ the subject itself nor yet by the ~~[illegible]~~ action of anything upon it, but which is true of A by virtue of ~~the~~ and ~~ipso facto in~~ in its junction with, the ~~[illegible]~~ fact that B and C, two subjects independent of it, ~~subjects, are some~~ ~~[illegible]~~ have some relation to each other. Now, were A matter, and the ~~that wh~~ fact that B was related to ~~[illegible]~~ with C acted upon A, that relation would be ipso facto ~~matter~~ a material substance. For example if the mutual gravitation of two bodies B and C were to be found to ~~produce~~ ~~produced~~ any effect upon an intervening body, A, independent of the ~~several~~ actions of B and C severally upon A, every physicist would take that as a perfect demonstration that gravitation was not an action at a distance but that there was a medium, for if ~~when~~ there is action upon A, there is ipso facto matter; for the ~~matter~~ subject of action is by definition matter. But if there is any such intervening matter

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explains the action between A and the relation of B to C, there is no indecomposible triplicity. For in that case, calling the intervening subject D, the fact is nothing more than this, that B acts upon C by acting upon D, which in turn acts upon C; and D, when so affected, acts upon A. In order, then, that elemental triplicity should be possible, A must be something upon which no material thing can exert any action but of concerning which the fact, — or true proposition — that two independent things B and C react upon one another, should, quite fact, render something true. But even so that fact becomes the subject of a dual relation with A, ~~between~~ and the triple relation merely consists in this dual relation between A ~~to~~ this fact together with the dual relation between B and C. The only way of in which ~~preventing~~ this decomposibility can be avoided is by A having such an action upon the relation between B and C that to be so related to any relation implies that the subject is so related to its relation to that relation. In that case, there is no ultimate relation by means of which the triplicity can be decomposed into dualities. Thus, elemental triplicity requires a subject of such a kind that it is capable of such a relation to a fact ~~between~~ concerning independent subjects that whatever is so related to a relation is so related to its own relation to that relation. [We know no subjects of this kind except thought or reason. To do anything deliberately, or as we may say, to ~~intend~~ mean to do anything is virtually ~~to intend~~ mean to ~~intend~~ mean to do it, and to ~~mean~~ mean to mean to mean to do it, and so on ad infinitum. To know and to know that we know, are says Hamilton, the same thing. If he means that to imply is to imply that we imply, he is right.] In order that this should be possible, since an endless series of performances cannot be accomplished, while yet none of these actions can take place until the one

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next preceding it has made that relation which is to serve the succeeding one for a subject; it is necessary that A should hold the whole series within itself, in the sense that the whole series cannot but exist as soon as A exists, or exists such as it has to be the subject of the triplicity. This is to say that A is therefore made by the relation between B and C. Yet it is also to say that not the whole series of relations are [illegible] adjuncts to As existence. But B and C are independent of A. Therefore, it is not B and C as they are within themselves or for each other which is adjunct to A; but there must be something which stand for B and C and replace them for A, which are adjuncts of and dependent upon A.

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Art. 3. Logic discriminates between true and false propositions. That is the final end of logic; and this is the reason why a philosophical development of logic must begin with the proposition, not the term. From a mathematical point of view, we may say that being true and being false are the two values one of which every proposition has. In probable logic there is a continuous line of values from certain truth to certain falsity. In the logic of continua, a proposition may be true, or the truth may lie at the limit of what the proposition admits, or the proposition may be quite away from the truth. But in the syllogistic logic with which we are alone concerned at the outset, the system of logical values has but two grades and is thus the most rudimentary of all systems of quantity.

A term may conform to the reality or not; that is, it may signify a kind of which there are examples in the universe of discourse or it may not. It is therefore not accurate to define a proposition as that which is either true or false. A proposition should be defined as that which professes to be true, or assigns a logical value to itself. The truth is defined as that logical value which a proposition assigns to itself. Whether or not there really is such value, whether there is any truth is a question, not of definitions, but of fact.

The first postulate of the mathematics of logic is that every proposition really has a logical value; namely, there are propositions, called facts, which really have the value that every proposition assigns to itself. Calling any fact, x, x has the value it says it has, that is x is x. This is the principle of identity.

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The second postulate of the mathematics of logic is that there are more (that there is a ... logical value, that) logical values than one. There is falsity as well as truth; and falsity differs from truth in this sense, that the same proposition cannot have both these logical values. This is the principle of contradiction.

