The English language fails to discriminate in precise terms between law in the abstract, and a particular law, ordained by the political sovereign, which are represented in most other languages by distinct terms: e.g. *ius* and *lex*. 'Right' and 'law' present these notions inadequately, because although formerly the former was used by the Anglo-Saxons like the German *Recht*, as in folc-riht, with us it now includes the whole domain of morals.

Literature: MAINE, Ancient Law (law is developed from the unwritten to the written; from the formal to the equitable; from the personal to the territorial); BRUNNER, Deutsche Rechtsgesch., i. 3, §§ 33, 38; SMITH, Right and Law, chap. ii. 2; BENTHAM, Mor. and Legisl., chaps. xvii, xxiii; RATTO, Sociologia e Filosofia del Diritto (Rome, 1894; *subjective* and *objective* law well contrasted in chap. vi); FILOMUSI GUELFI, Del Concetto del Diritto Naturale e del Diritto Positivo (Naples, 1874). (Cf. ADJECTIVE LAW, ADMINISTRATIVE LAW, CANON LAW, CASE LAW, CIVIL LAW, COMMON LAW, CONFLICT OF LAWS, CONSTITUTIONAL LAW, LEGAL, PRIVATE LAW, ROMAN LAW. (S.E.B.)

Law (moral): Ger. *Sittengesetz*; Fr. *loi morale*; Ital. *legge morale*. A rule of conduct resulting from the application of the moral ideal to life, or laid down by the moral authority, however this may be conceived.

The influence upon ethics, both of theology and of positive law, has led to the statement of morality as in essence a system of moral rules. See DUTY. (W.R.S.)

Law of Parcimony: see PARCIMONY.

Laws of Thought: Ger. *Denkgesetze*; Fr. *lois de la pensée*; Ital. *leggi del pensiero*. The three formulas of identity, contradiction, and excluded middle have been widely so known, though the doctrine that they are three co-ordinate and sufficient laws of all thought or of all reasoning has been held by a comparatively small party which hardly survives; and it is not too much to say that the doctrine is untenable. But the designation is so familiar and convenient that those formulas may very well be referred to as 'the so-called three laws of thought.' The formulas have usually been stated by those who upheld the doctrine as follows:—

I. *The Principle of Identity*: *A* is *A*.

II. *The Principle of Contradiction*: *A* is not not-*A*.

III. *The Principle of Excluded Middle* or *Excluded Third*: everything is either *A* or not-*A*.

It is noticeable that two of these propositions are categorical and the third disjunctive, a circumstance demanding explanation for those who hold the distinction of categorical, conditional, and disjunctive propositions to be fundamental.

The meaning of the formula of identity presents only one small difficulty. If the copula 'is' be taken in the sense of 'is, if it exists,' then the meaning of the formula is that no universal affirmative proposition having the same term as subject and predicate is false. If, however, the copula be understood to imply existence, the meaning is that no universal affirmative proposition is false in which the same term is subject and predicate, provided that term denotes any existing object. Or, the meaning may be that the same thing is true when the subject and predicate are the same proper name of an individual. In any case, it may properly be required that the precise meaning attached to the copula should be explained; and this explanation must in substance involve one or other of the above three statements; so that in any case the principle of identity is merely a part of the definition of the copula.

In like manner, if the word 'not' is to be used in logical forms, its force should be explained with the utmost precision. Such an explanation will consist in showing that the relation it expresses belongs at once to certain classes of relations, probably not more than two, in view of the simplicity of the idea. Each of these two statements may be embodied in a formula similar, in a general way, to the formulas of contradiction and excluded middle. It has, therefore, seemed to Mill and to the 'exact' logicians that these two formulas ought together to constitute a definition of the force of 'not.'

Other writers have regarded all three laws as 'practical maxims.' But practically nobody needs a maxim to remind him that a contradiction, for example, is an absurdity. It might be a useful injunction to tell him to beware of latent contradictions; but as soon as he clearly sees that a proposition is self-contradictory, he will have abandoned it before any maxim can be adduced. Seeing, then, that such formulas are required to define the relation expressed by not, but are not required as maxims, it is in the former aspect that their true meanings are to be sought.

