Wednesday Lectures open to all interested.

December 8. Wednesday.	Rev. Theophilus Parsons Sawin, D. D., of Troy, New York. Ethics of Altruism.
December 12.	Mr. William Potts, of New York, Vice-President of the National Civil Service Reform Association. Social Conditions in Town and Country.
December 15. Wednesday.	Mr. Frank B. Sanborn, of Concord, Mass. Walks With Emerson and Thoreau.
December 19.	Edward Atkinson, LL. D., Ph.D., of Boston. The Ethics of Trade or Commerce.
December 22. Wednesday.	Mr. Leo Wiener, Instructor in Russian, Harvard University. The Popular Poetry of the Russian Jews. With Recitations by Mr. Morris Rosenfeld from his own Dialect Poems.
December 26.	Colonel Thomas Wentworth Higginson, of Cambridge. England Twenty Years After.
December 29.	*(HOLIDAY INTERMISSION.)*

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January 2.	Miss Mary White Ovington, Head Worker of the Pratt Institute Neighborship Settlement, Brooklyn, New York. Neighborhood Ethics.
January 5. Wednesday.	Mr. Virchand R. Gandhi, B.A., M.R.A.S., of Bombay, India. The Education of Women in India.
January 9.	Hon. Carroll D. Wright, United States Commissioner of Labor. A Study of the Divorce Question.
January 12. Wednesday.	Mrs. Ellen M. Mitchell, President of "The Round Table," Syracuse, New York. The Social Philosophy of Dante.
January 16.	Mr. Edwin D. Mead, President of the Twentieth Century Club, Boston. Ethics of Citizenship.
January 19. Wednesday.	Mrs. N. D. Macdaniel, of New York. Brook Farm.
January 23.	Mrs. B. J. Harnett, of New York. Social and Religious Life of Hindu Women.
January 26. Wednesday.	Hon. Albion A. Perry, Mayor of Somerville, Mass. Theodore Parker.
January 30.	Rev. Thomas R. Slicer, of New York. Ethical Problems in Education.
February 6.	Prof. Josiah Royce, Ph.D., of Harvard University. First Lecture of Course on Aspects of Social Psychology.
February 13.	Mrs. Frederick Nathan, President of the New York Consumers' League. Ethics of Domestic Service.
February 20.	Prof. Josiah Royce, Ph.D., of Harvard University. Second Lecture on Aspects of Social Psychology.
February 27.	Prof. Franklin H. Giddings, Ph.D., of Columbia University. Poverty as a Social Problem.
March 6.	Prof. Josiah Royce, Ph.D., of Harvard University. Third Lecture on Aspects of Social Psychology.
March 13.	Mr. Henry Hoyt Moore, President of the Brooklyn Ethical Association. Ethical Aspects of the Saloon Problem.
March 20.	Mr. Joseph G. Thorp, President of the Cambridge Social Union. The Norwegian System of Regulating the Sale of Intoxicants.
March 27.	Mr. John S. Clark, Treasurer of the American Statistical Association. Ethics of Business Life.
April 3.	Prof. Josiah Royce, Ph.D., of Harvard University. Fourth Lecture on Aspects of Social Psychology.
April 10.	Mr. Edward King, of New York. The Rights and Duties of Labor.
April 17.	Prof. Josiah Royce, Ph.D., of Harvard University. Fifth Lecture on Aspects of Social Psychology.
April 24.	Mr. Walter S. Logan, of New York. The Rights and Duties of Capital.
May 1.	Prof. Josiah Royce, Ph.D., of Harvard University. Concluding Lecture on Aspects of Social Psychology.
May 8.	Dr. Lewis G. Janes, M.A., Director of the Conferences. Industrial Conciliation.

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The success of the "scientific temperance" fanatics in capturing the schools of the country in the interest of their peculiar propaganda has roused another set of cranks. The believers in teaching "patriotism" are now to the fore with a proposition to incorporate "patriotism" in the elementary school curriculum. This scheme is too absurd for discussion, but, if pushed, it may be trusted to pass one legislature after another by overwhelming majorities. We may next be on the lookout for the anti-vivisectionists, the woman suffragists, the reformed-dress brigade, the anti-gravitation societies, and all the other admirable organizations that make life really worth living. Let them all "enrich" the curriculum with their pet exercises: then let us "shorten" it to the vanishing point and begin all over again.

In the recent organization, for 1898, of the Philadelphia Board of Education, Mr. Simon Gratz, whose long and devoted services to public education are known and appreciated far beyond the bounds of his own city, declined a re-election to the presidency, and Mr. Samuel B. Huey was chosen his successor.