Let $\breve{x}$ be any algebraical expression of a proposition. Then, a heavy dash, called an obelus, drawn over $\breve{x}$, has, in the general algebra of logic, the effect of negativing $\breve{x}$, that is, of converting the whole into some proposition, — if there be any such proposition, — called a negative of $\breve{x}$, which asserts something which necessarily renders its logical value different from that of $\breve{x}$, and asserts nothing more. The second postulate ~~may be stated as a little further on than the above statement, so as to assert that every~~ asserts, then, that every proposition has a negative; that is there is some proposition which has a logical value other than ~~is that of~~ the first proposition. The first proposition, ~~has a negative.~~ The negative being other than it, the latter is other than this negative. The postulate is, then, that any proposition, $\breve{x}$, is a negative of a negative of itself, that is, has a logical value other than some logical value other than its own. That is,

(1) ~~$\breve{x}$ is $\breve{\breve{x}}$.~~ If $\breve{x}$ is true, $\breve{\breve{\breve{x}}}$ is true.

~~The third~~ This would be true even of the system of values of probable logic and of that of the logic of continuity.

The third postulate of the mathematics of logic is, that there is no third logical value, so that every proposition is either true or false. This is the principle of excluded middle (principium exclusi tertii.) That is, if a proposition have a logical value other than the value other than a given proposition's own (neither of which values does this postulate assert to be possible) then the former proposition must have the logical value of the latter. Further, if B asserts no

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more than is necessary in order to assert that the ~~valu~~ logical value of A is different from its own, that is, is different from that logical value which it, B, assigns to itself, namely the truth, B ~~simply~~ asserts no more than that the logical value of A is something other than that ~~assigns to~~ ~~asserts that A is not true~~; A and it, ~~then~~ ~~thereupon~~, thereupon C asserts no more than is necessary in order to assert that the logical value of B is different from its own, it only ~~asserts that~~ assigns to A ~~has~~ a different logical value from that logical value assigned to it by B. This ~~pro~~ third postulate, then, consists in saying that C under those circumstances must ~~assert~~ assign ~~the~~ A the logical value, the truth; so that if C is true, A must likewise be true, or

(2) If $\overline{\overline{x}}$ is true, x is true.

Art. 4. If ~~one quant~~ the value of one quantity y depends upon the values of other quantities, x, x', x'', etc., the former is said to be a function of the latter, ~~as its~~ which are called its variables. Let us consider ~~a~~ functions of single variables. If y can have every value for every value of x, they are said to be independent variables: y is not a function of x. If y can have every value, ~~but~~ while x is restricted to a single value, x is a constant, and y is ~~independent~~ a variable; but y is not a function of x. ~~Neither is~~ If y has the same value whatever be the value of x, y is a constant and is not a function.

It is easy to enumerate all the functions of single variables in the system of logical values. Of single-valued, ~~functio~~ unequivocal functions, there are but two. One of these is that function whose value is the same as that of its variable; the other is that function whose value is the negative of that of its variable. If a proposition Y asserts so much as is necessary to render its value the former function of a proposition, X, and asserts no more than is necessary to fulfil that condition, X and Y are said to be logically equivalent, and we write

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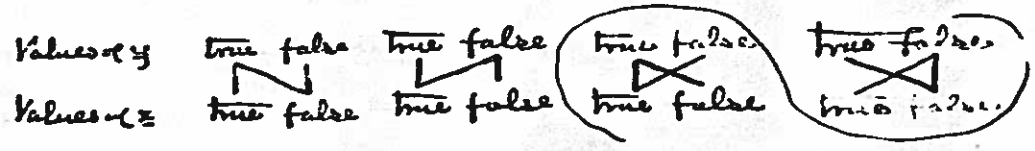
$Y = X$. If a proposition, N, asserts so much as is necessary to render it false in the latter function of the value of a proposition, M, and asserts no more, N is, as we have seen, the negative of M; $N = \bar{M}$.

The other functions of single variables are exhibited in the following diagrams.



In the first diagram y has that mode of dependence upon x that a consequent has upon its antecedent; that is, if x is true y is true, but if x is not true y may or may not be true. The second diagram shows the converse relation, The third the negative of the first, and the fourth the negative of the converse relation. There is thus but one fundamental function here among the four.

In this general algebra of logic the sign $\prec$, is called the copula of inclusion, is used to denote mean that the proposition written after it is consequent upon the proposition written before it as antecedent, in the sense that their logical values are connected as shown in the diagram.

The relation of antecedent to consequent, in this sense, may be defined in terms of the same ideas of Article 33 as follows:

First, $x \prec y$ implies that x virtually assigns to the proposition y the same logical value, that it assigns to itself;

Secondly, $x \prec y$ implies nothing which does not necessarily follow from the first clause of this definition.

In connection with this definition three more postulates of the mathematics of logic present themselves.

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The fourth postulate is that if the logical value which a proposition assigns to itself is correct, then, ipso facto, the logical value which it virtually assigns to any other proposition is correct. This may be stated thus: if A virtually assigns its own logical value to B, then it virtually assigns that value to every proposition to which B virtually assigns its own value. Or,

(3) If $x < y$ and $y < z$, then $x < z$.

The name of this principle is ~~This is called~~ the Nota notae (~~that is, No~~ from the first words of the formula "Nota notae is nota rei ipsius".) It is also called the principle of the transitiveness of the copula. It is ~~essentially~~ substantially the same as the Dictum de omni; and it has borne several other designations in logic.