If it is admitted that they constitute a definition, they must conform to the rules of definition. Considered as part of a definition, one of the commonest statements of the principle of contradiction, 'A non est non-A,' offends against the rule that the *definitum* must not be introduced into the definition. This is easily avoided by using the form 'A est non non-A,' 'A is not not-A,' or every term may be subsumed under the double negation of itself. If this form is adopted for the principle of contradiction, the principle of excluded middle ought to be 'What is not not-A is A.' If, however, we prefer to state the principle of excluded middle as 'Everything is either A or not-A,' then we should state the principle of contradiction as 'What is, at once, A and not-A is nothing.' There is no vicious circle here, since the term 'nothing,' or 'non ens,' may be formally defined without employing the particle 'not' or any equivalent. Thus, we may express the principle of contradiction as follows:

> Whatever there may be which is both A and not-A is X, no matter what term X may be.

In either formula, A may be understood to be restricted to being an individual, or it may be allowed to be any term, individual or general. In the former case, in order to avoid conflict with the fundamental law that no true definition asserts existence, a special clause should be added, such as 'if not-A there be.' In the latter case, it should be stated that by 'not-A' is not meant 'not some A,' but 'not any A,' or 'other than whatever A there may be.'

Bearing these points in mind, the formula 'A is not-not-A,' or 'A is other than whatever is other than whatever is A,' is seen to be a way of saying that the relation expressed by 'not' is one of those which is its own converse, and is analogous to the following:

> Every rose is similar to whatever is similar to whatever is a rose;

which again is similar to the following:

> Every man is loved by whatever loves whatever is a man.

But if we turn to the corresponding formula of excluded middle, 'Not-not-A is A,' or 'Whatever is not anything that is not any A is A,' we find that its meaning cannot be so simply expressed. Supposing that the relation r is such that it is true that

> Whatever is r to whatever is r to whatever is A is A,

it can readily be proved that, whether the multitude of individuals in the universe be finite or infinite, each individual is either *non-r* to itself and to nothing else, or is one of a pair of individuals that are *non-r* to each other and to nothing else; and conversely, if the universe is so constituted, the above formula necessarily holds. But it is evident that if the universe is so constituted, the relation r is converse to itself; so that the formula corresponding to that of contradiction also holds. But this constitution of the universe does not determine r to be the relation expressed by 'not.' Hence, the pair of formulas,

> A is not not-A,
> Not not-A is A,

are inadequate to defining 'not,' and the former of them is mere surplusage. In fact, in a universe of monogamously married people, taking any class, the A's,

> Every A is a non-spouse to whatever is non-spouse to every A,

and

> Whatever is non-spouse to whatever is a non-spouse to every A is an A.

No such objection exists to the other pair of formulas:

> Whatever is both A and not-A is nothing,
> Everything is either A or not-A.

Their meaning is perfectly clear. Dividing all ordered pairs of individuals into those of the form $A : B$ and those of the form $A : A$,

The principle of contradiction excludes from the relation 'not' all of the form $A : A$,

The principle of excluded middle makes the relation of 'not' to include all pairs of the form $A : B$.

From this point of view, we see at once that there are three other similar pairs of formulas defining the relations of identity, coexistence, and incompossibility, as follows:

> Whatever is A is identical with A; i.e. Identity includes all pairs $A : A$.
> Whatever is identical with A is A; i.e. Identity excludes all pairs $A : B$.
> Whatever is A is coexistent with A; i.e. Coexistence includes all pairs $A : A$.
> Everything is either A or coexistent with A; i.o. Coexistence includes all pairs $A : B$.
> Whatever is both A and incompossible with A is nothing; i.e. Incompossibility excludes all pairs $A : A$.
> Whatever there may be incompossible with A is A; i.e. Incompossibility excludes all pairs $A : B$.