Mr Huey is a graduate of Princeton and a lawyer of wide repute. As chairman of the Central High-School Committee he has already rendered valuable service to the city. Under his guidance the high school has doubled in numbers in the last four years, the attendance now being over thirteen hundred. The number of students who complete the course moreover, has steadily increased, while with the increase in the faculty has come an increase in the number of professors with a collegiate training, and, as a consequence, a decided upward change in the level and character of the instruction. Largely through Mr. Huey's efforts, the study of education established in connection with the High School was recently rearranged and provision made for a two-years' course of instruction. Besides a long and valuable experience, Mr. Huey brings to the presidency an intelligent and sympathetic interest in educational affairs that is certain to be productive of the best results.

EDUCATIONAL REVIEW

MARCH, 1898

I

THE LOGIC OF MATHEMATICS IN RELATION TO EDUCATION

§ 1 *Of mathematics in general*

In order to understand what number is, it is necessary first to acquaint ourselves with the nature of the business of mathematics in which number is employed.

I wish I knew with certainty the precise origin of the definition of mathematics as the science of quantity. It certainly cannot be Greek, because the Greeks were advanced in projective geometry, whose problems are such as these: whether or not four points obtained in a given way lie in one plane; whether or not four planes have a point in common; whether or not two rays (or unlimited straight lines) intersect, and the like—problems which have nothing to do with quantity, as such. Aristotle names, as the subjects of mathematical study, quantity and continuity. But though he never gives a formal definition of mathematics, he makes quite clear, in more than a dozen places, his view that mathematics ought not to be defined by the things which it studies but by its peculiar mode and degree of abstractness. Precisely what he conceives this to be it would require me to go too far into the technicalities of his philosophy to explain; and I do not suppose anybody would to-day regard the details of his opinion as important for my purpose. Geometry, arithmetic, astronomy, and music were, in the Roman schools of the

fifth century [1] and earlier, recognized as the four branches of mathematics. And we find Boethius (A.D. 500) defining them as the arts which relate, not to quantity, but to *quantities*, or *quanta*. What this would seem to imply is, that mathematics is the foundation of the minutely exact sciences; but really it is not worth our while, for the present purpose, to ascertain what the schoolmasters of that degenerate age conceived mathematics to be.

In modern times projective geometry was, until the middle of this century, almost forgotten, the extraordinary book of Desargues [2] having been completely lost until, in 1845, Chasles came across a MS. copy of it; and, especially before imaginaries became very prominent, the definition of mathematics as the science of quantity suited well enough such mathematics as existed in the seventeenth and eighteenth centuries.

Kant, in the *Critique of pure reason* (Methodology, chapter I, section 1), distinctly rejects the definition of mathematics as the science of quantity. What really distinguishes mathematics, according to him, is not the subject of which it treats, but its method, which consists in studying constructions, or diagrams. That such is its method is unquestionably correct; for, even in algebra, the great purpose which the symbolism subserves is to bring a skeleton representation of the relations concerned in the problem before the mind's eye in a schematic shape, which can be studied much as a geometrical figure is studied.

But Rowan Hamilton and De Morgan, having a superficial acquaintance with Kant, were just enough influenced by the *Critique* to be led, when they found reason for rejecting the definition as the science of quantity, to conclude that mathematics was the science of pure time and pure space. Notwithstanding the profound deference which every mathematician must pay to Hamilton's opinions and my own admiration for De Morgan, I must say that it is rare to meet with a

[1] Davidson, *Aristotle and the ancient educational ideals.* Appendix: The Seven Liberal Arts. (New York: Charles Scribner's Sons.)

[2] Brouillon, *Proiet d'une atteinte aux événemens des rencontres du cône avec son plan*, 1639.

careful definition of a science so extremely objectionable as this. If Hamilton and De Morgan had attentively read what Kant himself has to say about number, in the first chapter of the *Analytic of principles* and elsewhere, they would have seen that it has no more to do with time and space than has every conception. Hamilton's intention probably was, by means of this definition, to throw a slur upon the introduction of imaginaries into geometry, as a false science; but what De Morgan, who was a student of multiple algebra, and whose own formal logic is plainly mathematical, could have had in view, it is hard to comprehend, unless he wished to oppose Boole's theory of logic. Not only do mathematicians study hypotheses which, both in truth and according to the Kantian epistemology, no otherwise relate to time and space than do all hypotheses whatsoever, but we now all clearly see, since the non-Euclidean geometry has become familiar to us, that there *is* a real science of space and a real science of time, and that these sciences are positive and experiential—branches of physics, and so not mathematical except in the sense in which thermotics and electricity are mathematical; that is, as calling in the aid of mathematics. But the gravest objection of all to the definition is that it altogether ignores the veritable characteristics of this science, as they were pointed out by Aristotle and by Kant.