The fifth postulate is that if all the logical values which a proposition virtually assigns to other propositions are in fact correct, that makes ~~then~~ the value which it assigns to itself correct. ~~Or if~~ This may be stated thus: if ~~the proposition A virtually assigns its own value to every proposition to which the proposition B virtually assigns its own value, then A virtually assigns its own value of to B.~~ the logical value which A assigns to itself belongs in fact (so far as it is correct) equally to itself belongs (so far as it is correct) to B. Or

(4) If for every proposition, z, such that essentially $y < z$, it is a fact that $x < z$, then $x < y$.

In other words, of whatever ~~If~~ everything that a predicate means is true, that predicate is true. This is the Principle of reasoning from definition to definitum. It also governs the second Aristotelian figure of syllogism. In the domain of probable logic, it becomes very important as the principle of hypothetic syllogism, or inference from consequent to antecedent.

The sixth postulate is that ~~if none of the propositions which virtually assign their values to a given proposition assign correct values, then that proposition does not have a correct value to itself.~~ that if all the values which ~~propositions~~ are virtually assigned ~~refused~~ to a proposition propositions ~~are rightly refused then~~ assigning those same values to themselves are correctly assigned, then the value

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which is assigned to the proposition by itself is correctly assigned. This may be more clearly stated thus: If the logical value assigned by A to itself belongs [in fact] equally, if that is correct, in fact equally to every proposition which virtually assigns its own value to B, then if the logical value assigned by A to itself is correct this same value belongs to B. Or

(5) If for every proposition, x, such that essentially $x \prec y$, it is a fact that $x \prec x$, then $y \prec x$.

This is the principle of the dilemma, or reasoning from enumer division to divisum. It also governs in a negative way the Aristotle's third figure of syllogism. In the domain of probable reasoning, it becomes very important as the principle inductive syllogism, or inference from a sample to the whole class sampled.

It might be doubted whether (4) and (5) are independent postulates; but it is easy to show by an example that one might be true without the other. Namely, if all the rivers (streams from one junction of streams to another) tributary to one river-stretch, P, are also tributary to another river stretch Q, it does not follow that P is tributary to Q; yet it does not follow that because if all the river-stretches which receive the waters of S receive the waters of T therefore S must receive the waters of T. For Fig. 5 shows that if P were placed below Q there must be some branch tributary to P that is not tributary to Q (besides Q itself); while Fig. 6 shows how every river-stretch which receives the waters of S may receive the waters of T without S receiving the waters of T. So it is one thing to say if every antecedent of M is an antecedent

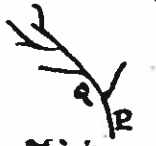

Fig. 5

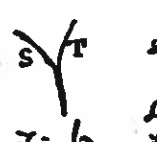

Fig. 6

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Let $\underset{=}{x}$ be any algebraical expression of a proposition. Then, a heavy dash, called an obelus, drawn over $\underset{=}{x}$, has the effect of negativing it, that is of converting it into a proposition which essentially, that is, in its meaning, asserts what is necessary to make its logical value different from that of $\underset{=}{x}$ and asserts nothing more. Taking $=$ as the sign of logical equality, that is, or essential sameness of value, and $\neq$ as the sign of essential otherness, or secondness, of logical value, we have.

(1) $$\bar{\underset{=}{x}} \neq \underset{=}{x}$$

(2) $\bar{\underset{=}{x}}$ asserts nothing which is not ~~implied in (1)~~ logically necessitated by (1).

The third postulate of the mathematics of Logic is, that there is no third logical value, so that every proposition has the value either of $\underset{=}{x}$ or of $\bar{\underset{=}{x}}$, – is either true or false. This is the principle of excluded middle, often call (Principium exclusi tertii.) As Boole remarked, it is as much as to say that every proposition satisfies one fixed quadratic equation. For if two numbers $\underset{=}{t}$ and $\underset{=}{f}$, represented the true and the false, we might in ordinary algebra write $(\underset{=}{t}-\underset{=}{x})\cdot(\underset{=}{x}-\underset{=}{f}) = 0$. If, as Boole did, we borrow from the Calculus of probabilities the expression of truth by 1 and of falsity by 0, we get Boole's expression for the fundamental quadratic $\underset{=}{x}(1-\underset{=}{x})=0$. But in the general algebra of Logic here described, we can express this postulate thus:

postulate thus

(3) $$\bar{\bar{\underset{=}{x}}} \text{ is } \underset{=}{x}.$$

That is, if a proposition were to have a value other than a value other than a value of $\underset{=}{x}$ (and though this postulate does not assert that any such values are possible) then it must have the value of $\underset{=}{x}$; and not asserting anything that it does not need in order to conform to that condition, it must assert $\underset{=}{x}$.

need to assert

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