Much has been written concerning the

relations of the three principles to forms of syllogism. They have even been called Die Principien des Schliessens, and have often been so regarded. Some points in reference to the meanings they have borne in such discussions require mention. Many writers have failed to distinguish sufficiently between reasoning and the logical forms of inference. The distinction may be brought out by comparing the moods Camestres and Cesare (see MOOD, in logic). Formally, these are essentially different. The form of Camestres is as follows:

> Every P is an M,
> Every S is other than every M;
> ∴ Every S is other than every P.

This form does not depend upon either clause of the definition of 'not' or 'other than.' For if any other relative term, such as 'lover of,' be substituted for 'other than,' the inference will be equally valid. The form of Cesare is as follows:

> Every P is other than every M,
> Every S is an M;
> ∴ Every S is other than every P.

This depends upon the equiparance of 'other than.' For if we substitute an ordinary relative, such as *loves*, for 'other than' in the premise, the conclusion will be

> Every S is loved by every P.

(See De Morgan's fourth memoir on the syllogism, *Cambridge Philos. Trans.*, x. (1860) 354.) The two forms are thus widely distinct in logic; and yet when a man actually performs an inference, it would be impossible to determine that he 'reasons in' one of these moods rather than in the other. Either statement is incorrect. He does not, in strict accuracy, reason in any form of syllogism. For his reasoning moves in first intentions, while the forms of logic are constructions of second intentions. They are diagrammatic representations of the intellectual relation between the facts from which he reasons and the fact which he infers, this diagram necessarily making use of a particular system of symbols—a perfectly regular and very limited kind of language. It may be a part of a logician's duty to show how ordinary ways of speaking and of thinking are to be translated into that symbolism of formal logic; but it is no part of syllogistic itself. Logical principles of inference are merely rules for the illative transformation of the symbols of the particular system employed. If the system is essentially changed, they will be quite different. As the Boolians represent Cesare and Camestres, they appear, after literally translating the algebraic signs of those logicians into words, as follows:

> A that is B is nothing,
> C that is not B is nothing;
> ∴ A that is C is nothing.

The two moods are here absolutely indistinguishable.

From the time of Scotus down to Kant more and more was made of a principle agreeing in enunciation, often exactly, in other places approximately, with our principle of contradiction, and in the later of those ages usually called by that name, although earlier more often *principium primum, primum cognitum, principium identitatis, dignitas dignitatum*, &c. It would best be called the *Principle of Consistency*. Attention was called to it in the fourth book of Aristotle's *Metaphysics*. The meaning of this, which was altogether different, at least in post-scholastic times, from our principle of contradiction, is stated in the so-called *Monadologie* of Leibnitz (§ 31) to be that principle by virtue of which we judge that to be false which involves a contradiction, and the denial of the contradiction to be true. The latter clause involves an appeal to the principle of excluded middle as much as the former clause does to the formal principle of contradiction. And so the 'principle of contradiction' was formerly frequently stated. But, in fact, neither is appealed to; for Leibnitz does not say that the contradiction is to be made explicit, but only that it is to be recognized as an inconsistency. Interpreted too strictly, the passage would seem to mean that all demonstrative reasoning is by the *reductio ad absurdum*; but this cannot be intended. All that is meant is that we draw that conclusion the denial of which would involve an absurdity—in short, that which consistency requires. This is a description, however imperfect, of the procedure of demonstrative REASONING (q. v.), and deos not relate to logical forms. It deals with first, not second, intentions. (C.S.P.)

It is unfortunate that 'contradictory' and 'principle of contradiction' are terms used with incongruent significations. If α and β are statements, they are mutually contradictory, provided that one or the other of them must be true and that both cannot be true; these are the two marks (essential and sufficient) of contradiction, or precise denial, as it might better be called. If a and b are terms, b is the precise negative of a (or the contradictory term to a), provided it takes in *all* of that which is *other than a*—that is, if everything must be one or the other (a or b) and if nothing can be both. These two properties constitute the definition of a pair of contradictories (whether terms or propositions), namely, they are mutually exclusive, and they are together exhaustive; expressed in the language of 'exact logic,' these properties are (writing $\bar{x}$ for the negative of x and $+$ for *or*):

(1) $x\bar{x} < 0$,	(2) $\infty < x + \bar{x}$,
what is at once x and $\bar{x}$ does not exist, or, in the language of propositions, the conjoint occurrence of x and $\bar{x}$ does not take place.	everything is either x or $\bar{x}$, or, in the language of propositions, what can occur is either x or $\bar{x}$, or, reality entails x or $\bar{x}$—there is no *tertium quid*.