Of late decades philosophical mathematicians have come to a pretty just understanding of the nature of their own pursuit. I do not know that anybody struck the true note before Benjamin Pierce, who, in 1870,[3] declared mathematics to be "the science which draws necessary conclusions," adding that it must be defined "subjectively" and not "objectively." A view substantially in accord with his, though needlessly complicated, is given in the article Mathematics, in the ninth edition of the *Encyclopædia Britannica*. The author, Professor George Chrystal, holds that the essence of mathematics lies in its making pure hypotheses, and in the character of the hypotheses which it makes. What the mathematicians mean

[3] In his *Linear associative algebra.*

by a "hypothesis" is a proposition imagined to be strictly true of an ideal state of things. In this sense, it is only about hypotheses that necessary reasoning has any application; for, in regard to the real world, we have no right to presume that any given intelligible proposition is true in absolute strictness. On the other hand, probable reasoning deals with the ordinary course of experience; now, nothing like a *course of experience* exists for ideal hypotheses. Hence to say that mathematics busies itself in drawing necessary conclusions, and to say that it busies itself with hypotheses, are two statements which the logician perceives come to the same thing.

A simple way of arriving at a true conception of the mathematician's business is to consider what service it is which he is called in to render in the course of any scientific or other inquiry. Mathematics has always been more or less a trade. An engineer, or a business company (say, an insurance company), or a buyer (say, of land), or a physicist, finds it suits his purpose to ascertain what the necessary consequences of possible facts would be; but the facts are so complicated that he cannot deal with them in his usual way. He calls upon a mathematician and states the question. Now the mathematician does not conceive it to be any part of his duty to verify the facts stated. He accepts them absolutely without question. He does not in the least care whether they are correct or not. He finds, however, in almost every case that the statement has one inconvenience, and in many cases that it has a second. The first inconvenience is that, though the statement may not at first sound very complicated, yet, when it is accurately analyzed, it is found to imply so intricate a condition of things that it far surpasses the power of the mathematician to say with exactitude what its consequence would be. At the same time, it frequently happens that the facts, as stated, are insufficient to answer the question that is put. Accordingly, the first business of the mathematician, often a most difficult task, is to frame another simpler but quite fictitious problem (supplemented, perhaps, by some supposition), which shall be within his powers, while at the same time it is sufficiently like the problem set before him to an-

swer, well or ill, as a substitute for it.[4] This substituted problem differs also from that which was first set before the mathematician in another respect: namely, that it is highly abstract. All features that have no bearing upon the relations of the premises to the conclusion are effaced and obliterated. The skeletonization or diagrammatization of the problem serves more purposes than one; but its principal purpose is to strip the significant relations of all disguise. Only one kind of concrete clothing is permitted—namely, such as, whether from habit or from the constitution of the mind, has become so familiar that it decidedly aids in tracing the consequences of the hypothesis. Thus, the mathematician does two very different things: namely, he first frames a pure hypothesis stripped of all features which do not concern the drawing of consequences from it, and this he does without inquiring or caring whether it agrees with the actual facts or not; and, secondly, he proceeds to draw necessary consequences from that hypothesis.

Kant is entirely right in saying that, in drawing those consequences, the mathematician uses what, in geometry, is called a "construction," or in general a diagram, or visual array of characters or lines. Such a construction is formed according to a precept furnished by the hypothesis. Being formed, the construction is submitted to the scrutiny of observation, and new relations are discovered among its parts, not stated in the precept by which it was formed, and are found, by a little mental experimentation, to be such that they will always be present in such a construction. Thus, the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the conditions of whose being we know all about.

But Kant, owing to the slight development which formal logic had received in his time, and especially owing to his total ignorance of the logic of relatives, which throws a brilliant light upon the whole of logic, fell into error in supposing

[4] See this well put in Thomson and Tait's *Natural Philosophy*, § 447.

that mathematical and philosophical necessary reasoning are distinguished by the circumstance that the former uses constructions. This is not true. All necessary reasoning whatsoever proceeds by constructions; and the only difference between mathematical and philosophical necessary deductions is that the latter are so excessively simple that the construction attracts no attention and is overlooked. The construction exists in the simplest syllogism in Barbara. Why do the logicians like to state a syllogism by writing the major premise on one line and the minor below it, with letters substituted for the subject and predicates? It is merely because the reasoner has to notice that relation between the parts of those premises which such a diagram brings into prominence. If the reasoner makes use of syllogistic in drawing his conclusion, he has such a diagram or construction in his mind's eye, and observes the result of eliminating the middle term. If, however, he trusts to his unaided reason, he still uses some kind of a diagram which is familiar to him personally. The true difference between the necessary logic of philosophy and mathematics is merely one of degree. It is that, in mathematics, the reasoning is frightfully intricate, while the elementary conceptions are of the last degree of familiarity; in contrast to philosophy, where the reasonings are as simple as they can be, while the elementary conceptions are abstruse and hard to get clearly apprehended. But there is another much deeper line of demarcation between the two sciences. It is that mathematics studies nothing but pure hypotheses, and is the only science which never inquires what the actual facts are; while philosophy, although it uses no microscopes or other apparatus of special observation, is really an experimental science, resting on that experience which is common to us all; so that its principal reasonings are not mathematically necessary at all, but are only necessary in the sense that all the world knows beyond all doubt those truths of experience upon which philosophy is founded. This is why the mathematician holds the reasoning of the metaphysician in supreme contempt, while he himself, when he ventures into philosophy, is apt to reason fantastically and not solidly, be-