Together these properties constitute the requirements of contradiction or of exact negation; it is a very inelegant piece of nomenclature (besides that it leads to actual confusion) to refer to (1) alone as the 'principle of contradiction.' Better names for them are (1) exclusion and (2) exhaustion (in place of excluded middle). In the common phraseology we are obliged to commit the absurdity of saying that two terms or propositions may satisfy the 'principle of contradiction' and still not be contradictory (since they may lack the quality of being exhaustive). The mere fact that (1) has been called the principle of contradiction has given it a pretended superiority over the other which it by no means deserves; they are of equal importance in the conducting of reasoning processes. In fact, for every formal argument which rests upon (1) there is a corresponding argument which rests upon (2): thus in the case of 'the fundamental law of TRANSPOSITION (q.v.), which affirms the identity of these two propositions, (m) the student who is not a citizen is not a voter; (n) every student is either a citizen or not a voter; that (m) follows from (n) depends upon one of these principles, and that (n) follows from (m) depends upon the other. These two names, exhaustion and exclusion, have the great advantage that they permit the formation of adjectives; thus we may say that the test for the contradictoriness of two terms or propositions which are not on their face the negatives one of another is that they should be (1) mutually exclusive and (2) together exhaustive.

It may be noticed that if two terms are exhaustive but not exclusive, their negatives are exclusive but not exhaustive. Thus within the field of number, 'prime' and 'even' are exclusive (no number can be both) but not exhaustive (except in the limiting case of two, some numbers can be neither), while 'not even' and 'not prime' are exhaustive and not exclusive.

In the case of propositions, 'contrary' and 'subcontrary' are badly chosen names for the OPPOSITION (q.v.) of A and E, O and I, respectively, of the traditional logical scheme; they do not carry their meaning on their face, and hence are unnecessarily difficult for the learner to bear in mind. A and E should be said to be mutually exclusive (but not exhaustive), O and I to be conjointly exhaustive (but not exclusive). This relation of qualities is then seen to be a particular case merely of the above-stated general rule.

Again, 'no a is b' and 'all a is b' are exclusive but not exhaustive, while 'some a is b' and 'some a is not b' are exhaustive but not exclusive (provided in both cases that a exists).

Laws of thought is not a good name for these two characteristics; they should rather be called the laws (if laws at all) of negation. Properly speaking, the laws of thought are all the rules of logic; of these laws there is one which is of far more fundamental importance than those usually referred to under the name, namely, the law that if a is b and b is c, it can be concluded that a is c. This is the great law of thought, and everything else is of minor importance in comparison with it. It is singular that it is not usually enumerated under the name. Another law of thought of equal consequence with those usually so called is, according to Sigwart, the law that the double negative is equivalent to an affirmative, $\bar{\bar{x}} = x$, or

(3) $x < \bar{\bar{x}}$,	(4) $\bar{\bar{x}} < x$.

But these are not fundamental, for from the principles of

Exclusion, (1) $x\bar{x} < 0$,	Exhaustion, (2) $\infty < x + \bar{x}$,

it follows

by (2) that $x < \bar{\bar{x}}$,	by (1) that $\bar{\bar{x}} < x$.

(C.L.F.)

Literature: for the history of these principles see UEBERWEG, Syst. d. Logik, §§ 75–80; PRANTL, Gesch. d. Logik (see 'principium' in the indices to the four volumes). There are additional notes in an appendix to HAMILTON, Lects. on Logic.

(C.S.P.)

END OF VOL. I.