cause he does not recognize that he is upon ground where elaborate deduction is of no more avail than it is in chemistry or biology.

I have thus set forth what I believe to be the prevalent opinion of philosophical mathematicians concerning the nature of their science. It will be found to be significant for the question of number. But were I to drop this branch of the subject without saying one word more, my criticism of the old definition, "mathematics is the science of quantity," would not be quite just. It must be admitted that quantity is useful in almost every branch of mathematics. Jevons wrote a book entitled *Pure logic, the science of quality*, which expounded, with a certain modification, the logical algebra of Boole. But it is a mistake to regard that algebra as one in which there is no system of quantity. As Boole rightly holds, there is a quadratic equation which is fundamental in it. The meaning of that equation may be expressed as follows: Every proposition has one or other of two *values*, being either *true* (which gives it one value) or *false* (which gives it the other). So stated, we see that the algebra of Boole is nothing but the algebra of that system of quantities which has but two values—the simplest conceivable system of quantity. The widow of the great Boole has lately written a little book[5] in which she points out that, in solving a mathematical problem, we usually introduce some part or element into the construction which, when it has served our purpose, is removed. Of that nature is a scale of quantity, together with the apparatus by which it is transported unchanged from one part of the diagram to another, for the purpose of comparing those two parts. Something of this general description seems to be indispensable in mathematics. Take, for example, the Theorem of Pappus concerning ten rays in a plane. The demonstration of it which is now usual, that of von Staudt, introduces a third dimension; and the utility of that arises from the fact that a ray, or unlimited straight line, being the intersection of two planes, these planes show us exactly where the ray runs, while, as long as we confine our-

[5] *The Mathematical psychology of Boole and Gratry.*

selves to the consideration of a single plane, we have no easy method of describing precisely what the course of the ray is. Now this is not precisely a system of quantity; but it is closely analogous to such a system, and that it serves precisely the same purpose will appear when we remember that that same theorem can easily (though not *so* easily) be demonstrated by means of the barycentric calculus. Although, then, it is not true that all mathematics is a science of quantity, yet it is true that all mathematics makes use of a scaffolding altogether *analogous* to a system of quantity; and quantity itself has more or less utility in every branch of mathematics which has as yet developed into any large theory.

I have only to add that the hypotheses of mathematics may be divided into those *general hypotheses* which are adhered to throughout a whole branch of mathematics, and the *particular hypotheses* which are peculiar to different special problems.

CHARLES S. PEIRCE

MILFORD, PA.

(*To be continued*)

II

THE PROBLEM OF OCCUPATION FOR COLLEGE WOMEN[1]

When the college woman sets out to earn a living, the chances are that she will think first, as a matter of course, of teaching, and, in nine cases out of ten, that she will finally decide to take up that pursuit.

The records of Bryn Mawr College show that of the 57 per cent. of its graduates who have at some time engaged in paid occupations, 86 per cent., or 49 per cent. of the whole number graduated, have been or are now teachers. These records were chosen for reference in the study of the present problem notwithstanding the comparatively brief period they cover, because they have been kept with unusual care, and afford an unusual amount of detail; and that they present a case fairly typical, not merely of collegiate but of general conditions, may be inferred from the testimony of general statistics. The census of 1890 gives 311,682 women following professional pursuits,—a class roughly comparable to the total class of wage-earners shown in college records,—and of these, 245,230, or about 80 per cent., are teachers, a figure very near to the 85 per cent. which represents the proportion of teachers to all occupied among the graduates of the College just mentioned.

The records of a man's college would show a different state of things. Not 57 per cent., but practically all graduates engage in some remunerative occupation, and, of those occupied, a far lower proportion than among women-graduates are engaged in teaching. General statistics, also, show a reversal between the proportions of men and women teachers to all employed of each sex respectively. From the census of 1890 it is found that, of the 632,641 men in professional pursuits at that time (again comparing this class to that of

[1] A paper presented to the Association of Collegiate Alumnæ.